verse-momentum, differential processes is explicitly realized. We should stress that the classical configuration (1), which we chose to be crypto-Abelian for simplicity, is by no means unique; indeed, all that is required of it is that its $\vec{B}$ field satisfy

$$\lim_{\rho\to 0}\int_0^{2\pi} d\varphi\, B_\varphi(x) \neq 0. \tag{16}$$

We expect that this class of configurations can be generated dynamically, thus demonstrating the self-consistency of our approach. This very interesting possibility is presently under investigation.

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[1]M. Voloshin and V. Zakharov, DESY Report No. 80/41 (unpublished), have also recently studied some effects of nonperturbative gluonic fields on cross sections.

[2]G. Tiktopoulos, Nucl. Phys. B147, 371 (1979); G. Sterman, Phys. Rev. D 19, 3185 (1979).

[3]F. R. Ore and G. Sterman, Nucl. Phys. B165, 93 (1980).

Neutral-Current Weak Interaction without Electroweak Unification

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It is shown that it is possible to obtain the neutral-current weak interaction as the third component of the SU(2) group without electroweak unification, provided that fundamental doublets contain in general a mixture of left-handed and right-handed fields. The main experimental consequence is the relation $m_{W_3} \simeq \sqrt{2}\, m_W$, with the further possibility that m_W could be lower than that predicted by the standard model.

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The existence[1] of the neutral-current weak interaction is regarded as an evidence for electroweak unification in terms of the standard gauge model[2] based on the local symmetry group SU(2) ⊗ U(1). In fact, the standard model predicted such an interaction before its experimental discovery. The question which I wish to study in this note is whether this prediction is unique to the standard model and whether in fact the unification of weak and electromagnetic interactions, which is no doubt a very profound idea, is necessary for the existence of the neutral-current weak interaction. In other words, is it possible to predict this interaction on a symmetry group of purely weak interactions? Previous attempts to identify the neutral current of the weak interaction with the third component of an SU(2) group,[3] the other two components giving the charged currents, failed in the sense that the resulting neutral-current weak interaction does not agree with the experiments. In this note, I show that one can identify the charged and neutral currents of the weak interaction with, respectively, $1\pm i2$ and three components of the SU(2) group in agreement with experiments, provided that (i) the fundamental doublets under SU(2) contain in general a mixture of left-handed and right-handed fields; (ii) the basic interaction is axial vector rather than vector. We illustrate our basic idea first by considering leptons ν_l, l (e.g., ν_e, e^-), where we take ν_l to be pure left handed and massless. We now assume that

$$L = \begin{pmatrix} \nu_{lL} \\ (\cos\alpha) l_L + (\sin\alpha) l_R \end{pmatrix} \tag{1}$$

is a doublet under SU(2) while $-(\sin\alpha) l_L + (\cos\alpha) \times l_R$ is a singlet. Consider the interaction

$$\mathcal{L}_{\rm int}{}^l = i g_W \bar{L}\gamma_\mu\gamma_5 W_\mu L, \tag{2a}$$

where

$$W_\mu = \tfrac{1}{2}\vec{\tau}\cdot\vec{W}_\mu = \frac{1}{2}\begin{pmatrix} W_{3\mu} & \sqrt{2}W_\mu{}^- \\ \sqrt{2}W_\mu{}^+ & -W_{3\mu} \end{pmatrix}. \tag{2b}$$

Clearly (2a) is invariant under global SU(2). Later we shall derive it when SU(2) is regarded as a local gauge symmetry. Since $\bar{\psi}_{L,R}\gamma_\mu\gamma_5\psi_{R,L}=0$, $\bar{\psi}_L\gamma_\mu\gamma_5\psi_L=\bar{\psi}\gamma_\mu\frac{1}{2}(1+\gamma_5)\psi$, $\bar{\psi}_R\gamma_\mu\gamma_5\bar{\psi}_R=-\bar{\psi}\gamma_\mu\frac{1}{2}(1-\gamma_5)\psi$, Eq. (2a) gives

$$\mathcal{L}_{\rm int}{}^{l}=\tfrac{1}{4}ig_W\{\sqrt{2}(\cos\alpha)\bar{\nu}_l\gamma_\mu(1+\gamma_5)lW_\mu{}^{-}+\text{H.c.}+[\bar{\nu}_l\gamma_\mu(1+\gamma_5)\nu_l-\bar{l}\gamma_\mu(1+\gamma_5)l-2(\sin^2\alpha)(-\bar{l}\gamma_\mu l)]W_{3\mu}\}. \qquad (3)$$

