Cosmological Density Fluctuations Produced by Vacuum Strings

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It is shown that vacuum strings produced in the early universe at the grand unification phase transition can generate density fluctuations sufficient to explain the galaxy formation.

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The origin of the density fluctuations leading to galaxy formation is one of the major unresolved problems in cosmology. Of course, one can take the point of view that explaining initial conditions is out of the scope of physics and postulate the spectrum of perturbations at $t=0$. For those who are not satisfied with such a "solution" there seem to be only two natural choices of the initial state: (i) Chaotic universe with $\delta\rho/\rho \sim 1$ and (ii) exactly homogeneous universe with $\delta\rho/\rho = 0$. In the present paper we shall discuss only the second possibility. Then one has to assume that at the beginning the universe was exactly Friedmannian and that the density fluctuations were generated later by some physical process (e.g., a phase transition). However, it can be shown that causal processes in the early universe (say, at $t<1$ s) cannot produce fluctuations of desired magnitude.[1,2] Modern broken-symmetric gauge theories of elementary particles have opened a new possibility. In such theories the symmetry can be restored[3] at sufficiently high temperatures, $T > T_c$. The phase transition at $T = T_c$ can give rise to extended structures—vacuum domain walls or strings, depending on the topology of the manifold of degenerate vacuum states.[4] Moving domain walls or strings produce density fluctuations on scales smaller than the horizon. In this scenario the fluctuation generating process continues for all times after the phase transition, extending to larger and larger scales.

The cosmological consequences of vacuum domain walls have been discussed in Refs. 4–6 with the conclusion that the existence of such walls contradicts observations. In the present paper it will be shown that vacuum strings produced at the grand unification phase transition can generate density fluctuations of the required magnitude.

The idea that vacuum strings can be responsible for cosmological density fluctuations has been first suggested by Zel'dovich.[2] I shall comment on his approach at the end of this paper.

The linear mass density of the strings is of the order[4]

$$\mu \sim \alpha^{-1} m^2, \tag{1}$$

where m and α are the characteristic mass and the coupling constant of the theory and we assume that all relevant masses and coupling constants are of the same order. (I use the system of units in which $\hbar = c = 1$.) Taking $\omega \sim 10^{-2}$ we obtain $\mu \sim 10^{-4}$ g/cm for electroweak strings ($m \sim 100$ GeV) and $\mu \sim 10^{22}$ g/cm for grand unification strings ($m \sim 10^{15}$ GeV). Only grand unification strings will be discussed below.

At the time of formation the strings are expected to have the shape of Brownian trajectories with a persistence length $\sim \xi_0$, where ξ_0 is the characteristic scale of variation of the Higgs vacuum expectation value. Causality requires that ξ_0 must not exceed the horizon size at the phase transition. Some aspects of the cosmological evolution of strings have been discussed in Refs. 2, 4, 7–9. However, a more complete analysis is needed for a quantitative estimation of the density perturbations.

Tension in convoluted strings results in oscillations on scales smaller than the horizon. In the course of expansion these oscillations are damped by various dissipation mechanisms, and the strings gradually straighten out. The following damping mechanisms have to be considered: (i) Friction due to interaction of strings with particles; (ii) cosmological red shift of oscillations as a result of expansion (this effect is similar to the red shift of the cosmic background radiation); (iii) intersecting strings changing partners and forming closed loops, thus decreasing their length; and (iv) gravitational radiation.

The force of friction on a moving string has been estimated by Kibble[4,7] assuming that the cross section for particle-string scattering is of the order of the string width, m^{-1}. He found that the effective damping time for the string velocity (assumed $\ll 1$) is

$$t_d \sim \mu m/NT^4 \sim G\mu m t^2, \tag{2}$$

where N is the equilibrium number of particle species at temperature T, t is the cosmic time, and I have used the relation $T \sim (NGt^2)^{-1/4}$. The damping time becomes of order t at

$$t \sim t_* \sim (G\mu m)^{-1}. \tag{3}$$

At $t \gg t_* \sim 10^{34}$ s friction can be neglected and the strings acquire relativistic speeds.

