Deep-Inelastic Electron Scattering and the Quark Structure of ^{3}He

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Deep-inelastic electron scattering from ^{3}He at momentum transfers $1 \leqslant Q^2 \leqslant 4$ (GeV/c)^2 is successfully described in a quark-cluster model of the nucleus. The quark-cluster probabilities and Fermi motion are obtained from a nuclear wave function consistent with low-energy data. A nucleon-bag (three-quark–cluster) radius of 0.45 fm provides an optimal description of the data. It fixes the geometrical probabilities at 0.83, 0.16, and 0.01 for three-, six-, and nine-quark clusters, respectively.

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Deep-inelastic electron scattering is a proven tool to investigate the structure of hadrons. Recently it has been applied to investigate light nuclei (e.g., ^{3}He) with momentum transfers of $Q^2 \geqslant 0.5$ (GeV/c)$^2 \approx 1/(0.08$ fm$^2)$. The data[1] indicate that the quark structure of the nucleus becomes more and more important with increasing Q^2. Here, we relate the quark structure and the experimental inelastic structure functions of ^{3}He. Indeed, the question of how quarks behave in nuclei has already received attention,[2] including examinations of portions of the data that we study.[3] However, these investigations have not explained the entire range of data that we reproduce.

We take a nucleon-bag radius R to determine the probabilities $p_i(R)$ for three-, six-, and nine-quark clusters ($i = 3, 6, 9$, respectively) from the geometrical overlap of nucleon bags. Two nucleons start forming a six-quark cluster when they are touching, i.e., a distance $2R$ apart, etc. In Table I, we give $p_i(R)$ for ^{3}He obtained from a harmonic-oscillator wave function chosen to have the measured nuclear rms radius.[4] The sum of the probabilities is normalized to one. The momentum distribution function of these i-quark clusters in the nucleus is $N_{i/^3\mathrm{He}}(\xi)$, where ξ is the fraction of the light-cone momentum $P^+ = E + P_z$ of the constituent, i.e.,

$$\xi_i(y) = \{(y^2P^2 + m_i^2)^{1/2} + yP\}/\{[P^2 + M^2(^3\mathrm{He})]^{1/2} + P\}, \tag{1}$$

where $P = M(^3\mathrm{He})\nu/Q$, ν is the laboratory energy loss of the electron, Q is the proton 4-momentum, and m_i is the cluster mass taken as $i/3$ times the nucleon mass. The ξ variable accounts[5] for finite-mass corrections to scaling. These corrections are sizable for deep-inelastic electron scattering on nuclei. The variable y specifies the momentum fraction of the cluster in the Breit frame of quark-photon scattering. We neglect transverse momenta in Eq. (1). We make an on-shell approximation for the multiquark clusters which is reasonable in the region where the respective wave functions contribute most, i.e., for momentum equipartition, $y_i = m_i/M$. It is easy to see that the mean value of ξ_i is $\langle \xi_i \rangle = i/9$ and the width $\Delta\xi$ is related to Δx, the momentum fraction spread in the Bjorken variable $x = Q^2/2M\nu$, via

$$\Delta\xi_{3,6} = \Delta x/(1 + \nu^2/Q^2)^{1/2}. \tag{2}$$

$\Delta x = 0.0384$ is obtained from our nuclear wave function and corresponds to $\langle \vec{p}^{\,2}\rangle^{1/2} = 0.180$ GeV/c. We also assume a Gaussian form for the ξ distributions $N_{i/^3\mathrm{He}}$ with the moments $\langle \xi_i \rangle$ and $\Delta\xi_i$ given above. *No* high-momentum components are added. It would be a complementary approach to analyze the experiment in terms of purely nucleonic components[6] with short-range correlations. Existing Fadeev calculations,[6,7] however, *cannot* reproduce the data. Instead of short-range nucleon-nucleon correlations our model replaces two nucleons at short distances by a six-quark cluster allowing for configurations not contained in the two-baryon subspace. Similar ideas

TABLE I. $p_i(R)$ for ^{3}He obtained from a harmonic-oscillator wave function chosen to have the measured nuclear rms radius (Ref. 4).

