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Observations of High-Beta Toroidal Plasmas

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A wide range of magnetohydrodynamically stable high-β plasmas is produced in the University of Wisconsin Levitated Toroidal Octupole. Near the single-fluid regime we obtain, in the bad-curvature region, $\beta = nk(T_e + T_i)8\pi/B^2 \approx 8\%$, twice the theoretical single-fluid ballooning instability limit of 4%. We also obtain stable plasmas at $\beta = 35\%$, nine times the theoretical limit, in a regime in which finite-ion-gyroradius (e.g., gyroviscosity) effects are important. Experimental spatial profiles of the equilibrium diamagnetism are compared with theory.

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The high-β [= (plasma pressure)/(magnetic pressure)] magnetohydrodynamic (MHD) ballooning instability is predicted to set a β limit for any average minimum-B system such as tokamaks,[1] tandem mirrors,[2] and multipoles.[3] Ballooning-mode study in a toroidal multipole contains simplifications in that multipoles may be operated without magnetic shear (unlike tokamaks) and possess an ignorable coordinate (unlike tandem mirrors). Thus high-β multipole experiments[4,5] can test ballooning-mode theory; if multipole β limits are incorrectly predicted, the present theory will likely be unreliable for other configurations. These β limits also influence the feasibility of a multipole advanced-fuel reactor.[6]

A broad range of extremely high-β MHD stable plasmas are obtained in the University of Wisconsin Levitated Toroidal Octupole. Plasma parameters vary from nearly the ideal single fluid to the kinetic regime. We have previously reported observation of $\beta \approx 12\%$ plasmas in a collisionless, kinetic regime with only 2 ion gyroradii within a pressure-gradient scale length.[4] In all cases, β is evaluated locally in the bad-curvature, high-field region (Fig. 1). β is higher at nearly all other locations.

Near the single-fluid regime, we have attained $\beta = nk(T_e + T_i)8\pi/B^2$ of 8%, twice the theoretical single-fluid ballooning limit of 4%. $L_p \sim 5\rho_i$, where $L_p = p/|\nabla p|$ is the pressure scale length and ρ_i is the ion thermal gyroradius. 25 ion gyroradii are contained between the ring and wall. The electron-ion Coulomb mean free path (~ 20 cm) is smaller than the magnetic connection length (~ 100 cm), eliminating kinetic-free-streaming and particle-trapping effects. The magnetic Reynolds number, S = (resistive skin time)/(Alfvén time), is about 1000, so that resistivity is ignorable. Since the predicted ballooning-mode amplitude varies along the field ($\vec{k}\cdot\vec{B}\neq 0$), which is shearless, resistive effects such as tearing and defeat of shear stabilization play no role.

To date, the highest β achieved, at lower magnetic field, is 35%, roughly nine times the fluid stability limit. The plasma is MHD stable. For this plasma both finite–ion-gyroradius and viscous effects are important since $2\rho_i \sim L_p$ and the ordinary Reynolds number $R = V_A l\rho/\mu \lesssim 1$ where V_A, l, ρ, and μ are the Alfvén speed, perpendicular scale length, mass density, and ordinary viscosity, respectively. Although not a reactorlike regime, examination of these apparently strong stabilizing effects is of value to determine their role in the theory and if they can be simulated in a reactor. For this $\beta = 35\%$ case, the measured equilibrium diamagnetism also differs dramatically from the fluid prediction, probably due to ion gyroviscosity, as indicated below.

The hydrogen plasma is created by simultaneous, cross-field injection from two coaxial Marshall guns into the 1.4-m–major-radius toroid containing four levitated internal rings. $\beta \approx 8\%$ is obtained, 400 μsec after injection, on the separatrix between the outer rings and wall (Fig. 1) with $n \approx 3.6\times 10^{13}$ cm^{-3}, $T_e \approx T_i \approx 20$ eV, and a purely poloidal magnetic field of 860 G (30% of present capability). $\beta \approx 35\%$ is obtained similarly with $B = 200$ G and $n \approx 2\times 10^{13}$ cm^{-3}, $T_e \approx T_i \approx 9$ eV. Density is measured with a vertical-path 70-GHz interferometer located at the midcylinder; this measures the separatrix density since most of its path samples constant density equal to the separatrix density. Density invariance along field lines is verified with Langmuir probes. Electron temperature is measured with a double Langmuir probe, C III line ratios, and time-dependent computer modeling of observed oxygen line radiation.[7] Ion temperature is measured with a $\frac{1}{4}$-in.-diam gridded electrostatic analyzer. T_e and T_i are equal, as expected, since the electron ion-equilibration time is $\simeq 50$ μsec. These diagnostics yield a measurement of β with a relative error of 0.25. Edge neutral pressure is $\sim 10^{-5}$ Torr.

