Transverse Effects in Superfluorescence

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Dynamic diffraction coupling is examined in superfluorescence with use of a semiclassical model in which diffraction and transverse density variations are rigorously included. The Cs data are correctly simulated for the first time.

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Superfluorescence[1] (SF) is the process by which coherent emission occurs from an ensemble of two-level atoms all intially in the upper state. An important question in SF experiments is why the output pulse is sometimes smooth, but at other times exhibits multiple structure or ringing. Strong ringing or pulsing has been observed by several groups, including the initial HF-gas studies.[2] Recent Cs experiments,[3] however, never show ringing at low densities, whereas at higher densities, highly fluctuating multiple pulsing is usually observed, and is believed to arise from transverse-mode competition. Strong Burnham-Chiao ringing[4] is predicted by plane-wave models,[5] which neglect variations transverse to the propagation direction. We find that inclusion of transverse effects, both *spatial averaging* and Laplacian *diffraction*, substantially alters the one-dimensional Cs predictions,[3b] leading to greater conformity with the Cs data.

The initial SF state is prepared by rapidly inverting a sample of three-level atoms by transferring population from the ground state to the upper state with a short light pulse, creating a cylindrical region of excited atoms.[2] SF pulse emission subsequently occurs between this excited state and the intermediate state. There is no optical cavity and stray feedback is negligible.

This study employs the semiclassical approach to explore the influence of transverse effects, using the average value[6] of the initial tipping angle.[4,5a] Both longitudinal fluctuations[6] and transverse flucutations, as influenced by diffraction, will be discussed elsewhere.

Transverse effects are expected to influence the pulse shapes in at least two ways, one of which is *spatial averaging.* In SF experiments the initial inversion density $n_0(r)$ is radially dependent since the pump light pulse typically has a Gaussian-like profile.[7] In the absence of diffraction this cylinder can be thought of as a set of concentric cylindrical shells, each with its own density, tipping angle, and delay time.[8] The radiation will be a sum of plane-wave intensities; when the entire output signal is viewed the ringing averages out, resulting in an asymmetric pulse with a long tail.[9]

A second transverse effect, *diffraction,* causes light emitted by one shell to affect the emission from adjacent shells. This coupling mechanism, which causes transverse energy flow, is more important for samples with small Fresnel numbers F.

SF is inherently a transverse-effect problem even for large-F samples since the off-axis modes are not discriminated against. This work is the first to correctly include this crucial element.

Our analysis adopts the coupled Maxwell-Schrödinger equations, which fully take into account propagation and transverse effects. Previous approaches examined transverse effects in the mean-field approximation[10] or included a loss term in the Maxwell equation to describe diffraction.[2,5] Thus our model possesses a long sought for degree of realism.[11]

The simulations are based upon an extension of a model[12] which describes transverse effects observed in self-induced transparency experiments.[13] For simplicity the influence of the backward wave, which is negligible,[14] is not considered, and cylindrical symmetry is assumed. The equations of motion are[12]

$$\partial\xi/\partial z - i(4FL)^{-1}\nabla_T^2\xi = (4\pi^2/\lambda)\mathcal{P}, \tag{1a}$$

$$\partial\mathcal{P}/\partial\tau + \mathcal{P}/T_2 = (\mu^2/\hbar)n\xi, \tag{1b}$$

$$\partial n/\partial\tau + n/T_1 = -\mathrm{Re}(\mathcal{P}\xi^*/\hbar), \tag{1c}$$

where ξ and $\mathcal{P}$ are the slowly varying complex amplitudes of the electric field and polarization, respectively, n is the inversion density, $\tau = t - z/c$ is the retarded time, μ is the transition dipole moment matrix element, and T_1 and T_2 are the population relaxation and polarization dephasing times. Diffraction is taken into account by the Laplacian term $\nabla_T^2\xi = (1/\rho)(\partial/\partial\rho)\rho\,\partial\xi/\partial\rho$, where $\rho = r/r_p$, with Fresnel number $F = \pi r_p^2/\lambda L$, r_p is the radius of the initial inversion density at half maximum, and L is the sample length. The boundary conditions are $\partial\xi/\partial r = 0$ on the axis ($r = 0$) and at $r = \infty$. To insure that (1) the entire field is accurately simulated, (2) no artificial reflections are introduced at the numerical boundary $r_m \gg r_p$, and (3) fine diffraction variations near the axis are resolved, the sample cross section is divided into nonuniform cells, and is surrounded by an absorbing shell.

Equations (1) are numerically integrated subject to the initial conditions $n = n_0\cos\theta_0$ and $\mathcal{P} = \mu n_0 \sin\theta_0$, which correspond to an initial tipping angle θ_0. The initial inversion density in the experiment is radially dependent; r dependence of n_0 and/or θ_0 is allowed for in the computations.