R (fm)	p_3	p_6	p_9
0	1.00	0	0
0.30	0.943	0.055	0.002
0.45	0.83	0.16	0.01
0.60	0.67	0.28	0.05
0.75	0.49	0.38	0.13
0.90	0.33	0.42	0.25

have been applied to the analysis of the baryon-baryon[8,9] and baryon-nucleus interactions.[10]

We remark that a proper quantum-mechanical treatment of overlapping quark bags must include quantum-chromodynamic (QCD) effects. However, in our phenomenological approach we *assume* a momentum distribution $n_{q/i}$ of quarks in an i-quark cluster ($i=6, 9$) from *counting rules*.[11] Let z be the quark momentum fraction of the momentum (yP) of the i-quark cluster [Eq. (1)]; then the improved scaling variable is

$$\tilde{\xi}_i = 2zP/[(y^2P^2 + m_i^2)^{1/2} + yP]. \tag{3}$$

The counting rules for large $\tilde{\xi}(<1)$, and Regge behavior for small $\tilde{\xi}$ give, for $i=6, 9$,

$$\tilde{\xi}_i n_{q/i}(\tilde{\xi}_i) = [B(\eta_i^{(2)}, \eta_i^{(1)}+1)]^{-1}(1-\tilde{\xi}_i)^{\eta_i^{(1)}}\tilde{\xi}_i^{\eta_i^{(2)}}, \tag{4}$$

with $\eta_i^{(1)} = 2(i-1)-1$, $\eta_6^{(2)}, \eta_9^{(2)} = 0.5$, and where $B(\ldots,\ldots)$ is Euler's β function. For the nucleon cluster (with $i=3$) we take the fit of Buras and Gaemers[12] to existing data. The η exponents have the approximate values $\eta_3^{(1)} \approx 3$ and $\eta_3^{(2)} \approx 0.65$ and are corrected because of QCD-scaling violation. To be consistent with the model proposed here it is probably worthwhile to reanalyze inelastic electron scattering on the deuteron. The corrections from six-quark clusters will be much smaller, however, than in the ^{3}He case, since the deuteron has a very large size. With these distribution functions it is possible to derive νW_2^{in} for ^{3}He from the photon-quark interaction:

$$\nu W_2^{\text{in}}(\nu, Q^2) = \sum_{\text{quarks}} e_j^2 \xi \mathcal{P}(\xi) = \tfrac{24}{9}\xi\mathcal{P}(\xi). \tag{5}$$

Note that ξ and x are related by $\xi = 2x/[1+(1+Q^2/\nu^2)^{1/2}]$.

$$\mathcal{P}(\xi) = \sum_{i=3,6,9} p_i \int_0^1 d\xi_i \int_0^{\xi_i^{\text{th}}} d\tilde{\xi}_i\, n_{q/i}(\tilde{\xi}_i) N_{i/^3\text{He}}(\xi_i)\,\delta(\tilde{\xi}_i\xi_i - \xi) \tag{6}$$

and

$$\xi_i^{\text{th}} = 2/[1+(1+4m_i^2/Q^2)^{1/2}]. \tag{7}$$

This formula has a very simple interpretation if one neglects the width of the cluster distributions and puts $N_{i/^3\text{He}} = \delta(\xi_i - i/9)$. Then it just shows that a ξ for ^{3}He corresponds to a three times bigger $\tilde{\xi}_3$ in the nucleon system $\tilde{\xi}_3 \approx 3\xi$ without nucleon motion and $Q^2 \to \infty$. The contribution of the quarks in the nucleon would end at $\xi(Q^2\to\infty) = x = \frac{1}{3}$. Above this kinematical limit the higher quark clusters take over. In fact the ξ variable has lower limits than x because of Eq. (7). For $Q^2 = 2\ (\text{GeV}/c)^2$ at $\xi = 0.26$ (or electron laboratory energy $E = 10.95$ GeV, $\nu = 1.0$ GeV) the unweighted contribution to the sum from the $i=6$ cluster in Eq. (6) is comparable to the nucleon ($i=3$) contribution. At $\xi = 0.31$ ($E = 10.95$; $\nu = 0.85$ GeV) the six- and nine-quark clusters dominate.

At smaller values of Q^2 the elastic scattering on the nucleon is still visible in the data. We propose to add this contribution to νW_2^{in}:

$$\nu W_2(\nu, Q^2) = \nu W_2^{\text{in}} + \nu W_2^{\text{el}}, \tag{8}$$

with

$$\nu W_2^{\text{el}} = 2\nu W_2^{\text{el}}(\text{proton}) + \nu W_2^{\text{el}}(\text{neutron})$$

and

$$\nu W_2^{\text{el}} = \left(\frac{G_E^2(Q^2) + (Q^2/4m_3^2)G_M^2(Q^2)}{1+Q^2/4m_3^2}\right) x N_{3/^3\text{He}}(x). \tag{9}$$

We employ a standard dipole form[13] for the nucleon elastic form factors. It is not clear how good this procedure is because some of the nonleading $1/Q^2$ corrections are already taken into account by the choice of the improved scaling variable.

