Rare Kaon Processes as a Probe of the Top-Quark Mass

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We study the K_L-K_S mass difference and the $K_L \to \mu^+\mu^-$ decay rate in the Kobayashi-Maskawa model. If the matrix element $\langle \bar{K}^0 | [\bar{s}\gamma_\mu(1-\gamma_5)d]^2 | K^0\rangle$ is sufficiently small (e.g., as estimated in the Massachusetts Institute of Technology bag model), then a stringent *upper* bound on the top-quark mass can be obtained.

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Seven years ago, Gaillard and Lee[1] made a quantitative prediction for the mass of the (at that time conjectured) charmed quark by studying the K_L-K_S mass difference in the four-quark version[2] of the standard model.[3] Soon after, their prediction was confirmed by experiment.[4] In this Letter we make a quantitative prediction for the mass of the (as yet undiscovered) top quark by studying the K_L-K_S mass difference and the $K_L \to \mu^+\mu^-$ decay rate in the six-quark version of the standard model due to Kobayashi and Maskawa (KM).[5] For a certain range of values for the matrix element $M \equiv \langle \bar{K}^0 | [\bar{s}\gamma_\mu(1-\gamma_5)d]^2 | K^0\rangle$, which enters the formula for $\Delta m = m_{K_L} - m_{K_S}$, a rather stringent *upper* bound on m_t can be obtained. In particular we shall compute the upper bound on m_t corresponding to M as calculated[6] in the Massachusetts Institute of Technology (MIT) bag model. Our estimates of the *short*-distance contributions to the relevant amplitudes are done both in the free-quark model and in quantum chromodynamics (QCD).

Our Letter is organized as follows. We shall first discuss the formulas for the K_L-K_S mass difference and the $K_L \to \mu^+\mu^-$ decay rate as obtained in the free-quark model and in QCD. Subsequently we shall combine these formulas to obtain an upper bound on m_t. Finally we shall briefly discuss how the upper bound in question might be affected by long-distance effects not included in our analysis.

The K_L-K_S mass difference can be quite generally written as follows:

$$\Delta m = 2\,\mathrm{Re}\,C\langle \bar{K}^0 | [\bar{s}\gamma_\mu(1-\gamma_5)d]^2 | K^0\rangle + \mathrm{L.D.}, \qquad (1)$$

where the first term on the right-hand side of (1) represents the short-distance contribution coming from the standard box diagrams[1] and L.D. stands for all long-distance contributions which cannot be absorbed into the matrix element which appears in the first term. Whereas the matrix element $\langle \bar{K}^0 | \cdots | K^0\rangle$ cannot be evaluated by perturbative techniques, the short-distance function C can be calculated, for instance, in the free-quark model or in the perturbative QCD. The evaluation of the coefficient C in the free-quark model involves the standard box diagrams[1] with $W^\pm$ gauge bosons and the unphysical scalar $\varphi^\pm$ exchanges. For the case discussed here $m_t/M_W > 0.25$ and the inclusion of unphysical scalar contributions[7] as well as an exact[8] evaluation of the box diagrams is necessary. In the literature, only Inami and Lim[9] have done such an exact calculation. We confirm their result.

The free-quark–model estimate of the coefficient C is modified by the QCD corrections. We have included these effects in our analysis by following the work of Gilman and Wise.[10]

Neglecting for the moment the L.D. contributions, the ratio of Δm to the kaon mass, which is measured[11] to be 0.71×10^{-14}, is given by

$$\frac{\Delta m}{m_K} = \frac{0.42}{R}\frac{G_F}{\sqrt{2}}\frac{2}{3}f_K^2\frac{\alpha}{4\pi}(\sin^2\theta_W)^{-1}F(x_i, \theta_j) = 1.6 \times 10^{-10} R^{-1}F(x_i, \theta_j) = 0.71 \times 10^{-14}, \qquad (2)$$

