Phase Transition in SU(5) Lattice Gauge Theory

Michael Creutz
Brookhaven National Laboratory, Upton, New York 11973
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I present Monte Carlo evidence for a first-order phase transition in pure SU(5) lattice gauge theory with Wilson's action in four space-time dimensions. Although less clear, a similar transition is strongly suggested already with SU(4).

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Conventional lore on four-dimensional lattice gauge theory is that for the non-Abelian groups SU(N) there should be no phase transition separating the strong-coupling or high-temperature regime from the weak-coupling or low-temperature domain. The latter should give an asymptotically free and confining field theory in a continuum limit. In contrast the U(1) gauge group has a separate weak-coupling phase containing massless photons as spin waves. Monte Carlo studies have verified this canonical behavior for U(1), SU(2), and SU(3) theories.[1] In this paper, however, I give numerical evidence that a new transition appears for SU(N) theory for large N. This behavior is clear at $N=5$ and probably begins at $N=4$. After presenting the evidence I speculate on the nature of this transition, which is unlikely to represent a loss of confinement.

The model is Wilson's lattice gauge theory[2] based on the gauge group SU(N). On each line $\{i,j\}$ joining nearest neighbors i and j of a four-dimensional hypercubic lattice is an $N\times N$ unitary unimodular matrix U_{ij}. The interaction of these matrices is described by the action

$$S=(2N/g_0^2)\sum_{\square}(1-N^{-1}\operatorname{Re}\operatorname{Tr}U_{\square}). \qquad (1)$$

Here the sum is over all "plaquettes" or elementary squares of the lattice and $U_{\square}$ is an ordered group product of the U_{ij} about the given plaquette. The path integral or partition function under investigation is

$$Z=\int\prod_{\{i,j\}}(dU_{ij})e^{-S(U)}, \qquad (2)$$

where the links are integrated with the group-invariant measure. To monitor the behavior of this statistical system, I use the internal energy or average plaquette

$$P=\langle 1-N^{-1}\operatorname{Re}\operatorname{Tr}U_{\square}\rangle, \qquad (3)$$

where the expectation is with the measure of Eq. (2). A strong-coupling expansion yields

$$P(g_0^2)=1-(g_0^2N)(1-\tfrac{1}{2}\delta_{N,2}) - \tfrac{3}{2}(g_0^2N)^{-2}\delta_{N,3}+O((g_0^2N)^{-3}), \qquad (4)$$

whereas in the weak-coupling limit

$$P(g_0^2)=\tfrac{1}{8}(1-1/N^2)g_0^2N+O((g_0^2N)^2). \qquad (5)$$

I have studied this system on periodic finite lattices using the Monte Carlo method suggested by Wilson.[3] A given link is multiplied by an SU(N) matrix randomly chosen from a table of fifty. If the resulting action is lowered, the link is replaced with this product. If the action is raised, the change is accepted with probability given by the exponential of the change in the action. This procedure is applied twenty times to a given link before proceeding to the next. A sequential pass through the entire lattice represents one Monte Carlo iteration in what follows. The SU(N) matrices in the table are selected randomly from the entire group but with a weighting toward the identity. This weighting is coupling dependent and selected to approximately optimize convergence. For each element, its inverse is also in the table. After each Monte Carlo pass through the lattice an entirely new table is generated. After each fifty iterations all link variables are renormalized onto the group in order to eliminate any accumulation of roundoff errors.

Figure 1 shows the average plaquette as a function of number of Monte Carlo iterations for the SU(5) model on a periodic 3^4-site hypercubical lattice. All these runs have $g_0^{-2}=1.67$ and represent four distinct initial conditions. The lowest (open circles) and highest (solid circles) sequences correspond to the variables initially completely ordered or disordered, respectively. The asymptotically lower of the intermediate runs (squares) started with a superheated lattice that had been ordered and Monte Carlo heating performed at $g_0^{-2}=1.4$ until the average plaquette exceeded 0.6. The coupling was then set to the value $g_0^{-2}=1.67$ and the plotted iterations begun.

