PHYSICAL REVIEW LETTERS

VOLUME 46 | 20 APRIL 1981 | NUMBER 16

Direct Experimental Reconstruction of the *pp* Elastic-Scattering Matrix at 579 MeV

E. Aprile, C. Eisenegger, R. Hausammann, E. Heer, R. Hess, C. Lechanoine-Leluc, W. R. Leo, S. Morenzoni, Y. Onel, and D. Rapin

Département de Physique Nucléaire et Corpusculaire, Université de Genève, CH-1211 Genève 4, Switzerland

and

S. Mango

Schweizerisches Institut für Nuklearforschung, CH-5234 Villigen, Switzerland

(Received 30 December 1980)

We have made, for the first time, a direct reconstruction of the *pp* elastic-scattering matrix at 579 MeV from a series of experiments performed at the Schweizerisches Institut für Nuklearforschung polarized-beam line. Fifteen observables consisting of the polarization, two-spin correlation and transfer parameters, and three-spin parameters were measured at seven angles between 66° and 90° (c.m.). The experimental results and reconstructed amplitudes are presented and compared to phase shift analysis.

PACS numbers: 13.75.Cs

A reconstruction of the *pp* elastic-scattering matrix directly from experiment[1] offers many advantages because of its inherent freedom from any theoretical assumptions, other than the basic symmetry principles. Since elastic *pp* scattering is described by five complex amplitudes, an experimental determination up to an undetermined overall phase[2] would, in principle, require the measurement of nine observable quantities, i.e., the differential cross section and eight polarization parameters. However, the complexity of the expressions for the observables in terms of the amplitudes leads to ambiguities which require measurements of additional observables for their resolution. In the past, several authors have examined this problem from both mathematical[3] and experimental[4,5] points of view and have suggested a number of possible observable sets for measurement. One attempt at a direct reconstruction with use of extant data at 90° (c.m.) has also been made but the results show large errors and ambiguities.[6]

We report, for the first time, an unambiguous direct reconstruction of the *pp* elastic-scattering matrix at 579 MeV from a series of experiments performed at the Schweizerisches Institut für Nuklearforschung (SIN) polarized-beam line. Fifteen observables were measured at seven angles between 66° and 90° (c.m.). These observables include the polarization P, the two-spin correlation parameters A_{00nn}, A_{00ss}, A_{00ks}, and A_{00kk}, and the two-spin polarization transfer and three-spin tensors D_{0n0n}, K_{n00n}, $D_{\omega 0s0}$, $K_{0\omega s0}$, $D_{\omega 0k0}$, $K_{0\omega k0}$, $M_{\omega 0sn}$, $N_{0\omega sn}$, $M_{\omega 0kn}$, and $N_{0\omega kn}$, where we have used the notation of Bystricky *et al.*[7] The experimental details of these measurements may be found elsewhere.[8-10] The results are shown in Figs. 1 and 2. The polarization index ω stands for the final transversal direction $\hat{\omega}$ as analyzed in the polarimeter. This direction is a mixture of the components s' and k' which results from the precession of the outgoing proton polarization vector about the vertical 25-kG field of the polarized target. A sixteenth observable, the differ-

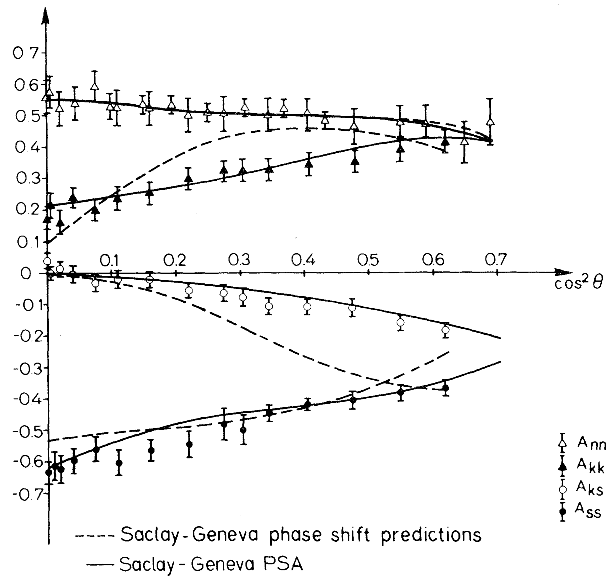

FIG. 1. Two-spin correlation parameters. Errors include statistical and systematic uncertainties.

