Magnetic Moment of a Massive Neutrino and Neutrino-Spin Rotation

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A massive Dirac neutrino has a magnetic moment, which causes its spin to precess in a magnetic field. This reduces the effective weak cross sections for relativistic neutrinos. An estimate on the basis of phenomenological considerations as well as the standard electroweak theory indicates that massive neutrinos from supernovae and neutron stars may contain significant mixtures of negative- and positive-helicity states.

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The possibility of nonzero neutrino masses and associated lepton mixing is still an open one.[1] Recent reactor neutrino experiments may, perhaps, indicate that neutrinos do have nonzero masses.[2] As is well known, a massless, chiral neutrino cannot have a nonzero magnetic (or electric) dipole moment. The same is true of a Majorana neutrino, whether massless or massive. However, a massive Dirac neutrino will, in general, have a magnetic moment. We denote this by $\vec{\mu}_{\nu_i} \equiv \mu_{\nu_i}\vec{\sigma}$, where the index i labels the neutrino *mass* eigenstate; $i=1,\ldots,n$ in the n-doublet version of the standard $[SU(2)]_L \otimes U(1)$ electroweak theory considered here.[3] (Where it is obvious, the index i will be suppressed.) In this paper we shall analyze the resultant phenomenon of neutrino-spin rotation in a strong magnetic field and shall show that, although μ_ν is quite small, this rotation may have a significant effect on neutrinos from supernovae and neutron stars.

The neutrino magnetic moment arises at the one-loop level, as does the weak contribution to the anomalous magnetic moment of a charged lepton.[4] The value of μ_ν in the standard theory can be read off from general formulas for the electromagnetic vertex, to one-loop order, of an arbitrary fermion.[5] To leading order in m_l^2/m_W^2, the results are independent of m_l and of the lepton mixing matrix U. We have[5]

$$\mu_\nu = \frac{3e G_F m_\nu}{8\pi^2\sqrt{2}} \tag{1}$$

(where e is positive). Observe that, whereas $\vec{\mu}_\nu$ and $\vec{\sigma}$ are antiparallel for the charged lepton l^-, they are actually parallel for the neutrino. Numerically,

$$\mu_\nu = 1.85\times 10^{-27}[m_\nu/(1\text{ eV})]\text{ eV G}^{-1}. \tag{2}$$

The magnetic moment $\vec{\mu}_\nu$ causes the neutrino spin to precess in the presence of a magnetic field $\vec{B}$. In the standard theory, a neutrino (mass eigenstate) ν is produced as a mixture of helicity states $\psi_\nu(\pm)$ with $h \equiv \hat{\sigma}\cdot\hat{p} = \pm 1$:

$$\psi_\nu(p) = (|a|^2 + |b|^2)^{-1/2}[a\psi_\nu(-) + b\psi_\nu(+)], \tag{3}$$

where $|b/a| \simeq m_\nu/(E + |\vec{p}|)$. A relativistic $\overset{(-)}{\nu}$ is thus predominantly in the $h = \pm 1$ state, respectively. A spin rotation from $\psi_\nu(-)$ toward $\psi_\nu(+)$ will reduce the resultant effective weak neutral- and charged-current scattering cross sections for ν which arise, in the relativistic case, predominantly from the helicity state $\psi_\nu(-)$, and similarly for $\bar{\nu}$.

Let us examine the behavior of a massive neutrino propagating in the presence of a magnetic field through vacuum. We specialize to the case of a field B which is essentially constant over the lengths and times relevant for the passage of a neutrino; this case should be a reasonable first approximation to the situations of astrophysical interest to be discussed below. The Hamiltonian for the neutrino is

$$H = \gamma_0(\vec{\gamma}\cdot\vec{p} - \vec{\mu}_\nu\cdot\vec{B} + m_\nu). \tag{4}$$

