Can There Be Low Intermediate Mass Scales in Grand Unified Theories?

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We show that simple grand unified theories, such as O(10), allow the mass scale at which parity conservation is presumably restored to be as low as 100–200 GeV.

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It is well known that the simplest grand unified theory, the SU(5) model of Georgi and Glashow,[1] predicts a desert in energies between M_W and the unification scale M_U. This, of course, is not true in general since any theory beyond SU(5) allows for the existence of intermediate mass scales. For example, the popular O(10) grand unified theory[2] can be imagined to be broken in stages

$$O(10) \underset{M_U}{\longrightarrow} [SU(2)]_L \otimes [SU(2)]_R \otimes [SU(4)]_C \underset{M_C}{\longrightarrow} [SU(2)]_L \otimes [SU(2)]_R \otimes [U(1)]_{B-L} \otimes [SU(3)]_C$$
$$\underset{M_R}{\longrightarrow} [SU(2)]_L \otimes U(1) \otimes [SU(3)]_C \underset{M_W}{\longrightarrow} [U(1)]_{em} \otimes [SU(3)]_C .$$

The important question can and should be raised as to whether some of the intermediate scales can be so low that the associated physical phenomena are observable in the near future. We show here that the answer *lies in the affirmative.* To make our point as clear as possible, we will work within the O(10) grand unified theory, and simply assume the above chain of symmetry breaking, in the limit $M_C = M_U$. In other words, we assume that the only mass scale above M_W is M_R, at which, presumably, parity conservation is restored.

Our sample is only illustrative of a general statement and is motivated by its physical interpretation. Left-right–symmetric gauge theories have been suggested by Mohapatra, Pati, Salam, and one of us (G. S.)[3] in order to account for parity nonconservation in weak interactions. According to these models, parity is spontaneously broken and its nonconservation at low energies is due to the heavier mass of right-handed gauge bosons, M_R. At energies above M_R, parity conservation is, therefore, expected to be restored. Also, the smallness of neutrino masses is tied up with parity nonconservation,[4] with the left-handed neutrino being a light Majorana particle and a right-handed neutrino being very heavy, of order M_R. Therefore, the only constraint on M_R comes from the neutral-current data and we find that M_R can be as low as 100–200 GeV.[5]

We demonstrate now that such low values of M_R can be made compatible with grand unification, which provides an alternative to the conventional picture according to which $M_R = M_U$. In order to find the constraint on M_R due to unification, we follow the program of Georgi, Quinn, and Weinberg[6] which treats the dependence of coupling constants with energy. In a straightforward manner we can derive the following equations for the SU(2), U(1), and $[SU(3)]_C$ coupling constants (at low energies), respectively,[7]

$$\frac{1}{g^2(M_W)} = \frac{1}{g^2(M_U)} + 2b_2 \ln\frac{M_U}{M_W}, \quad \frac{1}{g_1{}^2(M_W)} = \frac{1}{g_1{}^2(M_U)} + 2b_1 \ln\frac{M_R}{M_W} + 2\left(\frac{2}{5}b_1 + \frac{3}{5}b_2\right)\ln\frac{M_U}{M_R},$$
$$\frac{1}{g_s{}^2(M_W)} = \frac{1}{g_s{}^2(M_U)} + 2b_3 \ln\frac{M_U}{M_W}, \tag{1}$$

where b_N are well-known coefficients of the β function, given for the gauge group SU(N) by

$$b_N = -\frac{1}{16\pi^2}\left(\frac{11}{3}N - \frac{4}{3}\sum_f T_f(R) - \frac{1}{6}\sum_S T_S(R)\right). \tag{2}$$

The first term in (2) is the gauge-meson contribution and the second and third terms denote the fermionic and Higgs contribution, respectively. In what follows, we shall ignore the tiny Higgs contribution, since we do not want to restrict ourselves to a particular Higgs sector. This is certainly consistent with our approximation. We then arrive at

$$\sin^2\theta_w(M_W) = \frac{3}{8} - \frac{11}{3}\frac{\alpha(M_W)}{\pi}\left(\frac{5}{8}\ln\frac{M_U}{M_W} - \frac{3}{8}\ln\frac{M_U}{M_R}\right), \quad 1 - \frac{8}{3}\frac{\alpha(M_W)}{\alpha_S(M_W)} = \frac{11}{3}\frac{\alpha(M_W)}{\pi}\left(3\ln\frac{M_U}{M_W} - \ln\frac{M_U}{M_R}\right). \tag{3}$$

The existence of M_R clearly increases the values of $\sin^2\theta_w$ and for low M_R we seemingly run into conflict with the successful prediction of SU(5). However, for sufficiently low values of M_R that is simply not true. Namely, in that case $\sin^2\theta_w$ gets affected by the presence of M_R. Actually, the lower M_R, the higher $\sin^2\theta_w$ required to meet the existing data. That is the essence of our point: From (3) it is seen that low M_R increases $\sin^2\theta_w$ appreciably, but that is actually what is needed for the theory to pass the low-energy experimental tests.[5] In order to discuss the phenomenological consequences of the left-right-symmetric theory under consideration, we would have to specify all the detailed structure of the low-energy charged- and neutral-current Hamiltonians and compare it to the experiment. Since that is clearly beyond the scope of this Letter, we here only briefly outline the basic program of such an analysis; for details, the reader should see Ref. 5.