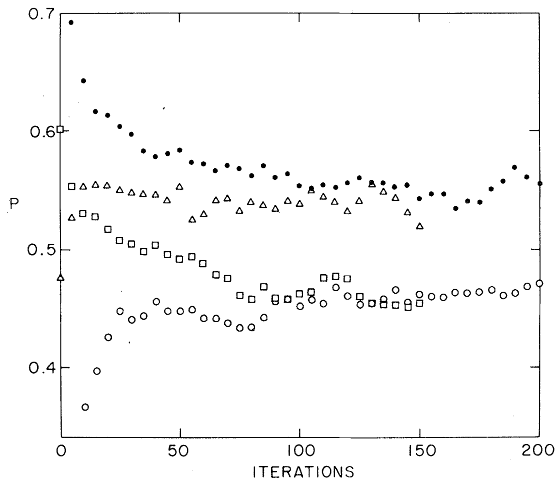

FIG. 1. Four Monte Carlo runs for SU(5) lattice gauge theory at $g_0^{-2}=1.67$. The various initial conditions are discussed in the text.

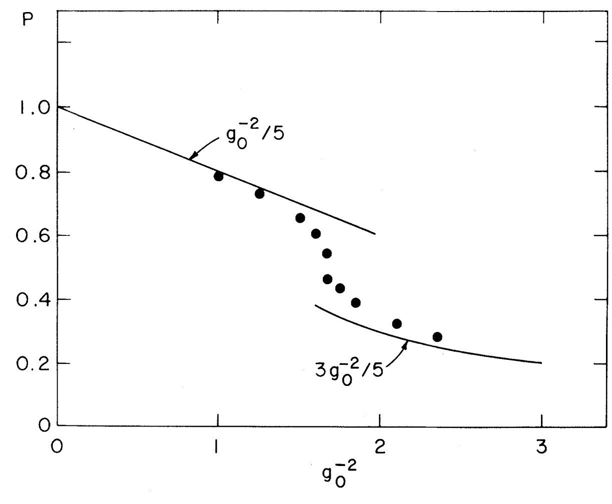

FIG. 2. The SU(5) internal energy as a function of g_0^{-2}.

The remaining run (triangles) was begun similarly except that initially a random lattice was supercooled at $g_0^{-2}=2.5$ until the average plaquette fell below 0.48.

Note that, of these four runs, two asymptotically approach a value of $P=0.54$ whereas the other two tend toward a different value, $P=0.46$. Note also the crossing of the two intermediate runs, showing that this system had been strongly supercooled and superheated. This figure represents the evidence for a first-order phase transition in SU(5) lattice gauge theory. From Monte Carlo runs where after each iteration g_0^2 was adjusted in an attempt to maintain $P=0.5$, halfway between the stable phases, I am led to quote

$$g_0^{-2}=1.66\pm0.03 \tag{6}$$

as the critical coupling.

A 3^4 lattice is rather small in linear extent. However, Fig. 1 shows that thermal fluctuations from the finite lattice size are smaller than the latent heat. A larger lattice should only reduce these fluctuations and make the latent heat larger.[8] As SU(5) has 24 generators, this system has 7776 degrees of freedom, of which 5832 remain after removing the gauge freedom. Thus in some sense the calculation is roughly comparable to a simple two-dimensional model on a 75×75 lattice. Of course, a finite lattice may be regarded as representing a finite physical temperature given by the inverse of the temporal lattice extent. To check that the effect is stable when the lattice size is changed, I made short runs of 100 iterations from both random and ordered initial states on a 4^4 lattice. These sequences followed closely the corresponding ones shown in Fig. 1, but with smaller thermal fluctuations, and suggested a slightly larger latent heat. In contrast, a finite-lattice effect should have sharply shifted the slow convergence to a new coupling regime.