Note that the axial-vector coupling in (2a) is necessary to have parity nonconservation in the neutral-current weak interaction. We can rewrite (3) as

$$\mathcal{L}_{\rm int}{}^{l}=(g_W/2\sqrt{2})\cos\alpha[J_\mu{}^W(l)W_\mu{}^{-}+\text{H.c.}+(\sqrt{2}/\cos\alpha)J_{3\mu}(l)W_{3\mu}], \qquad (4a)$$

where

$$J_\mu{}^W(l)=i\bar{\nu}_l\gamma_\mu(1+\gamma_5)l, \qquad (4b)$$

$$J_{3\mu}(l)=\tfrac{1}{2}\{i[\bar{\nu}_l\gamma_\mu(1+\gamma_5)\nu_l-\bar{l}\gamma_\mu(1+\gamma_5)l]-2(\sin^2\alpha)J_\mu{}^{\rm e.m.}(l)\}, \qquad (4c)$$

$$g_W{}^2\cos^2\alpha/8m_W{}^2=G_{\rm F}/\sqrt{2}, \qquad (4d)$$

$$g_W{}^2/8m_{W_3}{}^2=G_{\rm F}/\sqrt{2}. \qquad (4e)$$

The relation (4e) ensures the universality of coupling between charged- and neutral-weak-current interactions. The relations (4d) and (4e) imply

$$m_W/(\cos\alpha)m_{W_3}=1. \qquad (5)$$

The structure of Eqs. (4) and (5) is exactly the same as in the standard model with some important differences: (i) no necessity of electroweak unification and neutral current is just the third component of SU(2); (ii) experimentally[1] $\sin^2\alpha\simeq\frac{1}{2}$, i.e., 046 ± 0.03, and thus $m_{W_3}\simeq\sqrt{2}m_W$ instead of $m_Z\simeq(2/\sqrt{3})m_W$ in the standard model; (iii) since g_W is not constrained here, m_W is arbitrary in general. It may, however, be reasonable to take the semiweak coupling constant $g_W\lesssim e$. Then $m_W\lesssim 26.4$ GeV, $m_{W_3}\lesssim 37.3$ GeV. If $g\cos\alpha\lesssim e$, then $m_W\lesssim 37.3$ GeV, $m_{W_3}\lesssim 52.7$ GeV. It is interesting to note that the experimental value of $\sin^2\alpha\simeq\frac{1}{2}$ implies almost equal mixture of left-handed and right-handed fields in the doublet (1).

The extension to hadrons is straightforward if the quarks are integrally charged. Consider, for example, the Sakata-like triplet $\mathcal{P},\mathfrak{N},\lambda$ and a charm-carrying singlet $\mathcal{P}^c$ to obtain cancellation[4] of strangeness-changing neutral current. We take ($\mathfrak{N}',\lambda'$ denote Cabibbo-rotated $\mathfrak{N},\lambda$)

$$H=\begin{pmatrix}(\cos\alpha)\mathcal{P}_L+(\sin\alpha)\mathcal{P}_R\\ \mathfrak{N}_L'\end{pmatrix},\qquad H'=\begin{pmatrix}(\cos\alpha)\mathcal{P}_L{}^c+(\sin\alpha)\mathcal{P}_R{}^c\\ \lambda_L'\end{pmatrix} \qquad (6)$$

as doublets under SU(2), while $-(\sin\alpha)\mathcal{P}_L+(\cos\alpha)\times\mathcal{P}_R$, $\mathfrak{N}_R$, $-(\sin\alpha)\mathcal{P}_L{}^c+(\cos\alpha)\mathcal{P}_R{}^c$, λ_R are singlets. Then SU(2)-invariant

$$\mathcal{L}_{\rm int}{}^h=ig_W\bar{H}\gamma_\mu\gamma_5W_\mu H+ig_W\bar{H}'\gamma_\mu\gamma_5W_\mu H' \qquad (7a)$$

gives the interaction of the form (4a) with

$$J_\mu{}^W(h)=i\bar{\mathcal{P}}\gamma_\mu(1+\gamma_5)\mathfrak{N}'+i\bar{\mathcal{P}}^c\gamma_\mu(1+\gamma_5)\lambda', \qquad (7b)$$

$$J_{3\mu}(h)=\tfrac{1}{2}\{i[\bar{\mathcal{P}}\gamma_\mu(1+\gamma_5)\mathcal{P}-\bar{\mathfrak{N}}\gamma_\mu(1+\gamma_5)\mathfrak{N}-\bar{\lambda}\gamma_\mu(1+\gamma_5)\lambda+\bar{\mathcal{P}}^c\gamma_\mu(1+\gamma_5)\mathcal{P}^c]-2(\sin^2\alpha)J_\mu{}^{\rm e.m.}(h)\}. \qquad (7c)$$

