Is the $\Delta_{D35}(1925)$ Resonance Evidence for New Baryonic Degrees of Freedom?

K. C. Bowler and P. J. Corvi
Physics Department, University of Edinburgh, Edinburgh EH9 3JZ, Scotland

and

A. J. G. Hey and P. D. Jarvis[(a)]
Physics Department, University of Southampton, Southampton SO9 5NH, England
(Received 17 March 1980)

The possibility that the $\Delta_{D35}(1925)$ resonance is evidence for a $[\underline{56}, 1^-]$ multiplet corresponding to excitation of new gluonic degrees of freedom within baryons is examined in detail. A new group-theoretical analysis of a successful mass formula for the $N = 2$ multiplets of the nonrelativistic harmonic-oscillator quark model is also presented. Some results for the $N = 3$ level are also given together with an outline of the method for generalization to any degree of excitation of the system.

PACS numbers: 12.70.+q, 11.30.Ly, 12.40.Cc

The nonrelativistic harmonic-oscillator quark model is remarkably successful in accounting for many features of the baryon spectrum. In its modern guise,[1] including anharmonic perturbations, quark mass differences, and some effects of the nonrelativistic reduction of colored gluon exchange between quarks, it provides a good semiquantitative guide to negative- and positive-parity resonances up to about 2 GeV in mass.[2,3] Nevertheless, despite this huge success, the theoretical foundations of this nonrelativistic potential model are not well understood—certainly not from a fundamental standpoint like that of quantum chromodynamics. It is therefore important to examine possible deviations from this rather simple picture of baryons. In fact, there is a strong hint of a new degree of freedom being excited. Before we discuss the evidence for such an assertion, we briefly survey several possible realizations of such "extra" degrees of freedom.

The rigid, spherical-cavity approximation to the Massachusetts Institute of Technology bag model[4] gives a good description of the ground-state $[\underline{56}, 0^+]$ multiplet. However, in order to generate the negative-parity resonances, the rigidity of the surface must be relaxed to allow small surface oscillations. Rebbi[5] identified a $[\underline{70}, 1^-]$ multiplet together with extra $[\underline{56}, 1^-]$ multiplets which may be visualized as arising from oscillations of the three-quark system with respect to the bag walls. One of these $[\underline{56}, 1^-]$ multiplets is identified with the zero mode corresponding to translation of the ground state; the rest are presumably extra non–three-quark physical $[\underline{56}, 1^-]$ multiplets. The detailed calculations of Rebbi[5] suggest that the first such multiplet should be below 2 GeV. Similarly, a host of extra states are evidently possible in models where "constituent gluons," as well as quarks, are allowed as building blocks.[6] An alternative picture[7] of baryons is the string model in which quarks are attached to the ends of strings which meet at a common junction. In such models, there are clearly more than just quark degrees of freedom inside baryons and extra structure is to be expected, although there are no reliable estimates of masses.

So much for theory; what of experiment? In 1976, Cutkosky[8] announced the discovery of a new resonant state, the $\Delta_{D35}(1925)$. Since the quantum numbers of this state make it most plausibly assigned to a $[\underline{56}, 1^-]$ multiplet, Cutkosky suggested that this was evidence for gluonic degrees of freedom within the proton. However, before such a claim is to be believed, one must rule out a more conventional explanation, namely a $[\underline{56}, 1^-]$ multiplet which is a genuine three-quark excitation. Dalitz, Horgan, and Reinders[9] appeared to do just this, obtaining a sum rule relating the mass of the Δ_{D35} state to other, known Δ resonances, and predicted

$$M(\Delta_{D35}) = 2088 \pm 25 \text{ MeV},$$

some 160 MeV higher than the value 1925 ± 20 MeV quoted by Cutkosky *et al.*[8] On the face of it, therefore, it seems unlikely that the 1925 state can be this three-quark state. What reasons are there for re-examining this result? Firstly, a new phase-shift analysis by Cutkosky *et al.*[10] confirms the Δ_{D35} state, gives it $\underline{3}^*$ status, and quotes a mass of 1930 ± 20 MeV. Secondly, the sum rule derived by Dalitz, Horgan, and Reinders[9] was based on the specific assumption of neglecting SU(6) tensor forces. Such forces are now believed to be important,[1-3] more important certainly than the spin-orbit forces retained in the detailed SU(6) mass analyses performed by Horgan and others[11] which explicitly neglect spin-spin tensor forces of the type arising from one-gluon exchange.