Figure 1(a) displays results where spatial averaging is present but diffraction is absent, by setting $F = \infty$ in Eq. (1a). In this figure the emitted power of SF pulses is plotted for samples with uniform and Gaussian profiles of $n_0(r)$ and $\theta_0(r)$. First, we study ringing reduction due to spatial averaging of independent concentric shells, each emitting in a plane-wave fashion. The case in which θ_0 and n_0 are both constant (curve i), the uniform plane-wave limit, exhibits strong ringing.[4,5] In curve ii, in which n_0 is Gaussian $\{n_0(r) = n_0^0\exp[-\ln 2(r/r_p)^2]\}$ and θ_0 is uniform, the ringing is largely averaged out, resulting in an asymmetric pulse with a tail. An essentially identical result (curve iii) is obtained for the case in which n_0 and θ_0 are both Gaussian $\{\theta_0 = \theta_0^0 \times \exp[0.5\ln 2(r/r_p)^2]\}$, showing that the ringing is predominantly removed by a Gaussian n_0 regardless of the radial dependence of θ_0. This is expected, since the output-pulse parameters are all dependent only on $|\ln\theta_0|$.[8] As shown in Fig. 1(b), with uniform n_0 and θ_0 but with diffraction included, the output pulse is almost symmetrical

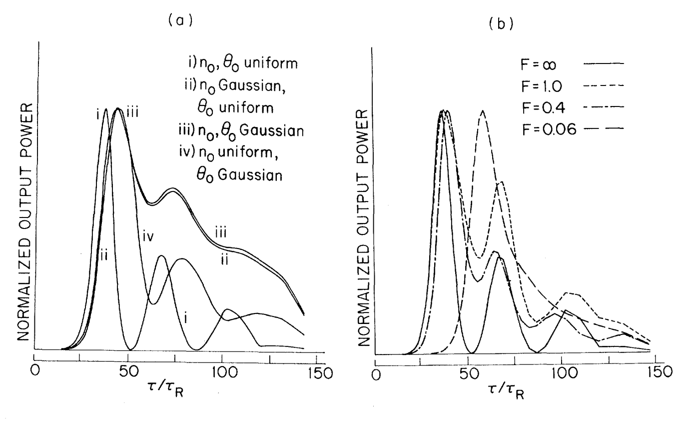

FIG. 1. (a) Normalized SF output power vs τ/τ_R, $\tau_R = \hbar\lambda/4\pi^2\mu^2 n_0^0 L = 8\pi\tau_0/3n_0^0\lambda^2 L$. ($\tau_R$ is the same as that defined in Ref. 5a. It appears smaller by a factor of 3 because it uses the "partial" radiative lifetime τ_0 instead of the observed one, T_{sp}.) $\theta_0^0 = 2\times 10^{-4}$, $T_1 = T_2 = T_2^* = \infty$, $L/c\tau_R = 3.9$, and $F = \infty$ (see text). (b) Same as (a) but with diffraction included and uniform $n_0(r)$ and $\theta_0(r)$.

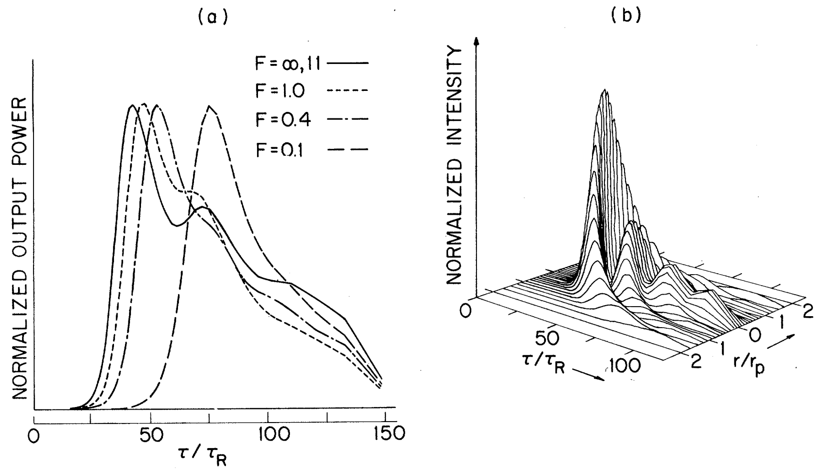

FIG. 2. Influence of diffraction on SF pulse shapes. Parameters are the same as in Fig. 1(a), with n_0 Gaussian and θ_0 uniform. (a) Emitted power; (b) isometric graph of intensity for the $F=1$ case of (a).

and also nearly free of ringing for $F \leqslant 0.4$.

Figure 2(a) studies the effect of diffraction on the SF pulse shapes by varying F, with use of a Gaussian n_0 as in Fig. 1(a), curve ii. Reducing F curtails the oscillatory structure and makes the output pulses more symmetrical, since the outer portions of the gain cylinder are stimulated to emit earlier because of diffraction from the inner portions. Thus diffraction becomes more important as F decreases.

Figure 2(b) is an isometric graph of the intensity buildup for a sample with $F=1$. The radial variations of intensity peaks, delay, and ringing illustrate how different gain shells contribute independently to the net power. Each shell exhibits a different Burnham-Chiao ringing pattern. Accordingly, their contributions to the net signal interfere and reduce the ringing. However, the central portion of the output pulse should exhibit strong plane-wave ringing. In fact, the ringing observed in the HF-gas experiments[2] may have been just that, since the detector viewed a small area in the near field of the beam.

