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Nuclear-Structure Effects Connected with Charge-Exchange Resonances

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The excitation energies and the (p,n) cross sections of the giant Gamow-Teller resonance as well as the isobaric analog state in ^{208}Pb are calculated. Similar to the results in electron scattering, the theoretical spin-flip strength overestimates the experimental one appreciably. The excitation energies of the spin-dependent excitations and the isobaric analog resonance can only be explained simultaneously if the effects of the "dynamical theory of collective states" and of the one-pion– and one-ρ–exchange potential are taken into account.

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Using a highly energetic proton beam, Goodman *et al.*[1,2] recently obtained detailed experimental results on the Gamow-Teller resonances (GTR) in many nuclei. These 1^+ resonances which have been reported[3] earlier in ^{90}Zr turned out to be the dominant reaction channel in a high-energy (p,n) reaction at very forward angles. The structure of these states is very similar to the well-known isobaric analog states (IAR). Both resonances can be described in the framework of the random-phase approximation (RPA) theory as a superposition of proton-particle, neutron-hole states.[4,5] In the case of the IAR the particle-hole pairs are coupled to 0^+, in the GTR case to 1^+. Both kinds of states are expected to be rather collective in heavy-mass nuclei.

In the following we investigate the situation in ^{208}Pb. With a (p,n) reaction one excites, in this case, states in ^{208}Bi. In order to obtain the structure of these resonances one has to solve the RPA equation:

$$\chi_{12}^{\mu} = \frac{n_1 - n_2}{\epsilon_1 - \epsilon_2 - \Omega_\mu} \sum_{3,4} F_{14,23}{}^{\text{p-h}} \chi_{34}{}^{\mu}. \quad (1)$$

The amplitudes χ^μ are connected with the scattering cross section and Ω_μ is the excitation energy of the corresponding state μ. In order to solve Eq. (1) one needs single-particle energies ϵ_ν and a particle-hole interaction $F^{\text{p-h}}$. The n_ν denote the occupation probability 0 or 1 of a given single-particle state ν.

Since the levels excited by a (p,n) reaction are connected with a change in isospin, they allow a selective investigation of the isospin-dependent part of the particle-hole interaction. In the following, we used the generalized Landau-Migdal interaction of Li and Klemt[6] which includes in addition to the zero-range terms also the one-pion– and one-ρ–exchange potentials explicitly. The isospin-dependent part of this interaction has the form

$$\begin{aligned} F^{\text{p-h}}(r_1,r_2) = {} & C_0(\tilde{f}_0'\vec{\tau}_1\cdot\vec{\tau}_2 + \tilde{g}_0'\vec{\sigma}_1'\cdot\vec{\sigma}_2\vec{\tau}_1\cdot\vec{\tau}_2)\delta(\vec{r}_1-\vec{r}_2) \\ & - f_\pi^2\cdot m_\pi[h_2^{(1)}(im_\pi r)\exp(-m_\pi r)S_{12}(\Omega) + \tfrac{1}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2\exp(-m_\pi r)/m_\pi r]\vec{\tau}_1\cdot\vec{\tau}_2 \\ & - f_\rho^2\cdot m_\rho[-h_2^{(1)}(im_\rho r)\exp(-m_\rho r)S_{12}(\Omega) + \tfrac{2}{3}\vec{\sigma}_1\cdot\vec{\sigma}_2\exp(-m_\rho r)/m_\rho r]\vec{\tau}_1\cdot\vec{\tau}_2. \end{aligned} \quad (2)$$

Here $\tilde{f}_0'$ and $\tilde{g}_0'$ are the zero-order Migdal parameters, corrected for the contributions of the direct (g'^{dir}) and exchange (g'^{exch}) terms of the one-pion–exchange potential (OPEP) and ρ-exchange potential[6]

$$\tilde{g}_0' = g_0' - g_\pi'^{\text{dir}} - g_\rho'^{\text{dir}} - \tfrac{1}{4}(g_{0\pi}'^{\text{exch}} + g_{0\rho}'^{\text{exch}}), \quad (2a)$$

where g_0' is now the usual Migdal parameter. $C_0 = \pi^2\hbar^2/k_F m^*$ is the inverse of the density of states at the Fermi surface and $f_\pi^2 = 0.081$ and $f_\rho^2 = 4.86$ are the corresponding coupling constants. The short-range correlations in the central part of the ρ-exchange potential are included in the same way as proposed by Anastasio and Brown.[7]