Our analysis is based on the following nonrelativistic oscillator model described by the Hamiltonian[2,3]

$$H=\sum_i m_i + H_0 + H_{\rm hyp},$$

with

$$H_0=\sum_{i=1}^{3}\frac{\vec{p}_i^{\,2}}{2m_i}+\sum_{i<j}V_{\rm conf}{}^{ij},$$

where the confining potential $V_{\rm conf}{}^{ij}$ is written

$$V_{\rm conf}{}^{ij}=\tfrac{1}{2}Kr_{ij}{}^2+U(r_{ij}).$$

Here, r_{ij} is the separation of quarks i and j and $U(r_{ij})$ is an unknown anharmonic perturbation, depending only on the magnitude r_{ij}, which we shall treat to first order in perturbation theory. We shall be less ambitious than Ref. 9 in that we shall only predict the mean masses of the various harmonic-oscillator multiplets, with total neglect of the one-gluon-exchange hyperfine interactions $H_{\rm hyp}$. This approach is already known to lead to an intriguing result for the $N=2$ multiplets.[2,3] Figure 1 shows the pattern of the degeneracy breaking induced by an arbitrary anharmonic perturbation: Apart from the overall choice of sign, the relative splittings are independent of the detailed form of U. This suggests that the result may be derived from purely group-theoretic considerations and corresponds to the breaking of a symmetry of the Hamiltonian by the perturbation U. The splitting at the $N=2$ level—with the lowering of the radially excited multiplet $[\underline{56},0^+]$ below the $[\underline{56},2^+]$ and the $\underline{70}$'s—seems to correspond rather well with the physical situation. It lends confidence to the belief that the pattern of splittings at the $N=3$ level will be of interest.

The results for the $N=2$ level[2,3] and some results[9] at the $N=3$ level were originally derived with the explicit oscillator wave functions and with use of the symmetry of the three-quark system to perform some of the Gaussian integrals. We therefore sketch an alternative derivation of these results via operator methods before briefly discussing the group-theoretical aspects of the problem.

In the absence of the anharmonic perturbation U, the Hamiltonian in the $S=0$ sector may be rewritten in terms of the standard three-body co-

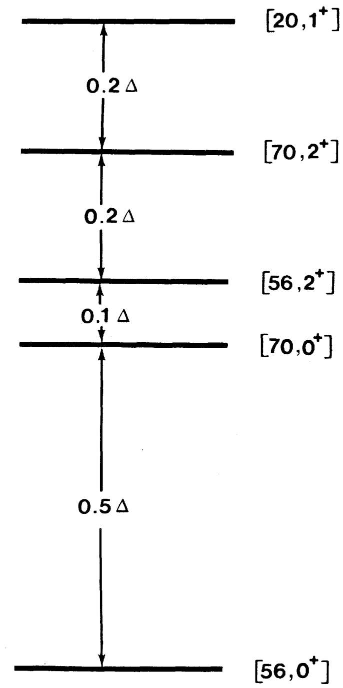

FIG. 1. The pattern of splitting of the $N=2$ supermultiplets under the influence of the anharmonic pertrubation U.

ordinates

$$\vec{\rho}=(\vec{r}_1-\vec{r}_2)/\sqrt{2},\quad \vec{\lambda}=(\vec{r}_1+\vec{r}_2-2\vec{r}_3)/\sqrt{6},$$

to yield

$$H_0=H_{\rm c.m.}+\frac{\vec{p}_\rho^{\,2}}{2m}+\frac{\vec{p}_\lambda^{\,2}}{2m}+\frac{3K}{2}(\vec{\rho}^{\,2}+\vec{\lambda}^{\,2}),$$

where the first term simply describes the free motion of the center of mass and we have taken $m_u=m_d=m$. We now introduce creation and annihilation operators for the two independent ρ and λ oscillators. In a spherical basis for the ρ mode we define

$$a_\pm{}^\dagger(\rho)=\mp 2^{-1/2}[a_x{}^\dagger(\rho)\pm i a_y{}^\dagger(\rho)],$$

$$a_0{}^\dagger(\rho)=a_z{}^\dagger(\rho),$$

obeying the usual commutation relations

$$[a_i,a_j{}^\dagger]=\delta_{ij}\quad(i,j=+,-,0;\hbar=1),$$

with similar expressions for the λ mode. The general excited state of H_0 with energy $E_N=(N+3)\omega$ and orbital angular momentum L may be expressed as an Nth-order monomial of creation operators acting on the ground state $|0\rangle=|0\rangle_\rho|0\rangle_\lambda$. For a given N and permutation symmetry $P(=S,A,$

M_ρ, or M_λ), the states may be written

$$|\psi_{LL_z}{}^{(P)}\rangle \equiv \hat{\psi}_{LL_z}{}^{(P)}|0\rangle,$$

where the relevant operator may be constructed by a systematic procedure given by Horgan.[12] For example, the $N=2$ [$\underline{56}$, 0^+] states are characterized by the operator

$$\hat{\psi}_{00}{}^{(S)} = (1/2\sqrt{3})\{[\vec{a}^\dagger(\rho)]^2 + [\vec{a}^\dagger(\lambda)]^2\}.$$