In Fig. 2, I show the average plaquette for SU(5) as a function of g_0^{-2}. The points at the critical coupling are extracted from Fig. 1 whereas the other points come from a rapid thermal cycle of the model. On this graph, I also plot the expansions in Eqs. (4) and (5). The approach to these

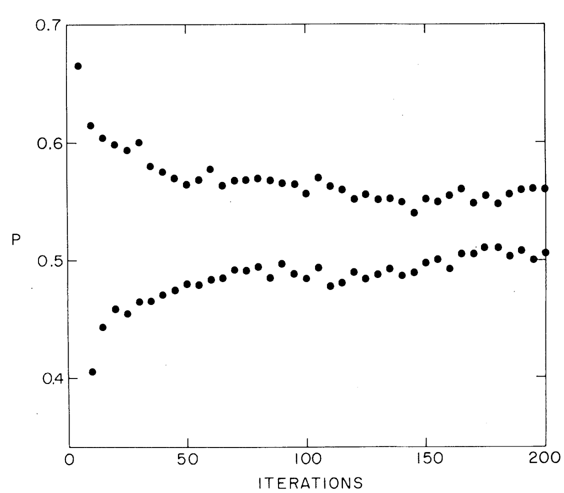

FIG. 3. Two Monte Carlo runs for SU(4) gauge theory at $g_0^{-2}=1.28$.

expansions is a check on the program.

Previous work[1] has shown that a similar transition does not occur for SU(2) or SU(3). Presumably this discontinuous behavior is a property of SU(N) for large N. In Fig. 3, I show runs for SU(4) on a 4^4 lattice with ordered and disordered starts at $g_0^{-2} = 1.28$. Convergence is unprecedentedly slow only very near this coupling. Although any surviving latent heat is smaller than for SU(5), a transition is strongly suggested with $N = 4$ at $g_0^{-2} = 1.28 \pm 0.04$.

Since SU(2) and SU(3) gauge fields should be unable to support a massless-gluon phase, presumably these large-N transitions are not conventional deconfinement as seen for U(1). A rather speculative possibility is that these large gauge groups undergo spontaneous symmetry breakdown along the lines of the currently popular unified theories but without the need for added Higgs fields. As a more mundane possibility, this phenomenon may be merely an artifact of the Wilson form of the action. In the limit $N \to \infty$ the two-dimensional Wilson theory develops a third-order phase transition which is not deconfining.[4] However, the existence of this transition is known to depend crucially on the precise action used.[5] Any invariant action depending only on the $U_\square$ can be expanded in characters. This suggests, as the simplest generalization of Wilson's theory, the two-parameter action

$$S = \sum_\square \{\beta(1 - N^{-1}\,\mathrm{Re}\,\mathrm{Tr}\,U_\square) + \beta_A[1 - (N^2 - 1)^{-1}\mathrm{Tr}_A U_\square]\}, \qquad (7)$$

where Tr_A represents a trace of the matrix corresponding to $U_\square$ but in the adjoint representation of the group. By adjusting β_A it may be possible to remove the SU(N) transition discussed here. If so, the rapid crossover in SU(2) and SU(3) theories is probably due to a nearby critical point in the (β, β_A) plane, and a small β_A can turn on a real transition in these models.[6] This is currently under investigation. From this point of view, the first-order transition in the SU(5) theory could be an accidental consequence of where a first-order line ends in this coupling constant space.

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[1]M. Creutz, Phys. Rev. Lett. 45, 313 (1980); C. Rebbi, Phys. Rev. D 21, 3350 (1980); D. Petcher and D. H. Weingarten, to be published; B. Lautrup and M. Nauenberg, Phys. Rev. Lett. 45, 1755 (1980); R. C. Edgar, L. McCrossen, and K. J. M. Moriarty, to be published.

[2]K. G. Wilson, Phys. Rev. D 10, 2445 (1974).

[3]K. G. Wilson, to be published.

[4]D. J. Gross and E. Witten, Phys. Rev. D 21, 446 (1980).

[5]N. S. Manton, Phys. Lett. 96B, 328 (1980); C. B. Lang, P. Salomonson, and B. S. Skagerstam, CERN Report No. TH-2995, 1980 (to be published).

[6]Note that if β_A is taken to infinity, the model becomes Z_N lattice gauge theory which is known to have a rich phase structure. A. M. Polyakov, private communication.