In order to compare the theoretical calculation with the experimental data, we recall the connection between the double-differential spin-averaged cross section $d^2\sigma/d\Omega\, dE$ and the structure functions W_2 and W_1,[14]

$$d^2\sigma/d\Omega\, dE = [4\alpha^2(E-\nu)^2/Q^4]\cos^2\tfrac{1}{2}\theta[W_2(\nu, Q^2) + 2W_1(\nu, Q^2)\tan^2\tfrac{1}{2}\theta], \tag{10}$$

where θ is the laboratory scattering angle. For the reduction of the given data at $\theta = 8°$ we can neglect the contribution of W_1 and directly calculate the experimental structure function νW_2, which is displayed in Figs. 1(a), 1(b), and 1(c) for the three different energies 7.257, 10.95, and 14.70 GeV. In the same graphs we show the results of the model for three different nucleon-bag radii of $R = 0.0$,

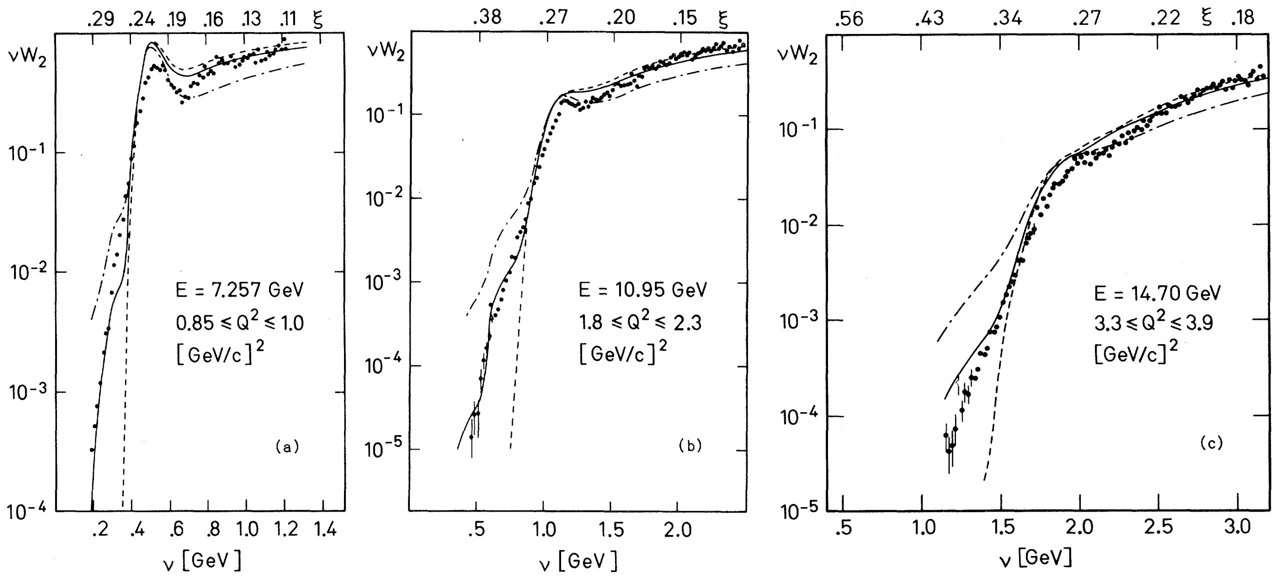

FIG. 1. Inelastic structure functions νW_2 of ^{3}He as functions of the electron laboratory-energy loss ν (lower scale) or ξ (upper scale) for three energies E. The dots correspond to the experiment in Ref. 1. We indicate the model predictions for νW_2 with a bag radius of $R = 0$ fm (dashed curve), $R = 0.45$ fm (full curve), and $R = 0.9$ fm (dash-dotted curve).