With the operator forms for the state vectors plus the commutation relations, it is now straightforward to calculate the first-order energy shift induced by U. First, the symmetry of the (flavor-spin) SU(6)$\otimes$O(3) three-quark states, $|\varphi\rangle$, is used to reduce the problem to a perturbation calculation for the ρ oscillator alone:

$$\sum_{i<j} \langle\varphi|U(r_{ij})|\varphi\rangle = 3\langle\varphi|U(\sqrt{2}\,\rho)|\varphi\rangle.$$

The remaining ρ-oscillator matrix elements of U are then quickly translated to the Gaussian moments of U defined in Ref. 3, and the pattern of Fig. 1 derived. The energy levels involve three parameters: an effective zero-point energy E_0 and oscillator frequency Ω common to the $N=0$, 1, and 2 (and 3) levels plus a splitting parameter Δ specific to the $N=2$ level.

A group-theoretical derivation of this result is both interesting in itself and leads to a new method for enumerating and constructing the state vectors of the multiplets at any given N which has some advantages over previous methods.[12] Contrary to what might be expected the relevant group is not the SU(6) dynamical symmetry group of the two-oscillator Hamiltonian but rather the associated spectrum-generating algebra—in this case a noncompact, real form of the symplectic group Sp(12). The group is realized on the twelve-dimensional vector

$$a_A = \begin{pmatrix} \vec{a}(\rho) \\ \vec{a}^\dagger(\rho) \\ \vec{a}(\lambda) \\ \vec{a}^\dagger(\lambda) \end{pmatrix},$$

in which creation and annihilation operators are associated to the *same* irreducible representation.

The state vectors are classified via the chain of embeddings

$$\mathrm{Sp}(12) \supset \mathrm{Sp}(6) \otimes \mathrm{O}(2),$$

$$\mathrm{O}(2) \supset S_3, \text{ and } \mathrm{Sp}(6) \supset \mathrm{SO}(3) \otimes \mathrm{Sp}(2).$$

The O(2) subgroup corresponds to rotations in (ρ, λ) space and is particularly convenient for the enumeration and construction of states of definite symmetry under the permutation group S_3. The SO(3) subgroup gives the angular momentum content; the compact generator of Sp(2) is proportional to the Hamiltonian. Allowed multiplets are therefore classified in finite-dimensional, nonunitary irreducible representations of Sp(12). The relevance of this group to the splitting induced by anharmonic perturbations is that *U is naturally classified under Sp(12)*. For example, at the $N=2$ level, there is a unique breaker of Sp(12) that can contribute, and one may explicitly construct an algebraic mass formula, involving the quadratic Casimir invariants of the various subgroup chains (plus one nonsubgroup invariant), that reproduces the pattern of Fig. 1. More details of both the group-theoretic and explicit methods will be presented in a future publication.[13]

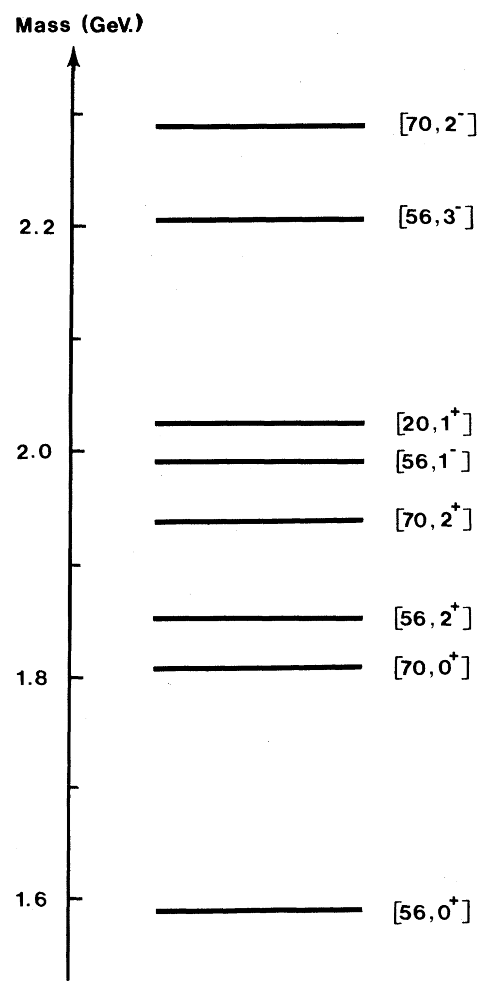

FIG. 2. The mass spectrum of the $N=2$ and some $N=3$ supermultiplets.