The helicity eigenstates $\psi_\nu(\pm)$ are expressed (with an accuracy up to a term linear in B) as

$$\psi_\nu(\pm) = 1/\sqrt{2}\,[(1\pm\rho)^{1/2}\varphi_\nu(-) \mp (1\mp\rho)^{1/2}\varphi_\nu(+)], \tag{5}$$

where $\varphi_\nu(\pm)$, respectively, denote the positive-energy eigenstates of the Hamiltonian (4) with eigenvalues $E_\pm = E \pm \Delta E(\theta)$, where

$$\Delta E(\theta) \equiv \mu_\nu B[\sin^2\theta + (m_\nu/E)^2\cos^2\theta]^{1/2} \tag{6}$$

and $E = (|\vec{p}|^2 + m_\nu^2)^{1/2}$. The angle θ and the parameter ρ are defined by

$$\cos\theta = \hat{p}\cdot\hat{B},$$

$$\rho = (m_\nu/E)\cos\theta/[\sin^2\theta + (m_\nu/E)^2\cos^2\theta]^{1/2}. \tag{7}$$

In the nonrelativistic limit, $E \to m_\nu$, $\rho \to \cos\theta$, and Eq. (5) reduces to

$$\psi_\nu(\pm) = (\cos\tfrac{1}{2}\theta)\varphi_\nu(\mp) \mp (\sin\tfrac{1}{2}\theta)\varphi_\nu(\pm), \tag{8}$$

with Eq. (6) replaced by $E_\pm = E \pm \mu_\nu B$. This is what one expects intuitively. For the relativistic case, $\rho \sim m_\nu/(E\tan\theta) \ll 1$ for $\tan\theta \gg m_\nu/E$, and the state mixing in Eq. (5) becomes nearly maximum, but the energy shift (6) now depends on the angle θ.

A neutrino (mass eigenstate) which starts as $\psi_\nu(-)$ at $t=0$ and propagates for a time t picks up a component along $\psi_\nu(+)$ because of its spin rotation, due to the phase difference in $\varphi_\nu(\pm)\exp\{-i[E \pm \Delta E(\theta)]t\}$. This effect is strongly dependent on the polar angle θ, as Eq. (6) indicates. Note also that the probability for a transition from $\varphi_\nu(+)$ to $\varphi_\nu(-)$ via the emission of a real photon is negligibly small. The effective weak charged- or neutral-current cross sections for a relativistic incident ν_i (with the index i now explicit) are then

$$\sigma(\nu_i,t) = |\psi_{\nu_i}{}^\dagger(-,0)\psi_{\nu_i}(-,t)|^2\sigma(\nu_i,0) \simeq \cos^2[\Delta E_i(\theta)t]\sigma(\nu_i,0). \tag{9}$$

A characteristic length for this rotation is the half-rotation length

$$L_i^{(1/2)}(\theta) = \pi/2\Delta E_i(\theta). \tag{10}$$

If the ν_i propagates a distance $\sim L_i^{(1/2)}$, then its average effective weak cross sections are reduced by a factor of the order of $\langle\cos^2[\Delta E_i(\theta)t]\rangle = \frac{1}{2}$. Numerically, for the symmetric value $\theta = \pi/2$,

$$L_i^{(1/2)}(\theta=\pi/2) = 1.67\times10^{17}[(1\text{ eV})/m_{\nu_i}][(1\text{ G})/B]\text{km}. \tag{11}$$

It should be stressed that neutrino-spin rotation would occur even if there were no lepton mixing and hence no neutrino oscillations.

In the general case of nonzero leptonic mixing, denote the neutrino weak gauge-group eigenstates as ν_a, with $l_a = e, \mu, \ldots, l_n$; in terms of the mass eigenstates these are given by $\nu_a = \sum_{i=1}^n U_{ai}\nu_i$. Then the probability that a ν_a emitted initially with $h=-1$ will, after traveling for a time t, develop a nonzero projection along ν_b with helicity $h=\pm1$ is

$$|\psi_{\nu_b}{}^\dagger(h,0)\psi_{\nu_a}(-1,t)|^2 = \left|\sum_{j=1}^n U_{bj}{}^*U_{aj}z_j\exp(-im_j{}^2t/2p)\right|^2, \tag{12}$$

where $z_j = \cos[\Delta E_j(\theta)t]$ or $\sin[\Delta E_j(\theta)t]$ for $h=-1$ or $+1$, respectively. In this formula we have assumed that the neutrinos are relativistic; it is straightforward to insert the factors of β, $(1 \pm\rho)^{1/2}$, etc., for the general case.