where

$$F(x_i, \theta_j) = [(\mathrm{Re}A_c)^2 - (\mathrm{Im}A_c)^2]B(x_c, x_c)\eta_1 + [(\mathrm{Re}A_t)^2 - (\mathrm{Im}A_t)^2]B(x_t, x_t)\eta_2 + 2[\mathrm{Re}A_c\mathrm{Re}A_t - \mathrm{Im}A_c\mathrm{Im}A_t]\tilde{B}(x_t, x_c)\eta_3, \qquad (3)$$

and

$$\mathrm{Re}A_c = -s_1c_2(c_1c_2c_3 - s_2s_3\cos\delta), \quad \mathrm{Im}A_c = -\mathrm{Im}A_t = s_1s_2s_3c_2\sin\delta, \quad \mathrm{Re}A_t = -s_1s_2(c_1s_2c_3 + c_2s_3\cos\delta), \qquad (4)$$

where $c_1 = \cos\theta_i$, $s_i = \sin\theta_i$, and δ are the standard KM parameters.[5] Furthermore $G_F = 1.1785 \times 10^{-5}$

GeV^{-2}, $f_K = 1.23 m_\pi$ is the kaon decay constant, and θ_W is the Weinberg angle ($\sin^2\theta_W \approx 0.23$). The parameter R (Ref. 12) depends on the estimate of the matrix element $\langle \bar{K}^0 | \cdots | K^0 \rangle$ in Eq. (1). In the MIT bag model $R = 1$,[6] whereas in the vacuum insertion approximation[1] $R = 0.42$.

The functions $B(x_i, x_i)$ and $\tilde{B}(x_i, x_j)$ are given by[9]

$$B(x_i, x_i) = x_i[\tfrac{1}{4} + \tfrac{9}{4}(1-x_i)^{-1} - \tfrac{3}{2}(1-x_i)^{-2}] + \tfrac{3}{2}[x_i/(x_i-1)]^3 \ln x_i, \tag{5}$$

$$\tilde{B}(x_i, x_j) = x_i x_j\{(x_j - x_i)^{-1}[\tfrac{1}{4} + \tfrac{3}{2}(1-x_j)^{-1} - \tfrac{3}{4}(1-x_j)^{-2}]\ln x_j + (x_j \leftrightarrow x_i) - \tfrac{3}{4}[(1-x_j)(1-x_i)]^{-1}\}, \tag{6}$$

where $x_i = m_i^2/m_W^2$. In obtaining (5) and (6) x_u has been set to zero. For small x_i (5) and (6) reduce to the formulas of Ref. 13.

The parameters η_i in Eq. (3) represent QCD corrections which depend very weakly on m_t. For the QCD scale parameter $\Lambda = 0.3$ GeV and $m_c = 1.5$ GeV (constituent quark mass) one finds[10] $\eta_1 \approx 0.90$, $\eta_2 \approx 0.62$, and $\eta_3 \approx 0.33$ for all values of m_t considered in this paper. For $m_c = 1.2$ GeV (current quark mass) we find $\eta_1 \approx 1.0$ with η_2 and η_3 unchanged. The free-quark model corresponds to $\eta_1 = \eta_2 = \eta_3 = 1$. η_i depend very weakly on Λ for $0.2 \le \Lambda \le 0.5$ GeV.

The evaluation of the short-distance contribution to the decay $K_L \to \mu^+\mu^-$ involves, in the free-quark model, the diagrams of Fig. 1, where the blob represents the induced $Z\bar{s}d$ coupling. The full list of diagrams contributing to this induced coupling can be found in Refs. 1, 9, and 14. In the approximation of neglecting the muon mass the box diagrams with $\varphi^\pm$ exchanges do not contribute. In the literature, exact calculations of the diagrams of Fig. 1 have been done only in Refs. 9 and 14. We confirm the results of these papers. We have also included QCD corrections in our analysis by making a straightforward generalization of the results of Ref. 15 to the six-quark model.

