Nonstatic Spin-Isospin Order in Light Nuclei

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(Received 19 September 1980)

A semimicroscopic model of the nucleus is presented in which spin-isospin order is realized as zero-point one-dimensional oscillations of spin-up protons and spin-down neutrons against spin-down protons and spin-up neutrons. This phase is favored by the one-pion-exchange potential in light deformed nuclei. The model is characterized by the lowering of the energy of the first excited state and enhancement of the $B(M2)$.

PACS numbers: 21.60.-n, 21.60.Ev

It has been suggested[1] that the one-pion-exchange (OPE) potential should give rise to a spin-isospin ordered phase. In infinite nuclear matter such an order requires some localization of nucleons. In a number of specific models[2] localization takes place along one direction only, the direction of spin quantization, giving rise to a laminated structure where spin and/or isospin alternate their orientation going from one layer to the next one.

The actual occurrence of this phase depends on the balance between the kinetic energy increase necessary to localize nucleons, the attraction coming from the OPE potential, and the change in the short-range interaction energy. This last effect is most difficult to evaluate and makes uncertain the determination of the critical density. It is therefore hard to establish theoretically whether this ordered phase is actually present in nuclei.

We want to explore the possibility of a realization of spin-isospin order in nuclei which does not require localization of nucleons in layers. For nuclei finite it is in fact sufficient to correlate nucleons with different spin-isospin in an appropriate way in order to get a nonvanishing contribution from the OPE potential. Such a correlation can be realized by allowing protons with up spin and neutrons with down spin to oscillate with respect to protons with down spin and neutrons with up spin. Spin-isospin order is thus obtained in the zero-point motion.

Oscillations can occur in one, two, or three dimensions. In the first case the spin can be quantized along the direction of oscillation or along a perpendicular direction, and analogous possibilities exist in the second case. Moreover, for deformed nuclei, which we assume for simplicity to have axial symmetry, additional possibilities arise according to the way in which the symmetry axis is related to the directions of oscillation and of spin quantization.

We have found that one-dimensional oscillations are always favored in energy, provided that their direction coincides with the direction of spin quantization. The possibilities are then reduced to oscillations and spin quantization along the symmetry axis or along a perpendicular direction. We will describe in some detail the first case, the second one being obtained by obvious modifications.

In analogy with the procedure adopted in Refs. 1 and 2, we introduce displaced creation operators

$$(C_{a\sigma_3\tau_3}')^\dagger = \exp[-(i/2\hbar)\sigma_3\tau_3 d p_z] C_{a\sigma_3\tau_3}{}^\dagger \exp[(i/2\hbar)\sigma_3\tau_3 d p_z], \tag{1}$$

where we label by a the spatial quantum numbers. Out of the operators (1) we construct a Slater determinant

$$\Lambda = \prod_{h,\sigma_3,\tau_3} (C_{h\sigma_3\tau_3}')^\dagger |0\rangle, \tag{2}$$

where h denotes occupied states.

Consistently with the traditional approach to the macroscopic description of collective vibrations,[3] we redefine d as a collective variable and assume a total wave function of the form

$$\Psi_n = \Phi_n(d)\Lambda(d). \tag{3}$$

The collective Hamiltonian in the harmonic approximation is

$$H = P_d^2/2M + \tfrac{1}{2}(C + K_{\rm OPE})d^2, \tag{4}$$

where M is the reduced mass, $K_{\rm OPE}$ the contribution to the restoring constant coming from the OPE, and C the contribution coming from the other components of the N-N potential.

In order to establish whether spin-isospin order in the zero-point motion is favored, we should compare the energy of this phase with the one of other possible ordered phases, like, for instance, pure isospin order, and of the disordered phase. A proper treatment of short-range correlations, however, is so difficult that this is practically out of the present possibilities. We therefore confine ourselves to the study of the characteristic predictions of the model which can be tested experimentally. Before doing this we evaluate $K_{\rm OPE}$. A necessary condition for spin-isospin order to be favored in the zero-point oscillation is in fact that $K_{\rm OPE}$ be negative.

The direct part of the expectation value of $V_{\rm OPE}$ is

$$\langle\Lambda|V_{\rm OPE}|\Lambda\rangle_D = \tfrac{1}{2}\int d^3r_1\int d^3r_2\langle\Lambda|S_{33}(\vec{r}_1)|\Lambda\rangle\langle\Lambda|S_{33}(\vec{r}_2)|\Lambda\rangle[V_c(r_{12}) + 3(z_{12}^2/r_{12}^2 - 1)V_T(r_{12})], \tag{5}$$

where

$$S_{ik} = \bar{\psi}\tau_i\sigma_k\psi \tag{6}$$

is the spin-isospin density operator and V_c and V_T are the central and tensor terms of the OPE. The contact term of V_c will be omitted. For a discussion of this point, see Palumbo.[4]