The implications of our analysis of neutrino-spin rotation will obviously be most important in situations where (1) the premise is valid that the medium is either vacuum or, if matter, that it is of sufficiently low density as to have a negligible influence on the passing neutrinos; and (2) there is an extremely strong, coherent (i.e., nonrandom) magnetic field. Such a situation may be realized in the region of space near a supernova or neutron star,[6] where $B \sim 10^{12}$–10^{13} G. The energies of escaping neutrinos are typically of the order of 10 MeV.

Consider first very early times, when T is sufficiently high that $\overset{(-)}{\nu}$'s make a significant contribution to the (anti)neutrino flux from the supernova. Given the bound[7] $m_{\nu_i} < 0.57$ MeV for all dominantly coupled ν_i contained in the gauge-group eigenstate ν_μ, a value such as $m_{\nu_i} = 100$ keV is allowed, at least by particle-physics data.[8] Then, with $B \simeq 10^{13}$ G, $L_i^{(1/2)}(\theta=\pi/2) \simeq 0.17$ km. As the temperature falls below $\sim m_{\nu_i}$ such heavy neutrinos cannot be produced. For the later stages of supernovae and for neutron stars, where $\overset{(-)}{\nu}_e$'s constitute the main component of the (anti)neutrino flux, $L_i^{(1/2)}(\theta)$ is much larger, since $m_{\nu_i} < 35$ eV for all dominantly coupled ν_i in ν_e.[7] However, fields as large as $B \sim 10^{15}$ G have been considered possible for neutron stars.[6] With $m_{\nu_i} = 10$ eV and $B = 10^{15}$ G, $L_i^{(1/2)}(\theta=\pi/2) = 17$ km. Since the scale size of the near magnetosphere of a neutron star is set roughly by its radius, typically ~ 15 km (and by the light-cylinder radius[6]) the above half-rotation length suggests that relativistic neutrinos from such objects could consist of significant mixtures of positive- and negative-helicity states, and similarly for antineutrinos. (This would, of course, be true trivially for nonrelativistic $\overset{(-)}{\nu}_i$.) If, indeed, the mixing is essentially complete, then one must divide the conventional weak cross sections for those (anti)neutrinos by a statistical weight factor of order 2.

Although our main calculations are naturally based on the standard model, it is useful to consider purely empirical bounds on μ_ν and $L^{(1/2)}(\theta)$. If, for example, fermions are composite, the internal structure of a neutrino might give rise to $\mu_\nu \gg \mu_\nu^{(\text{stand})}$ in Eq. (1). Moreover, μ_ν is very sensitive to possible right-handed currents; if present (at a phenomenologically allowed level), these could also lead to $\mu_\nu \gg \mu_\nu^{(\text{stand})}$. Upper bounds on μ_ν are usually quoted in terms of f_ν defined by $\mu_\nu = f_\nu(e/2m_e)$. An astrophysical bound is the most stringent[9]: $f_\nu < 0.85\times10^{-10}$ for $m_\nu \lesssim 10$ keV. From $\bar{\nu}_e e \to \bar{\nu}_e e$ and $\nu_\mu e \to \nu_\mu e$ reactions one has[10] $f_{\nu_e} \lesssim 1.9\times10^{-9}$ and $f_{\nu_\mu} < 0.81\times10^{-8}$. Indirect bounds can be obtained if one assumes that the possible internal structure or unknown dynamics produces comparable contributions to μ_l, $l=e$, μ, and to μ_ν. Then from the agreement[11] of quantum electrodynamic predictions and experimental measurements one again finds $f_\nu \lesssim 10^{-10}$. For comparison, Eq. (2) gives $f_\nu^{(\text{stand})} = 3.20\times10^{-19} m_\nu/(1\text{ eV})$. Thus, phenomenologically (independent of neutrino masses),

$$L^{(1/2)}(\theta = \pi/2) \gtrsim 0.6\times10^9[(1\text{ G})/B]\,\text{km}, \qquad (13)$$

that is, $\gtrsim 6$ cm for $B = 10^{13}$ G.