Combining our calculations with the upper bound on the short-distance contribution to $K_L \to \mu^+\mu^-$, as extracted by various authors[16,17] from the data, we are led[9,16] to the following inequality ($M_W = 80.5$ GeV):

$$|\mathrm{Re}A_t|\, G(x_t)\eta \le |s_1 c_3|\, 0.85 \times 10^{-2}. \tag{7}$$

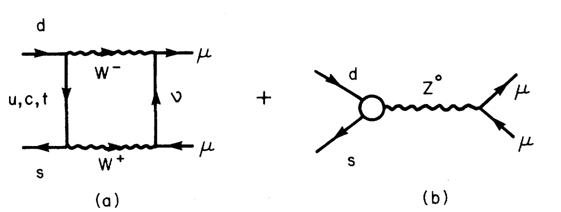

FIG. 1. Diagrams contributing to the left-hand side of the bound in Eq. (7); (a) box diagram, (b) induced Z^0 contribution.

Here[9,14]

$$G(x_t) = \frac{3}{4}\left(\frac{x_t}{x_t - 1}\right)^2 \ln x_t + \frac{x_t}{4} + \frac{3}{4}\frac{x_t}{1 - x_t} \tag{8}$$

and η, which represents QCD corrections, is equal to 0.9 for $\Lambda = 0.3$ GeV. The free-quark model corresponds to $\eta = 1$. The charm contribution to $K_L \to \mu^+\mu^-$ is smaller by two orders of magnitude than the upper bound of Eq. (7) and has been neglected. On the other hand, as noticed first by Shrock and Voloshin,[16] the bound of Eq. (7) is very useful for finding bounds on KM angles if the mass of the top quark is larger than 20 GeV.

Formulas like (2) and (7) (without accounting for QCD effects and in the small x_t approximation) have been used already by various authors with the aim to find bounds on the mixing angles θ_i and δ.[9,12,13,16,18,19] Here we shall use them to find an upper bound for m_t as a function of the parameter R which enters Eq. (2). It is probably useful to get a feel for the reason why a bound on m_t can be obtained *at all* from the formulas (2) and (7) alone. After all, these equations contain three unknown parameters θ_2, θ_3, and δ (θ_1 is known[20]), and it would appear that by making a suitable choice for their values an arbitrary large top-quark mass would be allowed. In order to see that this is not the case let us first make a very crude approximation (justified for large R) and neglect in Eq. (3) all terms but $\eta_2[\mathrm{Re}A_t]^2 B(x_t, x_t)$. Equations (2) and (7) lead then to the following inequality:

$$\frac{G^2(x_t)}{B(x_t, x_t)} \le \frac{1}{R}\frac{\eta_2}{\eta^2}[s_1 c_3]^2 \times 1.63. \tag{9}$$

Since the function $G^2(x_t)/B(x_t, x_t)$ increases monotonically with increasing x_t an *upper* bound on m_t can be obtained. Furthermore, since $\eta_2/\eta^2 \simeq 0.77$ the upper bound in question is reduced by QCD corrections relative to its free-quark-model value. The bound on m_t also decreases with increasing R. All these qualitative features remain valid when all the terms in Eq. (3) are retained. It should be emphasized that it is crucial for obtaining the bound that Δm and $K_L \to \mu^+\mu^-$ are considered *simultaneously*, and that R is larger than 0.42, the value used by Gaillard and Lee.[1]

In a numerical analysis of Eqs. (2) and (7) we have used $c_1 = 0.97$ and $|s_3| \leq 0.5$ as obtained in Ref. 20 from the data on nuclear β decay and hyperon decays, respectively. The upper bound on m_t can then be found for fixed values of R and m_c by varying s_2, s_3, and $\sin\delta$ in the *full* range $|s_2| \leq 1$, $|\sin\delta| \leq 1$, and $|s_3| \leq 0.5$. The result is shown in Fig. 2. We observe that QCD corrections substantially lower the bound. The bound is also lowered when the current quark mass $m_c = 1.2$ GeV is used instead of the constituent quark mass $m_c = 1.5$ GeV. We also observe that the strong dependence of $(m_t)_{\max}$ on the parameter R for $R<1$ is somewhat weakened for $R>1$. For $R = 0.42$ (not shown in Fig. 2), which corresponds to the vacuum insertion estimate of Ref. 1, no useful upper bound on m_t can be obtained. Even for $m_c = 1.2$ GeV and after the inclusion of QCD effects the upper bound on m_t corresponding to $R = 0.42$ is higher than m_W. Much more stringent bounds are obtained if the matrix element $\langle \bar{K}^0| \times |[\bar{s}\gamma_\mu(1-\gamma_5)d]^2|K^0\rangle$ is smaller by at least a factor of 2 than its vacuum-insertion estimate. For example, using the MIT–bag-model estimate[6] ($R=1$) of the matrix element in question, we find