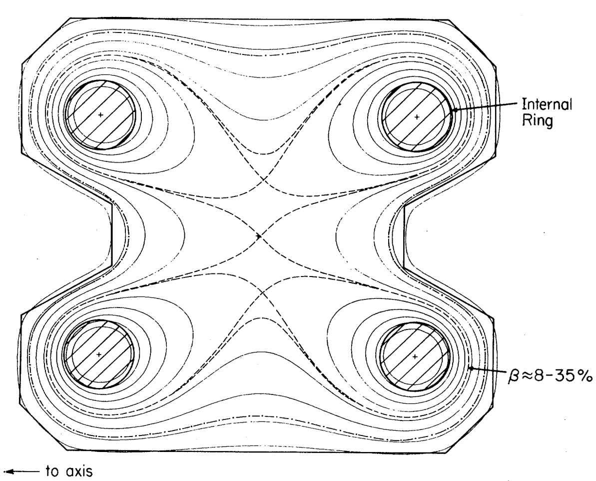

FIG. 1. Poloidal magnetic flux plot of the Levitated Octupole, indicating the bad-curvature region where β is evaluated. Plasma pressure peaks on the separatrix (dashed line). Major radius = 1.4 m.

Within 400 μsec after injection the plasma pressure becomes axisymmetric within 20% and adopts a steady profile peaked on the separatrix. Thereafter β decays from 8% (35%) with a 600 μsec (350 μsec) time constant which is ≈ 6000 (1000) Alfvén transit times and roughly the classical diffusion time. No β related fluctuations or disturbances are observed in magnetic field, density, or electrostatic potential. A coherent 30-kHz fluctuation in density and potential occurs in the large-gyroradius plasmas ($2\rho_i \sim L_p$). It peaks in the bad-curvature region with odd symmetry about the midplane.[8] However, the mode exists in identical form from $\beta \sim 0.1\%$ to $\beta \sim 10\%$ and has a negligible magnetic part ($\tilde{B}/B \approx 0.06\%$, $\tilde{n}/n \approx 30\%$).

The theoretical β limit for plasma stability to the ballooning mode is evaluated from linear single-fluid theory by numerically solving, in the Levitated Octupole geometry, the eigenvalue equation for marginal stability [Eq. (1), Ref. 4]. The critical β for instability is obtained by finding the minimum pressure gradient $\partial P/\partial\psi$, where ψ is the poloidal flux function, for which an eigenfunction exists. The experimental pressure profile is used as input and the eigenvalue equation may be evaluated with use of either the vacuum magnetic field or the self-consistent equilibrium values calculated from numerical solution of the Grad-Shafranov equation. The eigenfunction with even symmetry about the midplane (good-curvature) region is the most unstable mode with a threshold β value of 4%.[4] The eigenfunction peaks only gently in the bad-curvature region. Greater localization requires large energy expenditure in field-line bending.