We now discuss the gauge formulation of the above model. Consider the gauge transformation corresponding to the covariant derivative

$$D_\mu=\partial_\mu-ig_W\gamma_5\tilde{W}_\mu \qquad (8a)$$

and

$$F\to F'=(1+i\tilde{\epsilon})F, \qquad (8b)$$

where F is any fermion doublet. The notation is such that the tilde denotes that $W_\mu=\frac{1}{2}\vec{\tau}\cdot\vec{W}_\mu$ and $\epsilon=\frac{1}{2}\vec{\tau}\cdot\vec{\epsilon}$ are not only matrices in weak isospin space but also Dirac matrices, in this case unit Dirac matrices. Then the field tensor for $\tilde{W}_\mu$ is

$$\tilde{F}_{\mu\nu}=D_\mu\tilde{W}_\nu-D_\nu\tilde{W}_\mu=(\partial_\mu\tilde{W}_\nu-\partial_\nu\tilde{W}_\mu)-ig_W\gamma_5[\tilde{W}_\mu,\tilde{W}_\nu], \qquad (9)$$

which is also a Dirac matrix. The transformation law for $\tilde{W}_\mu$ deduced from the requirement that

$$(D_\mu F)'=(1+i\tilde{\epsilon})D_\mu F$$

is

$$\tilde{W}_\mu \to \tilde{W}_\mu' = \tilde{W}_\mu - i[\tilde{W}_\mu, \tilde{\epsilon}] + g_W^{-1}\gamma_5\partial_\mu\tilde{\epsilon}. \tag{10}$$

Also one can see that

$$\tilde{F}_{\mu\nu} \to \tilde{F}_{\mu\nu}' = (1+i\tilde{\epsilon})\tilde{F}_{\mu\nu}(1-i\tilde{\epsilon}). \tag{11}$$

Then the gauge-invariant Lagrangian is

$$\mathcal{L} = \tfrac{1}{4}\mathrm{Tr}_D\{-\tfrac{1}{2}\mathrm{Tr}\tilde{F}_{\mu\nu}\tilde{F}_{\mu\nu}\} - \bar{L}\gamma_\mu(\partial_\mu - ig_W\gamma_5\tilde{W}_\mu)L - \bar{H}\gamma_\mu(\partial_\mu - ig_W\gamma_5\tilde{W}_\mu)H - \bar{H}'\gamma_\mu(\partial_\mu - ig_W\gamma_5\tilde{W}_\mu)H', \tag{12}$$

where the first trace in (12) is with respect to Dirac matrices. In Eq. (12), we have not displayed the kinetic-energy terms for singlets under SU(2). Equation (12) gives the interaction Lagrangians (2a) and (7a).

To break the gauge symmetry spontaneously[5] so that W bosons acquire mass, we see that a Higgs triplet

$$\varphi = \tfrac{1}{2}\vec{\tau}\cdot\vec{\varphi} = \frac{1}{2}\begin{pmatrix} \varphi^+ & \sqrt{2}\varphi^{++} \\ \sqrt{2}\varphi^0 & -\varphi^+ \end{pmatrix} \tag{13a}$$

having the gauge-invariant coupling

$$\tfrac{1}{4}\mathrm{Tr}_D\{-\mathrm{Tr}(\partial_\mu\bar{\tilde{\varphi}} + ig_W[\bar{\tilde{\varphi}},\gamma_5\tilde{W}_\mu])(\partial_\mu\varphi - ig_W[\gamma_5\tilde{W}_\mu,\tilde{\varphi}])\} \tag{13b}$$

and with the vacuum expectation value (here the tilde denotes that φ is a unit Dirac matrix while the bar indicates Hermitian conjugate)

$$\langle\varphi\rangle = \frac{1}{2}\begin{pmatrix} 0 & 0 \\ \nu & 0 \end{pmatrix} \tag{13c}$$

gives

$$m_{W_3}^{\,2} = 2m_W^{\,2} \tag{14a}$$

the relation required. Now in general, if we do not assume the universality of couplings between the charged- and neutral-current weak interactions, the relation (5) is replaced by

$$\rho = m_W^{\,2}/(\cos\alpha)^2 m_{W_3}^{\,2}. \tag{14b}$$

With use of (14a) and noting that $\sin^2\alpha$ here corresponds to $2\sin^2\theta_W$ in the standard model, the above relation becomes

$$\rho = (2\cos^2\alpha)^{-1} = (2 - 4\sin^2\theta_W)^{-1}. \tag{14c}$$

This gives $\rho = 1$, i.e., the relation (5) for $\sin^2\alpha = \frac{1}{2}$ ($\sin^2\theta_W = \frac{1}{4}$), the values required by experimental data.