The average value of S_{33} is

$$\langle\Lambda|S_{33}|\Lambda\rangle = \sum_{\tau_3,\sigma_3}\tau_3\sigma_3\rho_{\tau_3\sigma_3}, \tag{7}$$

where

$$\rho_{\tau_3\sigma_3}(\vec{r},d) = \sum_h \varphi_{h\sigma_3\tau_3}{}^*(\vec{r},d)\varphi_{h\sigma_3\tau_3}(\vec{r},d), \tag{8}$$

$$\varphi_{h\sigma_3\tau_3}(\vec{r},d) = \exp[(-i/\hbar)\tau_3\sigma_3 p_z d]\varphi_{h\tau_3\sigma_3}(\vec{r}). \tag{9}$$

We disregard the contribution to $K_{\rm OPE}$ coming from the exchange term according to the estimates of Refs. 1 and 2 in nuclear matter. For simplicity we assume $N = Z$ and approximate the one-body density by a Gaussian of the form

$$\rho_{\tau_3\sigma_3}(\vec{r},d) = \rho_0\exp[-r_\perp^2/R_1^2 - (Z - \tfrac{1}{2}\sigma_3\tau_3 d)^2/R_3^2]. \tag{10}$$

R_1 and R_3 are related to the deformation parameter δ and the nuclear radius R through

$$R_1 = [2/5(1+\delta)]^{1/2}R;\quad R_3 = [2/5(1-\delta)]^{1/2}R. \tag{11}$$

We can now expand $\langle V_{\rm OPE}\rangle_D$ to first order in δ and second order in d and obtain

$$\langle V_{\rm OPE}\rangle_D \simeq \tfrac{1}{2}K_{\rm OPE}d^2 = \tfrac{1}{2}(K_0 + K_1\delta)d^2, \tag{12}$$

where

$$K_i = \frac{1}{4(5\pi)^{1/2}}\frac{v_0}{\mu}\frac{A^2}{R^3}\int_0^\infty dt\exp\left(-\frac{2}{\sqrt{5}}\mu R t - t^2\right)t p_i(t),\quad p_0 = 1 - 2/(2\mu R)^2 - [8/\sqrt{5}(2\mu R)]t - \tfrac{6}{5}t^2,$$
$$p_1 = 18/(2\mu R)^2 - \tfrac{1}{4} + [18/\sqrt{5}(2\mu R)]t + [1.7 - 40/7(2\mu R)^2]t^2 - \tfrac{8}{7}(\sqrt{5}/2\mu R)t^3 - \tfrac{18}{35}t^4,\quad t = \tfrac{1}{2}\sqrt{5}(r_{12}/R), \tag{13}$$

and μ = pion mass.

Evaluation of the above formulas shows that $K_{\rm OPE}$ is most attractive for not too heavy ($A < 60$) oblate nuclei. This cirtical value of A might change if a more realistic form for the density is used. We think, however, that the values of $K_{\rm OPE}$ obtained with the Gaussian approximation are reliable for light nuclei, as confirmed by preliminary results obtained for ^{12}C with a Slater determinant.

Analogous expressions are obtained for the case of oscillations and spin quantization along a direction perpendicular to the symmetry axis. In this case $K_{\rm OPE}$ is most attractive for not too heavy ($A < 60$) prolate nuclei. Numerical results for $A = 12$ and $A = 28$ are reported in Table I.

We now need an estimate of the part of the restoring force independent of spin and isospin. This is common to all the collective vibrations and can therefore be extracted, for instance, by a phenomeno-

TABLE I. Numerical results for $A = 12$ and 28. 1 W.u. is a Weisskopf unit.

A	δ	K_{OPE} (MeV fm^{-2})	$\hbar\omega$ (MeV)	$B(M2;0\leftarrow 0)$ (W.u.)	$B(M2;0\rightarrow 2)$ (W.u.)
12	−0.4	−13	12	21	0
	+0.4	−13	12	5	16
28	−0.4	−10	11	31	0
	+0.4	−9	11	8	23

logical analysis of the excitation energy of the electric-dipole giant resonance based on the assumption that the energy-weighted sum rule is exhausted by a single excitation mode, as reported by Bohr and Mottelson,[5] who obtain $C \sim 41A^{-5/3}$ MeV fm^{-2}. We must rescale this value by $(\frac{1}{2}A)^2$ because of a different definition of the mass parameter. The excitation energy $\hbar\omega$ reported in the table is therefore considerably lower than the energy of the electric-dipole giant resonance.