With this interaction magnetic moments and unnatural/parity states could successfully be described thus testing the momentum dependence of the interaction in the spin-isospin channel up to momentum transfers of $q \sim 3$ fm^{-1}. The momentum dependence of the interaction gives a qualitative reason why collective unnatural-parity states were never observed in nuclei: The interaction is strongly repulsive for small momentum transfer, but as the momentum transter increases, the one-pion exchange cancels the repulsive components and above $q = 1$ fm^{-1} the interaction is weakly attractive. Therefore, this interaction can build collective magnetic states only, if the Fourier components of the wave function are concentrated at low momentum transfers. This is the case for the GTR the energy of which is strongly pushed up above the unperturbed particle-hole energies. Therefore, this state promises to be a useful tool to investigate the particle-hole interaction in the spin-isospin channel. It turned out, however, that in a straightforward RPA calculation (with use of experimental single-particle energies) the energy of the GTR is found to be 2.7 MeV below the experimental value, whereas the IAS is nicely reproduced. If one bears in mind the success of the interaction [Eq. (2)] in the description of other magnetic states, this discrepancy is very unlikely due to the interaction. In the following we will show that the experimental single-particle energies used so far are not the appropriate ones in this case.

In connection with the giant dipole resonances (GDR), Brown and Speth[8] have pointed out that the phonon contributions to the single-particle energies, which give rise to a compression of the Brueckner-Hartree-Fock spectrum in the odd-mass nuclei,[9] depend on the excitation energy of the state in the even-mass system one wants to calculate. Therefore, the RPA has to be extended to a "dynamical theory of collective states."

The coupling of the phonons to a given collective state with energy $h\omega_\lambda$ can be expressed in terms of a self-energy $\Sigma(\hbar\omega_\lambda)$ and the real part of it can be written as a principal-value integral

$$\Sigma(\hbar\omega_\lambda) = \mathrm{P}\int dE_i [|M_\lambda(E_i)|\rho(E_i)/(\hbar\omega_\lambda - E_i)]. \quad (3)$$

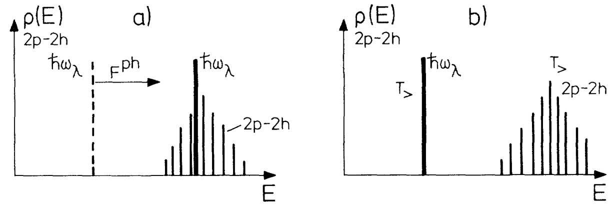

FIG. 1. Two limiting cases included in Eq. (3): (a) The collective resonance $\hbar\omega_\lambda$ is shifted, because of the particle-hole interaction, into the corresponding 2p-2h states (e.g., GDR and GTR); (b) the relevant 2p-2h states are higher in energy than the given collective resonance (e.g., IAR).

$M(E_i)$ is the matrix element which couples the given collective state $h\omega_\lambda$ to 2-particle, 2-hole (2p-2h) states (which includes also the coupling to two coherent 1p-1h vibrations). In Fig. 1 we show two limiting situations. In the GDR case [Fig. 1(a)] the repulsive particle-hole interaction pushes this resonance right in between the corresponding 2p-2h states. Therefore, the principal value, Eq. (3), is small, i.e., the effect of the phonons on the energy of the GDR is small. Therefore, the compression due to them in the empirical particle-hole energies must be removed.[8] Using the phenomenological formula derived by Brown and Speth[8] which interpolates between the single-particle spectrum calculated within the Brueckner-Hartree-Fock approach ($m^*/m \sim 0.65$) and the experimental one ($m^*/m \sim 1$), we obtain an effective mass of $m^*/m = 0.75$. Since the Skyrme-III interaction has an effective mass of $m^*/m = 0.76$, we used the corresponding single-particle spectrum in the calculation of the GDR, which now comes out at the observed energy (see Table I) removing the long-existing discrepancy[10] between empirical energy and the value calculated from empirical particle-hole energies.