The interesting new physics occurs at the $N=3$ level where the following eight multiplets are expected: $[\underline{56},3^-]$, $[\underline{56},1^-]$, $[\underline{70},3^-]$, $[\underline{70},2^-]$, $[\underline{70},1^-]$, $[\underline{70},1^-]$, $[\underline{20},3^-]$, $[\underline{20},1^-]$. Turning on the perturbation splits these multiplets and introduces a new parameter δ, specific to the $N=3$ levels. For example, for the $[\underline{70},3^-]$ we obtain[13]

$$E[\underline{70},3^-]=E_0+3\Omega-\tfrac{7}{10}\Delta+\tfrac{1}{2}\delta.$$

However, the important feature to note here is that the masses of three of the $N=3$ multiplets are independent of δ; *their masses are entirely determined by the $N=2$ level parameters:*

$$E[\underline{70},2^-]=E_0+3\Omega-\tfrac{2}{5}\Delta,$$

$$E[\underline{56},3^-]=E_0+3\Omega-\tfrac{3}{5}\Delta,$$

$$E[\underline{56},1^-]=E_0+3\Omega-\tfrac{11}{10}\Delta.$$

Reasonable phenomenology for the $N=0$, 1, and 2 levels was obtained[3] with $E_0\simeq 1150$ MeV and $\Delta \simeq \Omega \simeq 440$ MeV. Using these values, we predict the mass spectrum of Fig. 2. The mean mass of the $[\underline{56},1^-]$ is around 1985 MeV—close to the mass of the Δ_{D35} at 1930 ± 20 MeV. Given the simplicity of the model and our neglect of the hyperfine interactions, this is startlingly good agreement. We conclude that, in contrast to previous claims, the $\Delta_{D35}(1930)$ does *not* represent unambiguous evidence for new degrees of freedom inside baryons.

The authors thank Ken Barnes, Dave Fradkin, Ron King, Hans Reinders, and Patrick Walters for helpful discussions.

[(a)]Present address: Physics Department, University of Tasmania, Box 252 C, G.P.O., Hobart, Tasmania, Australia. 7001.

[1]For a review, see O. W. Greenberg, Annu. Rev. Nucl. Part. Sci. 28, 327 (1978).

[2]N. Isgur and G. Karl, Phys. Rev. D 18, 4187 (1978), and 19, 2653 (1979).

[3]N. Isgur, in Proceedings of the Sixteenth International School of Subnuclear Physics, Erice, 1978 (to be published).

[4]T. A. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D 12, 2060 (1975).

[5]C. Rebbi, Phys. Rev. D 14, 2362 (1976).

[6]D. Horn and J. E. Mandula, Phys. Rev. D 17, 898 (1978).

[7]E. Witten, Nucl. Phys. B160, 57 (1979).

[8]R. E. Cutkosky *et al.*, Phys. Rev. Lett. 37, 645 (1976).

[9]R. H. Dalitz, R. R. Horgan, and L. J. Reinders, J. Phys. G 3, L195 (1977).

[10]R. E. Cutkosky *et al.*, Phys. Rev. D 20, 2839 (1979).

[11]M. Jones, R. H. Dalitz, and R. R. Horgan, Nucl. Phys. B129, 45 (1977), and earlier references therein.

[12]R. Horgan, J. Phys. G 2, 625 (1976).

[13]K. C. Bowler, P. J. Corvi, A. J. G. Hey, and P. D. Jarvis, to be published.

Chiral-Symmetry Breakdown in Large-N Chromodynamics

Sidney Coleman[(a)]
Stanford Linear Accelerator Center, Stanford University, Stanford, California 93405

and

Edward Witten
Department of Physics, Harvard University, Cambridge, Massachusetts 82138

(Received 9 April 1980)

Chromodynamics with n flavors of massless quarks is invariant under chiral $U(n)\otimes U(n)$. It is shown that in the limit of a large number of colors, under reasonable assumptions, this symmetry group must spontaneously break down to diagonal $U(n)$.

PACS numbers: 12.20.Hx, 11.30.Ly, 11.30.Rd

In nature, the gauge group of chromodynamics is SU(3), and quarks are color triplets. Nevertheless, it is useful to consider generalizations in which the gauge group is SU(N) and quarks are color N-uplets. There are many observed properties of meson dynamics (e.g., Zweig's rule) that can be argued to be exact in the large-N limit; it is tempting to believe that this indicates that large-N chromodynamics is in some sense a good approximation to the real world.[1]

In this note we study large-N chromodynamics with n massless quark N-uplets. This theory is