The cosmological red shift of oscillations has been discussed in Ref. 8. It has been shown that the amplitude of small perturbations of wavelength λ on a straight string grows like $a(t)$ for $\lambda > t$ and remains constant for $\lambda < t$, while the wavelength always grows like $a(t)$. Here $a(t)$ is the cosmic scale factor. If we assume that large perturbations behave in a similar manner, then the strings are conformally stretched by the expansion on scales greater than the horizon and are straightened out on scales smaller than the horizon. In other words, the persistence length of the strings at time t is of order t.

The effect of closed-loop formation is expected to be of a comparable magnitude.[2,4] Loop formation, as well as gravitational radiation, can only speed up the straightening process. However, these mechanisms are effective only on scales smaller than the horizon and we conclude that the persistence length at $t > t_*$ is of order t. Then the energy density of the strings is

$$\rho_s \sim \mu t^{-2}. \tag{4}$$

The total energy density of the universe is $\rho \sim 1/Gt^2$, and thus

$$\rho_s/\rho = G\mu. \tag{5}$$

For grand unification strings $G\mu \sim 10^{-6}$. It should be noted that ρ_s does not include the contribution of closed loops smaller than the horizon.

Let us now discuss the evolution of closed loops. Large loops of size greater than the horizon behave in the same way as infinite strings: The loops are conformally stretched while small scale irregularities are smooth out. When a loop falls inside the horizon, it starts to collapse. Sufficiently large circular loops collapse to black holes (there seems to be nothing to prevent black hole formation if the Schwarzschild radius of the loop is greater than the width of the string: $GMm > 1$ or $M > 0.1$ g. Here M is the mass of the loop).

I have analyzed the behavior of small perturbations on a collapsing circular loop and have found that the amplitude of the perturbations remains constant, and thus deviations from circular shape grow like R^{-1}, where $R(t)$ is the radius of the loop. (Decreasing modes also exist, but to make all perturbations decrease would require a very special choice of initial conditions.) In the course of collapse, the radius of the loop changes by a factor of $G\mu \sim 10^{-6}$ and the loop remains approximately circular only if the initial perturbations are smaller than 10^{-6}. This suggests that the probability of black hole formation is very small.

Loops of irregular shape oscillate and lose their energy by gravitational radiation.[10] From time to time they can self-intersect and break into smaller pieces. If such self-intersections are frequent, then the loops rapidly deteriorate, eventually decaying into relativistic particles. The dynamics of the loops is not well studied and the frequency of self-intersections is difficult to estimate. In particular, it is possible that no self-intersections occur for a large class of initial conditions. Here we shall assume that self-intersections are rare enough, so that the gravitational radiation is the dominant energy-loss mechanism.

The energy loss by gravitational radiation for an oscillating loop is of the order $dM/dt \sim -GM^2 \times R^4\omega^6$, where M is the mass of the loop, ω is the frequency of oscillations, and R is the characteristic size of the loop. With $M \sim \mu R$ and $\omega \sim R^{-1}$ we find $dM/dt \sim -G\mu^2$. The lifetime of the loop is

$$\tau \sim M|dM/dt|^{-1} \sim (G\mu)^{-1}R. \tag{6}$$

Some closed loops are produced during the phase transition at $T \sim T_c$. Additional loops are produced by intersecting strings. The characteristic curvature radius of the strings at time t is $\sim t$ and we expect the typical size of the loops formed at that time to be also $\sim t$. (Large loops produced at earlier times are considered as "formed" when they fall within the horizon and start to oscillate. At that time their size is also $\sim t$.) We shall estimate the rate of loop formation assuming[2,4] that it is sufficient for straightening of the strings on scales smaller than the horizon (in other words, we assume that the effectiveness of loop formation is comparable to that of the cosmological red shift of oscillations). Then Eq. (4) yields $\mu t\, dn/dt \sim -d/dt(\mu t^{-2})$ or