The departure of the equilibrium magnetic field from its vacuum value has been measured and compared with the single-fluid equilibrium code with flux-conserving boundaries (experimental wall resistive-decay time is 100 msec). In the region between a ring and the wall the theoretical plasma self-current ($\nabla P \times \vec{B}/B^2$) reverses direction across the separatrix (peak pressure). Thus, there exist regions of local diamagnetism ($\Delta B < 0$) and local paramagnetism ($\Delta B > 0$) seen in Fig. 2.[4] For the $\beta \approx 8\%$ plasmas at $B = 860$ G with $5\rho_i \sim L_p$, good agreement exists between experiment and fluid theory, in magnitude and profile (Fig. 2). At the highest-β ($\approx 35\%$) case, in which $2\rho_i \sim L_p$ and viscosity is important, the measured field perturbation deviates from the single-fluid expectation by a factor of 7 (Fig. 3). As the field is increased, gyroradius and viscosity effects decrease and the experimental ΔB approaches the single-fluid value (Fig. 3). However, even at the high fields, the time behavior of β does not match that of $\Delta B/B$, which decays with a time constant ≈ 250 μsec. For the range of β considered, fluid theory predicts $\Delta B/B$ to be proportional to β; i.e., $\Delta B/B\beta$ is field independent.

To examine the cause of the experimental deviation from the single-fluid equilibrium, the region between the ring and wall is modeled by an infinite straight plasma annulus bounded by two coaxial cylinders, representing a ring and the wall. This model accurately describes the corresponding octupole region; the analytical single-fluid coaxial solution is nearly identical to the exact solution of the two-dimensional Grad-Shafranov equation for the octupole.

We include ion gyroviscosity, a finite-gyroradius effect, through the two-fluid equilibrium equations with the full pressure tensor[9]

$$nMv_{ir}v_{ir}' = neE_r - nev_{iz}B - p_i' - mn(v_{ir} - v_{er})\nu_{ei} + r^{-1}\{[2rp_i\tau/(1+4\omega^2\tau^2)][\tfrac{2}{3}(1+\omega^2\tau^2)v_{ir}' - \omega\tau v_{iz}']\}', \quad (1)$$

$$nMv_{ir}v_{iz}' = neE_z + nev_{ir}B - mn(v_{iz} - v_{ez})\nu_{ei} + r^{-1}\{[2rp_i\tau/(1+4\omega^2\tau^2)](\tfrac{1}{2}v_{iz}' + \omega\tau v_{ir}')\}', \quad (2)$$

$$nmv_{er}v_{er}' = -neE_r + nev_{ez}B - p_e' + mn(v_{ir} - v_{er})\nu_{ei}, \quad (3)$$

$$nmv_{er}v_{ez}' = -neE_z - nev_{er}B + mn(v_{iz} - v_{ez})\nu_{ei}, \quad (4)$$

where $' = \partial/\partial r$, and v_{ir}, v_{er}, v_{iz}, v_{ez}, ω, and τ are the ion and electron radial and axial velocities, ion cyclotron frequency, and ion-ion collision time, respectively. Electron viscous terms are absent since they are smaller by mass ratios. However, the electron velocity can deviate from its single-fluid value since electrons collisionally and electrostatically couple to ions. In the $\beta \approx 35\%$ plasmas, the ion viscous terms are of comparable magnitude to other terms.

If we assume radial velocities on the order of the collisional diffusion velocity, terms contain-

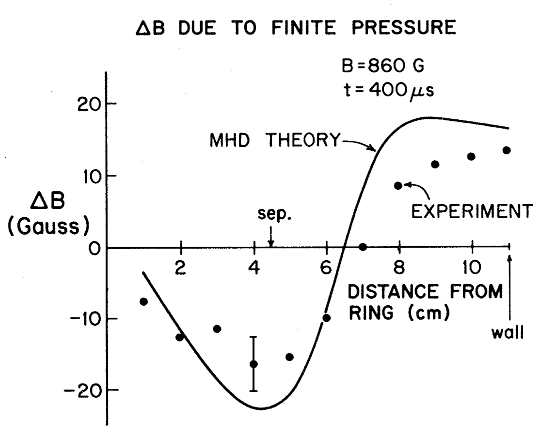

FIG. 2. Single-fluid theoretical and experimental spatial profile (between outer ring and wall) of perturbation, ΔB, to the poloidal magnetic field due to the equilibrium diamagnetic currents for $\beta \approx 8\%$. Plasma pressure peaks at the separatrix (Sep). Typical error bar is shown.