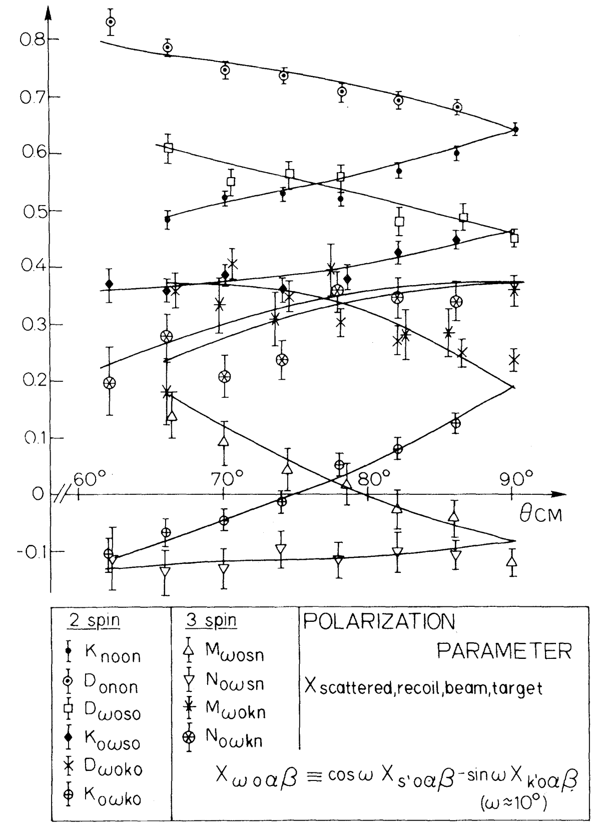

FIG. 2. Two-spin polarization transfer and three-spin parameters. Solid lines are the phase-shift fits of Bystricky *et al.* (Saclay-Geneva) after introduction of these data. Errors are purely statistical.

ential cross section, was used as a common normalization of the amplitudes (see Table I). Since this has been widely measured, its value was taken from the energy-dependent phase-shift analysis of Bystricky *et al.*[11]

For convenience, the *pp* elastic-scattering matrix of Ref. 7,

$$M(E,\theta)=\tfrac{1}{2}\{(a+b)+(a-b)(\vec{\sigma}_1\cdot\hat{n})\otimes(\vec{\sigma}_2\cdot\hat{n})+(c+d)(\vec{\sigma}_1\cdot\hat{m})\otimes(\vec{\sigma}_2\cdot\hat{m})$$
$$+(c-d)(\vec{\sigma}_1\cdot\hat{l})\otimes(\vec{\sigma}_2\cdot\hat{l})+e[(\sigma_1\otimes 1_2+1_1\otimes\vec{\sigma}_2)\cdot\hat{n}]\},\qquad(1)$$

was used since all observables are explicitly evaluated and given therein along with the transformations to equivalent formalisms. Equation (1) is the most general expression of an elastic-scattering matrix for two identical spin-$\frac{1}{2}$ particles subject to invariance under space rotations, parity, and time reversal. It is parametrized by the five complex amplitudes a, b, c, d, and e, which are functions of two kinematic variables, e.g., reaction energy E and center-of-mass angle θ. They contain all dynamical information concerning the process. Furthermore, application of the Pauli principle shows that these amplitudes need only be determined between 0° and 90° (c.m.).

In calculating the experimental observables one finds that they are bilinear expressions of these amplitudes. Moreover, the differential cross section

$$\sigma=\tfrac{1}{2}(|a|^2+|b|^2+|c|^2+|d|^2+|e|^2)\qquad(2)$$

normalizes the polarization parameters, e.g.,

$$P=\sigma^{-1}\operatorname{Re}(a^*e),\quad D_{\omega0s0}=\sigma^{-1}[\operatorname{Re}(a^*b)\cos(\alpha+\tfrac{1}{2}\theta)+\operatorname{Re}(c^*d)\cos(\alpha-\tfrac{1}{2}\theta)-\operatorname{Im}(b^*e)\sin(\alpha+\tfrac{1}{2}\theta)],\qquad(3)$$

where $\alpha=\frac{1}{2}\theta-\theta_{\rm lab}+\omega$. The precession angle ω can be added to the Wigner rotation since their rotation axes are the same[7] (the vertical axis $\hat{n}$).