$$dn/dt \sim t^{-4}. \tag{7}$$

This equation means that approximately one loop of size t per horizon scale is formed during the interval $\Delta t \sim t$. Equation (7) is basically of dimensional nature: The system of strings has no intrinsic scale. The density of loops with masses from M to $M+dM$ is given by

$$dn(t) \sim (a'/a)^3 t'^{-4} dt' \sim \mu^{-1} t^{-3/2} (M/\mu)^{-5/2} dM, \tag{8}$$

where $t' = M/\mu$ is the time of formation of the loops, $a \equiv a(t)$, $a' \equiv a(t')$, and I have used that in the radiation era $a(t) \sim t^{1/2}$. The mass spectrum (8) extends from $M_1(t) \sim G\mu^2 t$ to $M_2(t) \sim \mu t$. The lower cutoff is due to the finite lifetime of the loops [see Eq. (6)]. The total mass density of the loops is

$$\rho_L(t) \sim \int_{M_1}^{M_2} M(dn/dM)\, dM \sim (G\mu)^{-1/2} \mu t^{-2}$$

and

$$\rho_L/\rho \sim (G\mu)^{1/2}. \tag{9}$$

Note that Eqs. (8) and (9) apply only during the radiation era.

We now discuss the cosmological density fluctuations introduced by the strings. The density of infinite strings and closed loops greater than the horizon is too small to produce an interesting effect [see Eq. (5)]. We therefore concentrate on loops smaller than the horizon. Our analysis will be similar to that by Carr[11] who has studied the density fluctuations produced by primordial black holes. On scales greater than the horizon ($L > t$) the density fluctuations due to strings are balanced by the corresponding variations of the radiation density,[1,2,11] so that $\delta\rho_{L \gg t} = 0$. On scales smaller than the Jeans length, which in a radiation-dominated universe is of order t, the radiation density fluctuations are transformed into acoustical waves. The strings are not coupled to radiation and do not partake in the photon-gas oscillations. It can be shown[11,12] that the gravitational clustering of the loops is negligible until the decoupling time, $t_{\rm dec} \sim 10^{12} \Omega^{-1/2}$ s; thereafter the total density fluctuations grow like $t^{2/3}$. Here Ω is the density of the universe in terms of the critical density.

For estimations below we shall assume that $t_{\rm dec} \sim t_{\rm eq}$, where $t_{\rm eq}$ is the time of equal radiation and matter densities; this corresponds to $\Omega \sim 0.1$ and $t_{\rm dec} \sim 3 \times 10^{12}$ s.

At the time of decoupling, the mass distribution of the loops is

$$dn(t_{\rm dec}) \equiv \nu(M)\, dM \sim \mu^{-1} t_{\rm dec}^{-3/2} (M/\mu)^{-5/2} dM. \tag{10}$$

Under the assumption that the spatial distribution of the loops is random, the mass fluctuation on scale $L < t_{\rm dec}$ due to loops of mass $\sim M$ in the interval $\Delta M \sim M$ is

$$\delta\mu_M \sim [\nu(M) M^3 L^3]^{1/2} \propto M^{1/4}. \tag{11}$$

This implies that the dominant contribution to $\delta\mu$ is given by the largest loops which the region under consideration can be expected to contain: $\delta\mu \sim M_{\max}$, where $\nu(M_{\max}) M_{\max} L^3 \sim 1$. The total mass of matter on scale L is $\mu \sim \rho_{\rm dec} L^3 \sim L^3/G t_{\rm dec}^2$, and we obtain

$$(\delta\mu/\mu)_{\rm dec} \sim M_{\max}/\mu \sim G\mu (t_{\rm dec}/G\mu)^{1/3}. \tag{12}$$

Objects of mass μ bind at $t \sim t_B$ when $\delta\mu/\mu \sim 1$:

$$t_B \sim t_{\rm dec} (\delta\mu/\mu)_{\rm dec}^{-3/2} (G\mu)^{-3/2} (G\mu t_{\rm dec})^{1/2} \sim 10^3 (G\mu)^{-3/2} (\mu/M_\odot)^{1/2}. \tag{13}$$

For galactic mass scales ($\mu \sim 10^{12} M_\odot$) to bind at $t \sim 10^{16}$ s, we must have $G\mu \sim 10^{-5}$. According to Eq. (1), this corresponds to particle masses of the grand unification scale ($m \sim 3 \times 10^{15}$ GeV).