0.45, and 0.90 fm. At moderate ξ or large ν, the experiment is not very sensitive to the different quark-cluster probabilities p_i. Only the model with $R = 0.9$ fm underestimates the structure function. It is at large ξ (ν small and Q^2 large), however, that the various bag sizes predict νW_2's which differ by orders of magnitude. A model of the ^{3}He nucleus with $R = 0$ bags which therefore only includes nucleons as isolated $3q$ structures with Fermi motion is clearly excluded by the data. An intermediate bag radius of $R = 0.45$ fm allows for enough six-quark- and nine-quark-cluster structure to reproduce the experimental structure function at large ξ.

Currently an important debate[15] concerns the effect of quark degrees of freedom in nuclei. Its symbolic point of reference has become the radius of the nucleon bag R. Our analysis giving $R = 0.45$ fm indicates a (10–20)% probability of nonnucleonic quark structure with orders of magnitude consequences for selected regions of the data. This probability value is consistent with traditional nuclear-physics results. The radius $R = 0.45$ fm favored is smaller than the electromagnetic charge radius and the Massachusetts Institute of Technology–bag-model radius. Explicit calculations[8] indicate that the quarks remain localized at the nucleon positions at intermediate separations ($\delta \approx 2R \geqslant 0.8$ fm). These localized quarks still have the quark distribution functions $n_{q/3}(x)$ of three-quark clusters. Our model parameter R is compatible with this dynamical picture.

The theoretical results for $R \neq 0$ exhibit some shoulders in the structure function. These variations of νW_2 are due to the thresholds ξ_i^{th} in Eq. (9), which limit the quark momentum to the momentum of its cluster. A complete picture incorporating more binding effects than longitudinal Fermi motion may smear out these shoulders.

Finally, the description of the quasielastic peak is only fair. As we discussed before the simple addition of the noninteracting quark contribution and the nucleon quasielastic knockout is disputable. The quasielastic peak would be reduced with the probability p_3 and a quasielastic electron six-quark scattering contribution would be needed if long-lived dibaryon states exist.

The quark description of ^{3}He is more fundamental, but is it really necessary to have nonnucleonic components in ^{3}He? There is no doubt that the photon at large $Q^2 \geqslant 1$ (GeV/c)^2 resolves the quark degrees of freedom in the nucleus. In addition, the failure of the existing three-nucleon Fadeev calculations[6,7] to explain more than the quasielastic peak near its maximum value provides evidence for the type of quark *clustering* behavior in nuclei that we propose. Other inelastic electron-scattering experiments with Q^2 *and* ν larger than in the experiment of Ref. 1 but still $x > \frac{1}{3}$ can help to map out this new structure of the ^{3}He nucleus.

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Exit Doorway State in ^{12}C(^{16}O,^{8}Be)^{20}Ne

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We remark that resonances in the ^{12}C(^{16}O, ^{8}Be)^{20}Ne reaction are due to the "exit doorway state," that is ready for decay to the exit channel. We find a parameter-free formula that relates the resonance energy and angular momentum of the ^{12}C + ^{16}O system to those of the ^{12}C + ^{12}C system. If we use the experimental resonance energies of ^{12}C + ^{12}C, this formula yields more than a dozen one-to-one correspondences to the resonance energies of ^{12}C(^{16}O, ^{8}Be)^{20}Ne. We also find a parameter-free formula for ^{12}C(^{16}O, ^{4}He)^{24}Mg.

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The ^{12}C + ^{16}O reaction has been studied extensively by various workers in recent years.[1] In the energy region of $E_{\text{c.m.}} = 10$–25 MeV, this system is very rich in resonances. Although these resonances are strongly clustered in groups of the same angular momentum, experiments show that the intermediate structures in the ^{12}C + ^{16}O system are not consistent with each other for various exit channels.

For instance, Malmin *et al.*[2] report a sharp $J^\pi = 14^+$ resonance at $E_{\text{c.m.}} = 19.7$ MeV in the elastic scattering and later confirmed a rotational-band-like structure together with resonances of $J^\pi = 9^-$ and 15^- (16^+) at $E_{\text{c.m.}} = 13.6$ MeV and $E_{\text{c.m.}} = 22.0$ MeV.[3] Later on, Eberhard *et al.*[4] observed a $J^\pi = 10^+$ resonance at $E_{\text{c.m.}} = 18.8$ MeV in the reaction ^{12}C(^{16}O, ^{8}Be$_{\text{g.s.}}$)^{20}Ne. The spin value $J = 10$ is four units of angular momentum below the grazing value $J = 14$ obtained from the elastic scattering by Malmin *et al.*, who did not observe this reso-