$$m_t \leq \begin{cases} 47 \text{ GeV (free-quark model)} & \text{(10a)} \\ 33 \text{ GeV (QCD)} & \text{(10b)} \end{cases}.$$

It should also be remarked that the experimental lower bound[21] on m_t ($m_t \geq 19$ GeV) leads to an upper bound on R. This bound is roughly $R \approx 3$ and $R \approx 2$ for the free-quark model and QCD estimates, respectively.

So far in our analysis we have neglected the long-distance (L.D.) term which enters Eq. (1). As pointed out by Wolfenstein,[12] and recently by Hill,[22] the contribution of low-mass intermediate states (e.g., π, η), which are not accounted for by the box diagrams [first term in (1)], may give a sizable contribution to Δm.[23] Following Wolfenstein and Hill, we write L.D. $= -z\,\Delta m$ (z is a parameter), which together with the box contribution gives Eq. (2) with $R \to R(1+z)$. Thus for $z > 0$ the upper bound on m_t, which we found above, is lowered, but it is increased if $z<0$. In this respect the partially conserved axial-vector current estimate of the L.D. term by Hill,[22,24] who finds $z>0$, is very interesting. On the other hand, Wolfenstein[12] attaches greater unreliability to the estimate of z and considers also negative values of z, which would increase our bound. However, as is pointed out by Hill,[22] independently of the partially conserved axial-vector current estimate, a positive sign of z is preferred if the "penguin" diagram contributions to the CP nonconservation ratio ϵ'/ϵ are as large as claimed by Gilman and Wise.[25] Negative z together with the results of Ref. 25 would lead[22] to the nonconservation of the experimental bound on ϵ'/ϵ. Thus in the end it may well be that z is indeed positive.

In summary, we may conclude that if the matrix element of Eq. (1) is not larger than its MIT–bag-model estimate,[6] and if $z>0$ as suggested by Hill's paper,[22] then our analysis (within the six-quark version of the standard model[26]) indicates that the top quark should have a mass *not more than 30–40 GeV*. Experimentalists will tell us in the not too distant future whether this is indeed the case.

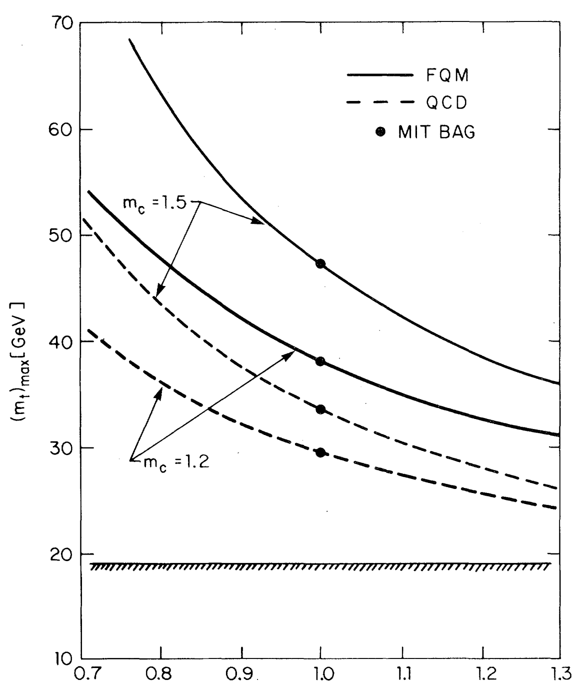

FIG. 2. The upper bound on m_t for various cases considered in the text as function of the parameter R. FQM stands for the free-quark model. The horizontal line shows the approximate experimental lower bound on m_t.

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