In the scenario just described, the galaxies condensate around oscillating loops of mass $M \sim 10^9 M_\odot$. Such loops are formed at $t \sim 10^9$ s and must have radiated away their energy by $t \sim 10^{14}$ s. By that time the matter density fluctuations are large enough and continue to grow independently.

The cosmological scenario described here is different from that suggested by Zel'dovich[2] who was the first to discuss vacuum strings as a possible source of density fluctuations. He envisions strings formed on a mass scale even higher than the grand unification scale, so that $\rho_s/\rho \sim G\mu \sim 10^{-3}$ ($m \sim 3 \times 10^{16}$ GeV) and argues that in this case the density of infinite strings is sufficient to produce fluctuations of required magnitude. Such a situation may be desirable if, for some reason, the rate of closed-loop formation is much smaller than estimated in the present paper (for example, if the probability of changing partners by intersecting strings is very small) or if the lifetime of the loops is very short (e.g., because of self-intersections). Then the density fluctuations on a comoving scale L are produced at $t \sim L$ in the form of acoustic waves. Zel'dovich assumes that the amplitude of the fluctuations, $\delta\rho/\rho$, is of the order $\rho_s/\rho \sim 10^{-3}$.

Finally, it should be noted that the string formation at the grand unification phase transition is possible only if the manifold of the degenerate vacua of the gauge theory is not simply connected. A discussion of the conditions for the existence of vacuum strings can be found in Refs. 4, 7, 9, 13, and 14. The results of the present paper suggest that theories leading to strings may be preferred, since they give a natural explanation for the galaxy formation.

[1]P. J. E. Peebles, Astron. Astrophys. 32, 391 (1974); W. H. Press and E. T. Vishniac, Astrophys. J. 239, 1 (1980).

[2]Y. B. Zel'dovich, Mon. Not. R. Astron. Soc. 192, 663 (1980).

[3]D. A. Kirszhnits and A. D. Linde, Phys. Lett. 42B, 471 (1972).

[4]T. W. B. Kibble, J. Phys. A 9, 1387 (1976).

[5]Y. B. Zel'dovich, I. Yu. Kobzarev, and L. B. Okun, Zh. Eksp. Teor. Fiz. 67, 3 (1974) [Sov. Phys. JETP 40, 1 (1975)].

[6]A. Vilenkin, Phys. Rev. D 23, 852 (1981).

[7]T. W. B. Kibble, Phys. Rep. 67, 183 (1980).

[8]A. Vilenkin, Phys. Rev. D, to be published.

[9]A. E. Everett, unpublished.

[10]Another energy-loss mechanism is the gravitational drag due to the gravitational field of the density perturbations left behind a moving string. It can be shown that this effect is neglible compared to that of gravitational radiation.

[11]B. J. Carr, Astron. Astrophys. 56, 377 (1977).

[12]P. Meszaros, Astron. Astrophys. 37, 225 (1974).

[13]A. M. Polyakov, Zh. Eksp. Teor. Fiz. 68, 1975 (1975) [Sov. Phys. JETP 41, 988 (1975)].

[14]A. S. Schwarz and Yu. S. Tyupkin, Phys. Lett. 90B, 135 (1980).

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NATURE OF MELTING AND SUPERIONICITY IN ALKALI AND ALKALINE-EARTH HALIDES. L. L. Boyer [Phys. Rev. Lett. 45, 1858 (1980)].

The calculated shear elastic instability at $a = 5.87$ Å in CaF_2 was mistakenly identified as the vanishing of $C_{11} - C_{12}$ when actually it was the vanishing of C_{44}. Therefore, on page 1860 beginning with the paragraph, "For CaF_2 the present theory..." and in all subsequent discussion, the expression, $C_{11} - C_{12}$, should be replaced by C_{44}. Clearly, none of the results or conclusions of the paper are affected by this labeling error.