At each angle the amplitudes were determined in a polar representation, e.g., $a=|a|\exp(i\varphi_a)$ where the phases are taken relative to e with the overall phase fixed by the condition $\varphi_e=0$. A minimum χ^2 fit was performed over our fifteen polarization measurements for the eight variables $|a|$, $|b|$, $|c|$, $|d|$, φ_a, φ_b, φ_c, and φ_d, with e kept fixed. The differential cross section was then used after the fit to de-

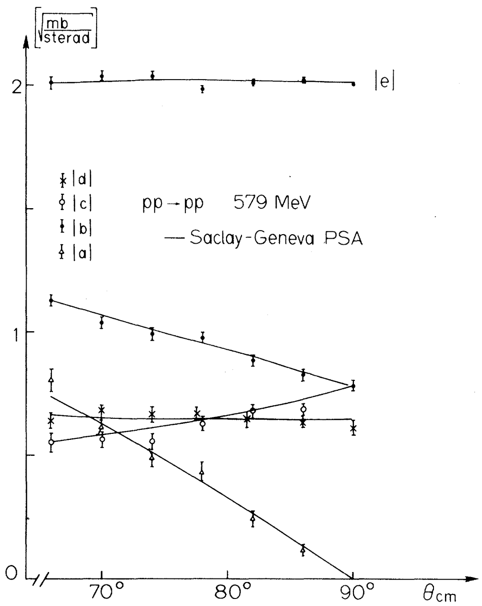

FIG. 3. Absolute values of the amplitudes.

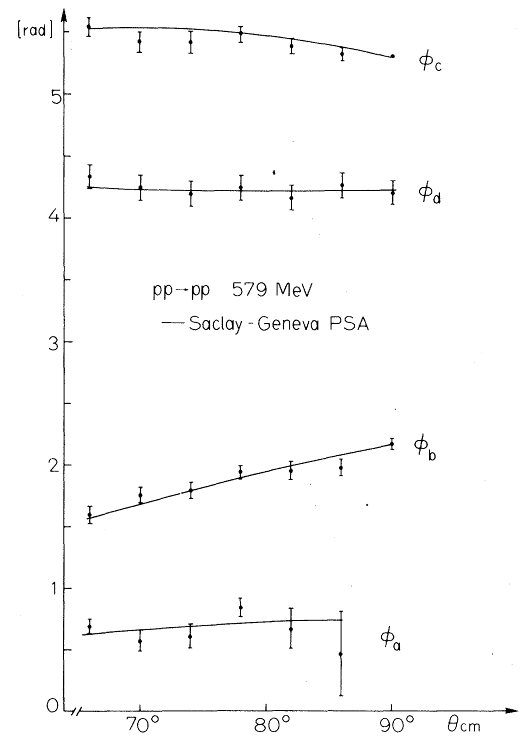

FIG. 4. Phases relative to e of the amplitudes.

termine e and renormalize the fitted moduli with the help of Eq. (2). At 90° (c.m.) only b, d, and e were determined, since the Pauli principle requires that $a = 0$ and $b = -c$. Figures 3 and 4 and Table I display the results of our fit together with the single energy phase-shift analysis of Bystricky *et al.* (solid lines) with use of this same data plus additional existing information at small angles.[12]

TABLE I. Moduli and phases relative to e of reconstructed amplitudes for pp elastic scattering. The differential cross sections used as normalization are also given.

θ_{cm} (degrees)	$\|a\|$ $(mb/sterad)^{1/2}$	$\|b\|$	$\|c\|$	$\|d\|$	$\|e\|$
66	.805 ± .045	1.127 ± .023	.552 ± .037	.641 ± .031	2.009 ± .024
70	.613 ± .038	1.038 ± .023	.565 ± .033	.676 ± .029	2.037 ± .018
74	.493 ± .036	.991 ± .023	.556 ± .033	.667 ± .031	2.038 ± .016
78	.435 ± .039	.976 ± .022	.627 ± .028	.669 ± .027	1.985 ± .015
82	.245 ± .033	.886 ± .023	.678 ± .027	.646 ± .029	2.012 ± .012
86	.116 ± .022	.827 ± .023	.686 ± .023	.638 ± .024	2.024 ± .010
90	.000 ± .000	.785 ± .020	.785 ± .020	.612 ± .029	2.008 ± .008