The first excited state is characterized by enhanced $M2$ transition probabilities. In fact,

$$B(\lambda, I=K=0 \to I,K) = [2/(1+\delta_{K0})]\,|\langle\Phi_{1K}|\mathfrak{M}(\lambda=I,\nu=K)|\Phi_0\rangle|^2, \tag{14}$$

where

$$\mathfrak{M}(I,K) = \langle\Lambda|M(I,K)\Lambda|\rangle \sim (i/2\hbar)d \sum_{h,\sigma_3,\tau_3} \sigma_3\tau_3\langle\varphi_{h\sigma_3\tau_3}|[p_3, M(I,K)]|\varphi_{h\sigma_3\tau_3}\rangle. \tag{15}$$

It is easy to show from this equation that the lowest nonvanishing multiple operator is the magnetic quadrupole one, whose expression is

$$\mathfrak{M}(M2,K) \simeq \frac{A}{8}\left(\frac{5}{\pi}\right)^{1/2}(g_p - g_n)d\,\frac{e\hbar}{2mc} \times \begin{cases} \delta_{k0} \\ -\frac{1}{2}\delta_{k0} + \frac{3}{8}\delta_{k2} \end{cases}, \tag{16}$$

where the upper case holds for oscillations along the symmetry axis and the lower case for oscillations along a perpendicular direction. Numerical values are given in the table.

Enhanced transition probabilities of different multipolarity to other states are also allowed. These can occur either because the excited states have a different order or because a different order coexists in the ground state.

The average value of the isospin in the ground state is given by $\langle T^2\rangle \sim 1$, and isospin seems to be broken as for static spin-isospin order.[6] In the present case, however, this is an artifact of the semiclassical approximation. In the same way one would obtain $\langle T^2\rangle \sim 1$ in the ground state of the Goldhaber-Teller model. This artifact is avoided in a fully microscopic formulation like the random-phase approximation, whose close relation to semiclassical descriptions is well known.[3]

In static spin-isospin order, isospin breaking is related to a nonvanishing average value of the pion field (pion condensation[7]), while in the present model the average value of the pion field vanishes due to the nonstatic character of the spatial correlation. In fact, the pion field is given by

$$\varphi_3(\vec{r}) \propto \langle d\rangle(\Delta + \mu^2)^{-1}\partial_3\rho_{\tau_3\sigma_3}(\vec{r},0),$$

and $\langle d\rangle = 0$. Our model is characterized by a nonvanishing average value of $\varphi_3^2 \propto \langle d^2\rangle$.

Needless to say, our numerical estimates can only be indicative. However, the predictions concerning the character of the deformation, the lowering of excitation energy, and the enhancement of the $B(M2)$ can be taken, in our view, as qualitatively significant.

We complete our analysis by mentioning that an attractive value of K_{OPE} is also obtained for light spherical nuclei. For O^{16}, for instance, $K_{OPE} \sim -5$ MeV fm^{-2}. This smaller value is due to the fact that the tensor force does not contribute in this case.

Let us conclude with a few remarks. First, other spin-isospin zero-point oscillations can coexist in the ground state, for instance, a zero-point breathing mode of spin-up protons and spin-down neutrons against spin-down protons and spin-up neutrons. This kind of correlation would presumably enhance $M1$ transitions. It seems, therefore, that the signature of nonstatic spin-isospin order is the lowering of the energy and the enhancement of the electromagnetic transition amplitudes of the levels of unnatural parity. This is considered a precursor to pion condensation,[8] i.e., to static spin-isospin order. Precursor phenomena are usually studied in the framework of the random-phase approximation, whose close relation to our semiclassical description has already been emphasized. The exact connection between precursor phenomena and the present model, however, is not straightforward and will be investigated separately. We further observe that correlations other than spin-isospin will occur in the ground state. Coexistence of different zero-point modes will, of course, reduce the transition amplitudes relative to our estimates.

Let us finally note that K_{OPE} is strongly increasing with the density. For instance, K_{OPE} increases from -13 to -19 MeV fm^{-2}, increasing the density by 20% in ^{12}C. Spin-isospin oscillations can become more pronounced with increasing density, and therefore acquire relevance to heavy-ion collisions.

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Energy Dependence of the Coupling Potentials in (p,n) Reactions

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(Received 20 November 1980)

We investigate the physical origin of the energy dependence of the isovector potentials V_τ and $V_{\tau\sigma}$ within a model which includes the one-pion and one-ρ—exchange potential as well as the second-order effects of the tensor force.

PACS numbers: 21.30.+y, 25.40.Ep

In the past year, new experimental information has been obtained on collective spin resonances in nuclei using highly energetic protons in (p,n) reactions.[1,2] The physical origin of this new development is connected with a strong energy dependence of the effective V_τ-coupling potential which has been found in those experiments. At incident proton energies below 50 MeV the "electric" charge-exchange resonances, like the well-known isobaric-analog resonances (IAR), which are connected with the isospin operator $\vec{\tau}$ only, dominate the experimental cross sections. The "magnetic" charge-exchange resonances, which are connected with the spin and isospin exchange