Finally, we comment on neutrino rotation in dense matter such as that *inside* a supernova core or neutron star. Given the bound (13), for $B \sim 10^{13}$ G, $L_l^{(1/2)}(\theta)$ can be much smaller than the neutrino mean free path $l_{\text{mfp}} \sim 0.1$ km in such an object.[6] However, the situation is complicated by possible coherent weak scattering effects,[12] which could be more important than those due to neutrino rotation, depending on μ_ν, B, E, and θ. If rotation were more important, then it could have a significant influence on the cooling of neutron stars.

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Note added.—Magnetic fields of order 2–3 μG may occur over distances of order kiloparsecs (kpc) in the interstellar medium in galaxies (see Smith, Ref. 6). The bound (13) with $B = 2\ \mu$G gives $L^{(1/2)} \gtrsim 0.01$ kpc, so that interstellar neutrinos may undergo a substantial rotation.

After submission of this work a related paper by Cisneros[13] came to our attention. Cisneros considered the different case of *massless* neutrinos with non-$(V-A)$ couplings in a nonrenormalizable–pregauge-theory framework, obtained an ill-defined, one-loop divergent μ_ν, and discussed the possible effect of spin rotation on solar neutrinos. We disagree with his claim that (for his case of massless neutrinos) "if we have a vector or axial-vector interaction... the resulting magnetic moment... is zero. Any combination of vector and axial vector will similarly give zero magnetic moment... ." [This is true only for massless chiral $(V\pm A)$ neutrinos.] We also disagree with his claim that "since the neutrino appears only on external lines in the process, permitting it to have a nonzero mass will not change the answer (for μ_ν)." We also received a preprint by Lynn and Feinberg[14] on a related subject.

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$$\mu_{\nu_i}1(\text{a}) = k(\tfrac{5}{6} - \tfrac{1}{4}\epsilon), \quad \mu_{\nu_i}1(\text{b}) = k(\tfrac{2}{3} - \tfrac{1}{2}\epsilon),$$

where $k \equiv eG_F m_{\nu_i}/(4\pi^2\sqrt{2})$, whence $\mu_{\nu_i} = \mu_{\nu_i}1(\text{a}) + \mu_{\nu_i}1(\text{b})$, with

$$\epsilon \equiv \sum_{a=1}^{n} (m_{la}^2/m_W^2)|U_{ai}|^2.$$

For the case of nonsinglet ν_R, *not* considered here, the additional contribution to μ_ν can be extracted, with appropriate redefinitions, from R. E. Shrock, Phys. Rev. D 9, 743 (1974), or directly from the general formulas mentioned above; for an exactly massless meutrino it was also given by J. Kim, Phys. Rev. D 14, 3000 (1976). See also W. Marciano and A. I. Sanda, Phys. Lett. 67B, 303 (1977).

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Study of High-Transverse-Momentum π^0 Pairs Produced at the CERN Intersecting Storage Rings

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Correlations of two π^0 mesons with transverse momenta p_T up to 13 GeV/c have been measured with large azimuthal acceptance. Results for cross sections, $p_{\rm out}$, and z distributions are compared in detail with models based on two-constituent scattering and fragmentation. The width of the constituent transverse-momentum Gaussian distribution would have to be doubled to match that from experiments with p_T below 5 GeV/c. A more likely explanation is the presence of processes with more than two constituents in the final state.

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Experiments at PETRA[1] studying jet structure in hadronic reactions from e^+e^- collisions report that Feynman-Field-type models,[2] which describe hadronic final states in terms of two-constituent (quark-antiquark) fragmentation, while fitting the data well below $\sqrt{s} \approx 10$ GeV, fail at higher energies. This failure has been attributed to the lack of a third constituent (gluon) in these models.

The analysis of jet structure in hadronic collisions is substantially more difficult, primarily because of the presence of "spectator" particles not associated with the jets, and the (not well understood) degrees of freedom of the confined constituents in the initial state, such as their transverse momentum and longitudinal rapidity. These parameters, not encountered in e^+e^- collisions, cause ambiguity in identifying the jet-associated particles.

Despite these problems, much progress has been made by studying the characteristics of high-transverse-momentum particles produced in high-energy hadron collisions.[3] Experimentally observed[4] correlations of such particles have been compared to predictions of quantum chromody-