θ_{cm} (degrees)	ϕ_a (rad)	ϕ_b	ϕ_c	ϕ_d	$d\sigma/d\Omega$ (mb/sterad)
66	.686 ± .057	1.597 ± .069	5.532 ± .075	4.329 ± .103	3.336
70	.575 ± .083	1.758 ± .064	5.420 ± .080	4.252 ± .099	3.190
74	.610 ± .097	1.792 ± .063	5.417 ± .084	4.197 ± .106	3.065
78	.846 ± .077	1.943 ± .060	5.483 ± .062	4.249 ± .096	2.962
82	.666 ± .169	1.955 ± .071	5.388 ± .062	4.172 ± .102	2.884
86	.461 ± .343	1.976 ± .069	5.331 ± .051	4.275 ± .103	2.836
90		2.174 ± .040	5.316 ± .040	4.209 ± .098	2.819

The errors indicated are statistical. The systematic global normalization errors are within the given standard deviations for all parameters, except for $|d|$, where the uncertainty amounts to 0.1 $(\mathrm{mb/sr})^{1/2}$. A χ^2 per degree of freedom of 1.15, averaged over the seven angles, was found in these fits and confirms the consistency of the data, which come from different and independent experiments. As a further check, the spin correlation parameters A_{00ss}, A_{00ks}, and A_{00kk}, a subgroup of experiments independent of the others, were taken out of the fit. The same result was found for the amplitudes, with, of course, some larger errors.

This study shows the consistency of our set of fifteen observables and shows that they are more than sufficient for an amplitude reconstruction. The reconstructed results strikingly demonstrate the important role played by all five amplitudes, in particular the spin-dependent part. The importance of measuring three-spin parameters with reasonable precision is also illustrated by the fact that an unambiguous reconstruction at 90° (c.m.) is impossible without them.

We would like to express our deepest gratitude to Professor R. Mermod, Professor J. P. Blaser, Professor E. Baumgartner, Professor J. Deutsch, Professor H. J. Gerber, Professor Ch. Haenny, Professor P. Huguenin, Professor C. Joseph, Professor D. Measday, Professor J. Rossel, Professor F. Scheck, Dr. J. Domingo, Dr. F. Lehar, and Dr. E. Michaelis for their unwavering support during the realization of this elaborate experiment. We are indebted to our previous collaborators, Professor A. Gersten, Dr. G. Greeniaus, Dr. B. Favier, Dr. S. Jaccard, Dr. J. C. Niklès, and especially to Dr. D. Besset, Dr. C. Weddigen, and Dr. D. W. Werren. We express our thanks to all the technical staff at SIN and Geneva University who have contributed to this research. This work was supported by the Swiss Institute for Nuclear Research, the Swiss National Science Foundation, and the Troixième Cycle de la Physique en Suisse Romande.

[1] L. Puzikov *et al.*, Nucl. Phys. 3, 436 (1957).

[2] The physical meaning of the overall phase is discussed by A. Martin, in *New Phenomena in Subnuclear Physics*, edited by A. Zichichi (Plenum, New York, 1977), pt. A.

[3] E.g., C. R. Schumacher and H. A. Bethe, Phys. Rev. 121, 1534 (1961); F. Lehar and P. Winternitz, Czech. J. Phys. A17, 158 (1967), and Fortschr. Phys. 15, 495 (1967).

[4] D. Besset *et al.*, Nucl. Instrum. Methods 148, 129 (1978).

[5] P. W. Johnson *et al.*, Phys. Rev. D 15, 1895 (1977).

[6] J. M. Fontaine, in *Proceedings of the Fourth International Symposium on Polarization Phenomena in Nuclear Reactions, Zürich, Switzerland,* edited by W. Gruebler and V. König (Birkhäuser-Verlag, Basel and Stuttgart, 1976).

[7] J. Bystricky *et al.*, J. Phys. (Paris) 39, 1 (1978).

[8] D. Besset *et al.*, to be published.

[9] E. Aprile *et al.*, in *Polarization Phenomena in Nuclear Physics—1980,* edited by G. G. Olsen, AIP Conference Proceedings No. 69 (American Institute of Physics, New York, 1981).

[10] E. Aprile *et al.*, in *Proceedings of the 1980 International Symposium on High-Energy Physics with Polarized Beams and Polarized Targets, Lausanne, 1980,* edited by C. Joseph (Birkhäuser, Basel, 1981).

[11] J. Bystricky *et al.*, Centre d'Etudes Nucléaires de Saclay Report No. DPhPE-79-01 (to be published).

[12] D. Besset *et al.*, Phys. Rev. D 21, 580 (1980).