Figure 3 compares the normalized Cs SF data of Refs. 3 and 11b (for which $F \simeq 0.7$ with uncertainty ranging from 0.35 to 1.4) to the theory (including relaxation terms). The data were fitted with use of a Gaussian n_0 and a uniform θ_0 with nominal value[6] $\theta_0 = 2(n_0{}^0 \pi r_p{}^2 L)^{1/2}$, $n_0{}^0$ being adjusted to yield the observed delays (1.6–2.8 times the experimental n_0 values). However, in Ref. 3 the curve published at each density was the one with the *shortest* delay. The *average* delay is ~30% greater at each density.[15] Thus the *effective* ratios of our computed densities to the experimental ones range from 1.2 to 2.2, compared with the +60%, −40% quoted experimental uncertainties.

The quantum calculations[6] actually yield $\theta_0 = (2/\sqrt{N})[\ln(2N)^{1/8}]^{1/2}$, a 9% correction which further reduces the range to 1.14–2.0. If one sets $\theta_0 = 6/\sqrt{N}$, as suggested by the small injection experi-

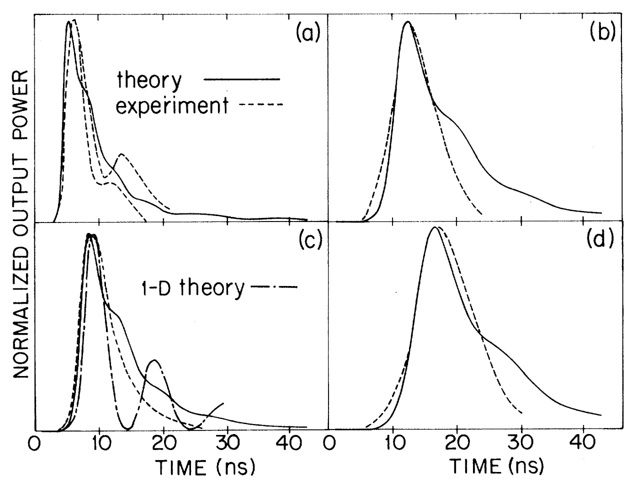

FIG. 3. Theoretical fits to Cs data of Ref. 3. The two dashed-line curves in (a) indicate typical experimental shot-to-shot variations. $F=1$, $L=2$ cm, $T_1 = 70$ ns, $T_2 = 80$ ns, $\lambda = 2.931\ \mu$m, $\tau_0 = 551$ ns, θ_0 is uniform or Gaussian, and $n_0(r)$ is Gaussian. The following give $\theta_0{}^0$(fit), $n_0{}^0$(fit), $n_0{}^0$(exp), with $\theta_0{}^0$ in units of $10^{-4}/\mathrm{cm}^3$ and $n_0{}^0$ in units of $10^{10}/\mathrm{cm}^3$: (a) 1.07, 31, 19; (b) 1.37, 18, 7.6; (c) 1.69, 11.9, 3.8; (d) 1.96, 8.85, 3.1. The broken-line curve in (b) is the one-dimensional fit of Ref. 3b, with $\theta_0{}^0 = 1.69$ and $n_0{}^0 = 12$.

ment,[15] the range is 1–1.8, in still better agreement.

The calculated shapes are in good agreement with the data, and are within the range of shot-to-shot fluctuations [Fig. 3(a)]. The only discrepancy is that the simulations predict more of a tail than observed in the experiments. For comparison, Fig. 3(b) also plots the fit in Ref. 3b of the one-dimensional Maxwell-Schrödinger theory.[4] As can be seen, the present theory gives a more accurate fit, illustrating the necessity of including transverse effects. The pulse tails are further curtailed by reducing F within the range of experimental uncertainties[11b] (which used a $1/e$ rather than a half width at half maximum definition of r_p). Note that often a Fresnel number F', defined as $r_p^2/\lambda L$, is used; diffraction effects become important when $F'=1$ (i.e., when $F=0.36$).

In conclusion, SF experiments are described much more accurately by including transverse effects. Our calculations do not include short-scale-length phase and magnitude fluctuations in θ_0, which result in multiple transverse-mode initiation of the SF process, leading to multidirectional output emission with hot spots. This effect, which is expected to be important only for large-F samples (since diffraction singles out a smooth phase front in small-F samples), is under study.

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[1]This effect if also known as *superradiance* and *Dicke superradiance*, although these terms are also used to describe coherent emission from samples with initial polarization.

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[8]See Ref. 5a, Eq. (49); J. A. Hermann, Phys. Lett. 69A, 316 (1979); McCall (Ref. 4); Burnham and Chiao (Ref. 4); Polder, Schuurmans, and Vrehen (Ref. 6).

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[15]Footnote 13 of Q. H. F. Vrehen and M. F. H. Schuurmans, Phys. Rev. Lett. 42, 224 (1979).