The behavior of the GTR is similar to the GDR because the GTR is the $T_<$ state. The GTR couples strongly to the 2p-2h ($T_<$) states which are in the same energy region, i.e., the self-energy [Eq. (3)] is small. Therefore, we used also in this case the single-particle spectrum calculated with the Skyrme-III interaction. Since the single-particle spectrum is now fixed from the GDR calculation, the GTR in ^{208}Pb can be used to adjust the g_0' parameter of the Landau-Migdal force. From a previous investigation of magnetic states in ^{12}C and ^{16}O, we obtained a value of $g_0' = 0.75$.[6] In the present context we used $g_0' = 0.65$. This is in agreement with a theoretical result using the Reid soft-core potential.[11] If one bears in mind

TABLE I. Comparison between theoretical and experimental energies of the electric dipole resonance (GDR), isobaric analog resonance (IAR) and Gamow-Teller resonance, respectively. The energies refer to the ground state of ^{208}Pb.

	GDR (^{208}Pb)		IAR (^{208}Bi)		GTR (^{208}Bi)	
	theor.	expt.	theor.	expt.	theor.	expt.[a]
E (MeV)	13.7	13.7	17.0	18.0	18.9	18.4 ± 0.2
Single-particle spectrum	Skyrme III		expt. energies		Skyrme III	
Force parameter	$f_0'^{\mathrm{in}} = 0.60$; $f_0'^{\mathrm{ex}} = 1.8$				$g_0' = 0.65$	

[a]Refs. 1 and 2.

the uncertainties connected with the single-particle spectrum, our theoretical result shown in Table I is in fair agreement with the experimental one. The wave function which we obtain from our calculation shows a fairly collective behavior; nevertheless, there are two dominant configurations (i) $\chi^{1^+}(\pi i_{11/2}\nu i_{13/2}^{-1}) = 0.68$ and (ii) $\chi^{1^+}(\pi h_{9/2}\text{-}\nu h_{11/2}^{-1}) = 0.46$. The total Gamow-Teller strength concentrated in the GRT at 18.9 MeV is about 82%.

For consistency we also calculated the other, well-known collective charge-exchange resonance, the IAR. The GDR and IAR depend on f_0'. This parameter is strongly density dependent.[11] Therefore, we used the density-dependent version of the Landau-Migdal parameters: $f_0'(\rho) = f_0'^{\mathrm{ex}} + (f_0'^{\mathrm{in}} - f_0'^{\mathrm{ex}})\rho(r)$, where $\rho(r)$ is the nuclear density. The parameter $f_0'^{\mathrm{in}} = 0.6$ follows from the symmetry energy. Our f_0' corresponds to $\lambda = 36$ MeV (with an effective mass of $m^*/m = 0.82$).[12] The factor of 3 used for $f_0'^{\mathrm{ex}}$ is suggested by the result of Ref. 11. With use of these parameters and Skyrme III which fits the GDR, the energy of the IAR is 2 MeV too high compared with experiment. The solution of this discrepancy follows again from the dynamical theory.

The situation of the IAR is shown schematically in Fig. 1(b). Since the IAR is the $T_>$ state, the corresponding 2p-2h ($T_>$) states are much higher in energy (the coupling to the $T_<$ states is very weak). Therefore, the principal value integral (3) is large, which means that the phonon coupling is not removed and one has to use the experimental single-particle energies. The result of that calculation is shown in Table I. It is in reasonable agreement with experiment. In the IAR case more than 95% of the strength is concentrated in one state.