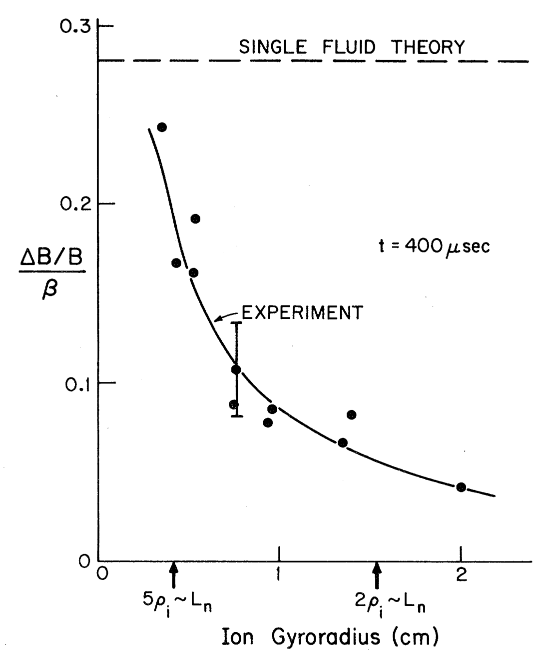

FIG. 3. Theoretical and experimental fractional change in the magnetic field (at the separatrix in the bad-curvature region) due to plasma diamagnetic currents vs thermal ion gyroradius. $\Delta B/B$ is normalized to the separatrix β value so that single-fluid theory yields a horizontal line.

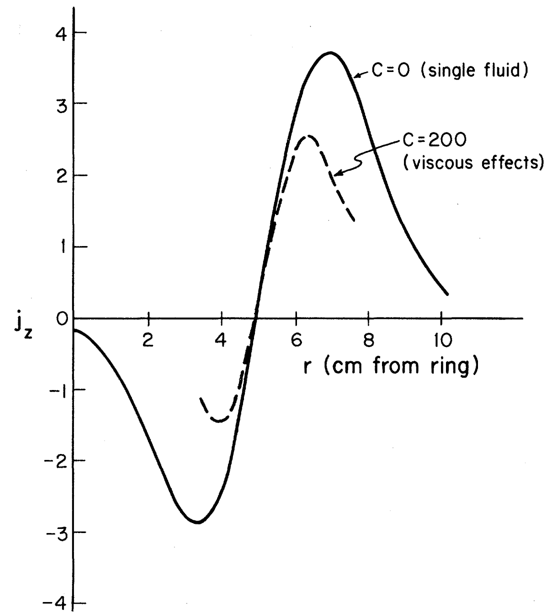

FIG. 4. Spatial profile of the equilibrium current density for the coaxial model of the Octupole from ideal single-fluid theory and viscous two-fluid theory. The two-fluid theory is not plotted near the boundary region where the method of solution breaks down. A Gaussian pressure profile is assumed.

ing v_{ir} and v_{er} are small (except near the edge where p becomes small). Thus we set $v_{ir} = v_{er} = 0$, permitting analytical solution for v_{iz}, v_{ez}, E_r, E_z, and, consequently, axial (toroidal) current density, j,

$$jB = p' - C(\omega\tau)'/r, \tag{5}$$

where C is an integration constant. The last term represents the desired deviation from single-fluid theory. For proper choice of C, this term diminishes the current density (Fig. 4) but maintains the overall shape, as observed experimentally. Detailed comparison with experiment awaits numerical solution of the full set of equations. Moreover, the theory is inaccurate in the highest-β case where the gradient lengths are comparable to the mean free paths.

In summary, a range of MHD stable high-β plasmas has been attained both in the kinetic regime with $\beta = 35\%$, nine times the fluid ballooning limit, and near the single-fluid regime with $\beta = 8\%$, twice the theoretical limit. MHD theory is inadequate and kinetic effects are apparently more powerful than is generally assumed. Thus, since a reactor (with trapped particles, large-gyroradii ions from neutral beams, etc.) may be no more fluidlike than the octupole experiments, their designs should perhaps not necessarily be constrained by the MHD ballooning instability β limit. Kinetic explanation of the stability results will be reported elsewhere. Diamagnetic current measurements roughly agree with single-fluid results in the fluidlike case, but depart sharply in the kinetic regime. The present experimental limit to β is a temporary limitation of the plasma sources.

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