We now briefly discuss the case of fractionally charged quarks u, d, s, and c for the hadronic sector. We form the following doublets:

$$H_1 = \begin{pmatrix} (\cos\beta)U_L + (\sin\beta)U_R \\ (\cos\gamma)d_L' + (\sin\gamma)d_R' \end{pmatrix}, \quad H_2 = \begin{pmatrix} -(\sin\beta)U_L + (\cos\beta)U_R \\ -(\sin\gamma)d_L' + (\cos\gamma)d_R' \end{pmatrix}, \tag{15}$$

with similar ones, H_1' and H_2', involving c and s'. Then the following interaction Lagrangian is invariant under the global SU(2):

$$ig_W\{\bar{L}\gamma_\mu\gamma_5W_\mu L + \cos^2\theta(\bar{H}_1\gamma_\mu\gamma_5W_\mu H_1 + \bar{H}_1'\gamma_\mu\gamma_5W_\mu H_1') + \sin^2\theta(\bar{H}_2\gamma_\mu\gamma_5W_\mu H_2 + \bar{H}_2'\gamma_\mu\gamma_5W_\mu H_2')\}. \tag{16}$$

Note that because of two hadronic doublets H_1, H_2, compared to one lepton doublet, it may be natural to implement the universality as above. Then to ensure the universality between leptonic and hadronic charged-current weak interactions and the absence of right-handed hadronic charged current [i.e., to have the form (7b)] we have, respectively, (note that θ here is in general different from the Cabibbo angle θ_C)

$$\cos^2\theta\cos\beta\cos\gamma + \sin^2\theta\sin\beta\sin\gamma = \cos\alpha, \tag{17a}$$

$$\cos^2\theta\sin\beta\sin\gamma + \sin^2\theta\cos\beta\cos\gamma = 0. \tag{17b}$$

Furthermore, in order that the hadronic neutral current has the same form as the leptonic one [i.e., to have the form (7c)], we must have

$$\cos^2\theta\sin^2\beta + \sin^2\theta\cos^2\beta = \tfrac{2}{3}\sin^2\alpha, \tag{18a}$$

$$\cos^2\theta\sin^2\gamma + \sin^2\theta\cos^2\gamma = \tfrac{1}{3}\sin^2\alpha. \tag{18b}$$

Equations (17) and (18) have the solution

$$\cos 2\theta = (1 - \tfrac{8}{9}\sin^2\alpha)^{1/2},$$
$$\cos 2\beta = (1 - \tfrac{4}{3}\sin^2\alpha)/(1 - \tfrac{8}{9}\sin^2\alpha)^{1/2},$$
$$\cos 2\gamma = (1 - \tfrac{2}{3}\sin^2\alpha)/(1 - \tfrac{8}{9}\sin^2\alpha)^{1/2}. \tag{19}$$

The gauge formulation of Eq. (16) is somewhat complicated and it would arise as an effective interaction by enlarging the SU(2) group to, for example, $SU_1(2) \otimes SU_2(2)$. I shall consider it in another publication.

To conclude, it is possible to have the neutral-current weak interaction similar to that of the standard model and in agreement with the experiments without electroweak unification. The main experimental consequence of the present model which differs from the standard model is the relation $m_{W_3} \approx \sqrt{2} m_W$ and that mass of charged W (and hence that of W_3) could conceivably be lower than that predicted by the standard model.

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[1]See, for example, C. Baltay, in *Proceedings of the Nineteenth International Conference on High Energy Physics, Tokyo, Japan, 1978,* edited by S. Homma, M. Kawaguchi, and H. Miyazawa (Physical Society of Japan, Tokyo, 1979).

[2]S. L. Glashow, Nucl. Phys. 22, 579 (1961); Abdus Salam and J. C. Ward, Phys. Lett. 13, 1681 (1964); S. Weinberg, Phys. Rev. Lett. 19, 1264 (1967); Abdus Salam, in *Proceedings of the Seventh Nobel Symposium,* edited by N. Svartholm (Almqvist and Wiksell, Stockholm, 1968), p. 367.

[3]N. Kemmer, Phys. Rev. 52, 906 (1937); Abdus Salam, Rev. Mod. Phys. 52, 525 (1980); S. Bludman, Nuovo Cimento 9, 433 (1958).

[4]S. L. Glashow, J. Iliopoulos, and L. Maiani, Phys. Rev. D 2, 1285 (1970).

[5]See, for example, Ling-Fong Li, Phys. Rev. D 9, 1723 (1974).