Using the wave functions of the IAR and GTR, we have calculated the (p,n) differential cross sections, the results of which are shown in Fig. 2. The effective projectile-target-nucleon interaction and the optical-model parameters are taken from Love and Petrovich.[13] Whereas the theoretical IAR cross section is in good agreement with the experimental one, our theoretical GTR cross section is too large by about a factor of 2. If we suppose that the coupling potentials $V_{\sigma\tau}$ and V_τ are known (which has to be checked independently) this deviation raises some interesting structure problems, which are connected with similar questions of magnetic resonances:

(i) Since we do not have a conservation law for

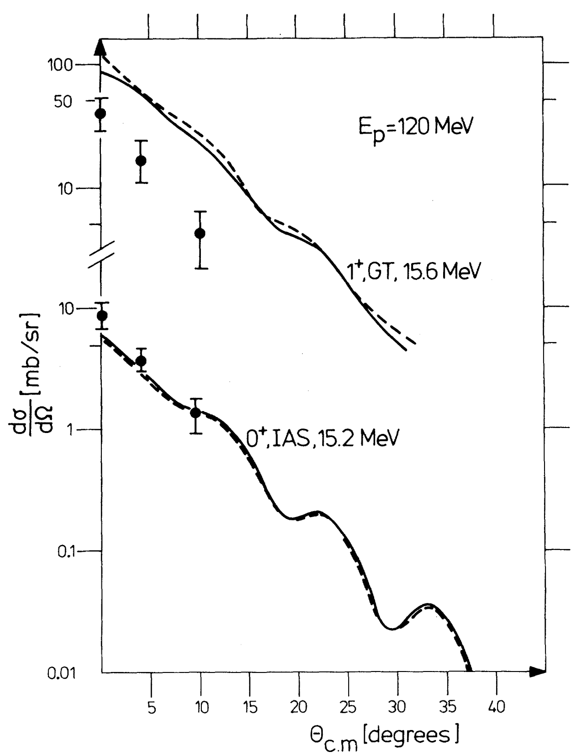

FIG. 2. Theoretical cross sections of the 0^+ (IAS) and 1^+ (GTR) of the reaction ^{208}Pb$(p,n)^{208}$Bi. The experimental results are taken from Horen *et al.* (Ref. 2). The dashed lines represent the results where the dynamical theory has been taken into account, whereas the full lines follow from a conventional RPA calculation.

the GT resonances (such as the charge conservation in the electric case), one measures in those experiments the amount of the single-particle strength of each single-particle and single-hole state, which is in any case smaller than unity. The residue at the quasiparticle pole was found by Li and Klemt[8] to be $Z_\lambda \simeq 0.6$, but not the full reduction $1-Z$ which should be applied. The dominant mixing with the lowest 3^- vibration gives rise to the spreading width, and so this strength will be found within the envelope of the GTR. This removes more than half of the reduction $1-Z_{\rm eff}$ and we end up with a factor of ~ 0.9 in the amplitude.

(ii) It is well known that the theoretical GT matrix elements always overestimate the experimental ones. Oset and Rho[14] have shown that the Lorentz-Lorenz effect gives rise to a 30% reduction of the theoretical matrix elements. Since this effect is density dependent, it should be smaller for the scattering cross sections (because the reaction takes place mainly on the surface). Since we have not calculated the effect of the surface, we use the full factor, thereby overestimating the reduction $R \simeq (0.9\times 0.7)^2 \simeq 0.4$.

In summary, we have investigated the physical consequences of the Gamow-Teller resonances in heavy nuclei, which are so far the best example of a collective magnetic resonance. It has been shown that the "dynamical theory of collective states" gives rise to an energy shift of about 2.5 MeV. If one would neglect this effect and adjust the force to the experimental energy, one would overestimate the repulsive character of the particle-hole interaction in the spin-isospin channel.

We have also demonstrated that the comparison of the GDR and IAR provides an excellent example of the importance of the "dynamical theory of collective states." In the first case it gives rise to a change of the single-particle energies, whereas in the case of the IAS the single-particle energies remain unchanged.

Last but not least, we found that, similar to the results in electron scattering, the theoretical spin-flip strength overestimates the experimental one appreciably. The theoretical understanding of the "missing" GT strength which has been found all over the periodic table is one of the exciting new phenomena connected with the GT resonances.

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