Because the familiar leptonic charged currents are constrained to be effectively left handed (recall that $m_{\nu_R} \sim M_{W_R}$) at low energies, the usual charged-current processes, such as μ and β decay, do not provide any limits on the mass of $W_R^{\pm}$. To be more precise, let us write down the charged-current Hamiltonian at low Q^2,

$$H_{cc} = (G_F/\sqrt{2})[J_L^\dagger J_L + \alpha(J_L^\dagger J_R + J_R^\dagger J_L) + \beta J_R^\dagger J_R], \tag{4}$$

where J_L and J_R are the conventional weak chiral currents and α and β are functions of various Higgs vacuum expectation values[5] and whose form is irrelevant for our argument. Since

$$J_{R\mu}^{\text{lept}} = \bar{e}_R \gamma_\mu \nu_R, \tag{5}$$

the experiment provides only a limit on the parameter α, i.e., the amount of W_L-W_R mixing: $|\alpha| \lesssim 0.1$ (or so). Therefore, it is only the neutral-current interactions which give the bounds on the masses of heavier charged and neutral gauge bosons. There are three different classes of neutral-current processes which can be used to constrain the four parameters of the model: $\sin^2\theta_w$, M_{W_L}, M_{W_R}, and α [notice, that the parameter β in (4) can be expressed as a function of these four]. These processes are (a) neutrino-hadron and neutrino-electron scattering, (b) parity-nonconserving electron-hadron interactions, and (c) forward-backward asymmetry in $e^+e^- \to \mu^+\mu^-$. Within this four-parameter space, there are regions for which the model agrees with the data known from (a) to (c), within the present experimental limits. The details of this procedure evidently require more space than what this Letter allows and are given in a following paper.[5] The main idea is very simple: One just confronts the predictions of the theory with the model-independent allowed values of the quark and lepton couplings.[8] The major result of this analysis is that for a range of the other parameters $\sin^2\theta_w$ can be as large as 0.25–0.31. For such large values of $\sin^2\theta_w$ it is found that W_R has to be quite light, with $M_{W_R} \simeq 100$–150 GeV. For the sake of illustration, Table I shows clearly the dependence of $\sin^2\theta$ with M_R.

We now present the results of our investigation; a few remarks are first in order. If, say, $M_R \geqslant 1000$ GeV, then clearly $\sin^2\theta_w$ is as in the standard model. But then (3) implies $M_R \geqslant 10^9$ GeV (see Table II). This result was known by several people[9] and has led to the claim that the intermediate mass scale in grand unified theories is so large that for practical purposes we would have the equivalent of a desert.

TABLE I. The values of $\sin^2\theta_w$ and M_R for which the left-right–symmetric models pass the neutral-current tests (recall that ν_R is very heavy and so W_R does not participate in β and μ decays).

$\sin^2\theta_w(M_W)$	M_{W_R} (GeV)
0.23	258
0.25	155
0.27	134
0.29	120
0.31	116
0.33	106

TABLE II. The values of M_U and M_R needed to arrive at $\sin^2\theta_w \simeq 23$. Therefore, if $M_R \gtrsim 10^3$ GeV (or so), then clearly (3) leads to $M_R \gtrsim 10^9$ GeV.

M_U (GeV)	M_R (GeV)
10^{14}	2×10^{12}
10^{15}	5×10^{11}
10^{16}	1×10^{11}
10^{17}	2×10^{10}
10^{18}	5×10^{9}
10^{19}	1×10^{9}

TABLE III. The values of $\sin^2\theta_w$, α_S, and M_U for low M_R (M_R = 100–200 GeV). Comparison with Table I shows that the model is perfectly consistent with the phenomenology.

M_U (GeV)	$\sin^2\theta_w(M_W)$	$\alpha_S(M_W)$
10^{14}	0.31	0.04
10^{15}	0.30	0.05
10^{16}	0.30	0.05
10^{17}	0.29	0.06
10^{18}	0.29	0.06
10^{19}	0.28	0.08

On the other hand, if M_R is low (few M_W), then clearly the above considerations do not apply. The point, as we mentioned before, is that $\sin^2\theta_w$ is substantially bigger than in the standard model as the data show. Equation (3) can now be satisfied. A clarification is needed first: We have set $M_R = M_W$ in (3), since to the leading log approximation, we cannot do any better. Table III then shows $\sin^2\theta_w$ and α_S for various values of the unification scale M_U. As far as $\sin^2\theta_w$ is concerned, the unification scale could be any number between 10^{14} and 10^{19} GeV (we obviously want to stay below the Planck mass). However, a precise knowledge of α_S would clearly determine M_U. If we take, for example, $\alpha_S(m_W) = 0.08$ as in SU(5), we would get $M_U \simeq 10^{19}$ GeV. Therefore, the model discussed offers an interesting alternative to the conventional picture of grand unification: There exists a mass scale, not far from M_W, above which parity is expected to become a good symmetry. On the other hand, the proton would be then practically stable ($\tau_p \gtrsim 10^{46}$ yr). Of course, a precise determination of M_U will be known only when Λ is determined precisely.

In any case, as our results show, there may be low intermediate mass scales. We have presented an example of a left-right–symmetric theory embedded in an O(10) grand unified theory. Obviously, the same would be true for any group G with the mentioned symmetry breaking. Our example, as much as pedagogical, is also realistic and allows the possibility of a unification of weak, electromagnetic and strong interactions, and simultaneously could enable us to find out, in the near future, the origin of parity nonconservation in β and μ decay.

Led by our example, one could envision the picture according to which there would be many intermediate mass scales, whose presence would affect both the phenomenological determination of $\sin^2\theta_w$ and the predictions of (3), as to make them compatible. There may not be a desert.

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