CP Nonconservation in Cascade Decays of B Mesons

Ashton B. Carter and A. I. Sanda
Rockefeller University, New York, New York 10021
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General techniques are introduced to expose new CP-nonconserving effects in cascade decays of B mesons. These effects are computed in the Kobayashi-Maskawa model. The CP asymmetries so obtained range from 2% to 20% if the parameters are in the favorable range $s_3 < s_2 \lesssim 0.1$. Effects of this size should be observable in upcoming experiments.

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Observation of CP nonconservation has so far been confined to the strange-neutral-meson system. There the symmetry-nonconserving effects are attributable solely to the presence of CP impurities in the K_L and K_S. There is *a priori* no reason to suppose that symmetry nonconservation will be exhibited so discreetly in other systems.[1] On the contrary, general expectation holds that, as new flavor thresholds are passed, CP nonconservation will reveal itself more pervasively and more boldly, extending to on-shell transitions and exceeding the asymmetries, of order $\epsilon_K \sim 10^{-3}$, observed in the kaon system. These expectations are particularly keen for transitions involving the bottom (b) quark.

Within the six-quark Kobayashi-Maskawa (KM) model,[2] these general expectations are fulfilled. In this model, CP nonconservation cannot occur in on-shell transitions below the charm threshold. The only way for symmetry-nonconserving phases to enter the strange sector is through the off-shell transitions to heavy flavors which occur during K_0-$\overline{K}_0$ mixing. Above the charm threshold, but below the bottom threshold, symmetry nonconservation in on-shell transitions is possible, but the asymmetries generated are necessarily of order $s_2 s_3 \sin\delta$ ($\cong 10^{-3}$ to 10^{-4}, as determined from ϵ_K), where $s_i = \sin\theta_i$, and θ_i and δ are the familiar KM mixing angles and phase, respectively.

Above the bottom threshold, on-shell effects can be large, of order $\sin\delta$. However, most authors[3] who have previously studied CP nonconservation in the bottom sector have confined themselves to estimates of CP admixture in the B_0-$\overline{B}_0$ system, in rather strict analogy to the K_0-$\overline{K}_0$ and D_0-$\overline{D}_0$ systems. [Here B_0 stands for either $B_s{}^0 = (b\overline{s})$ or $B_d{}^0 = (b\overline{d})$.] Yet it appears that the observable effects due to CP impurities in the mixed B_0-$\overline{B}_0$ states are in fact rather small.

In this paper, we emphasize the importance of studying on-shell transitions in the bottom sector, as opposed to virtual mixing effects. The former can, we argue, produce CP asymmetries of order 10^{-1}–10^{-2}, whereas the latter is known to produce only about 10^{-3}–10^{-4}. One method of exhibiting the KM phase, which we shall call Method (A), involves producing, through mixing, a coherent beam of B_0 and $\overline{B}_0$ and observing a decay channel to which both components contribute. The total rate samples interference between the two decay amplitudes, displaying the CP-nonconserving KM phase δ. In contrast to earlier suggestions for detecting CP nonconservation in the B_0-$\overline{B}_0$ system, this method makes use of the mixing, which might be strong, but does not rely solely upon the CP impurity in the mixing, which by itself cannot produce large symmetry-nonconserving effects. Rather, the CP-nonconserving phase enters in the on-shell transitions which make up the decay cascades of the B_0 and $\overline{B}_0$. In Method (B) one seeks two different coherent cascade decays of the bottom quark to a common hadronic state.[4] Interference of the two cascade amplitudes depends upon the CP-nonconserving phase δ. Method (A) produces CP asymmetries proportional to the parameters characterizing the B_0-$\overline{B}_0$ mixing. Method (B) does not depend upon mixing and can be applied to charged as well as neutral mesons, but it involves measurement of decay modes suppressed by one power of the mixing angles. In what follows we shall give examples of both methods and analyze the magnitude of the effects produced.

Method (A).—Denote the mass eigenstates of the B_0-$\overline{B}_0$ system by

$$B_{1,2} = pB_0 \pm q\overline{B}_0, \tag{1}$$

where p and q are complex numbers satisfying $|p|^2 + |q|^2 = 1$. Let $m_{1,2}$ and $\Gamma_{1,2}$ be the masses and widths of $B_{1,2}$. Then the time dependence of a state which is B_0 at $t=0$ is given by

$$|B_0(t)\rangle = f_+(t)|B_0\rangle + (q/p)f_-(t)|\overline{B}_0\rangle, \tag{2}$$

where

$$f_\pm(t) = \tfrac{1}{2}\{\exp[it(-m_1+\tfrac{1}{2}i\Gamma_1)]\pm\exp[it(-m_2+\tfrac{1}{2}i\Gamma_2)]\}.$$

The dominant decay channels of the components of this mixture are $B_0\to D_0+X$ and $\bar{B}_0\to\bar{D}_0+X$, where X stands for hadrons common to both decays. The two components can be made to interfere by selecting a common decay channel of D_0 and $\bar{D}_0$: $B_0\to D_0+X\to K_S+Y+X$; $\bar{B}_0\to\bar{D}_0+X\to K_S+Y+X$. Define $M=\langle K_S+X+Y|H|B_0\rangle$ and $\bar{M}=\langle K_S+X+Y|H|\bar{B}_0\rangle$, choose a phase convention $CP|B_0\rangle=-|\bar{B}_0\rangle$, and take $CP|K_S+X+Y\rangle=|K_S+X+Y\rangle$. (Here we set $\epsilon_K=0$ and use spherical symmetry.) If CP is a good symmetry, $M=-\bar{M}$. Denoting by $\Gamma,\bar{\Gamma}$ the width for $B_0(t),\bar{B}_0(t)\to K_S+Y+X$ we obtain, upon integration over time,

$$\left\{\begin{matrix}\bar{\Gamma}\\ \Gamma\end{matrix}\right\}\sim\left\{\begin{matrix}|p|^2\\ |q|^2\end{matrix}\right\}\{1\pm\alpha+|\lambda|^2(1\mp\alpha) - 2\,\mathrm{Re}(y\pm ix\alpha)\lambda\},\qquad(3)$$

where $y=(\Gamma_1-\Gamma_2)/(\Gamma_1+\Gamma_2)$, $x=2(m_1-m_2)/(\Gamma_1+\Gamma_2)$, $\alpha=(1-y^2)/(1+x^2)$, and $\lambda=pM/q\bar{M}$. In terms of the off-diagonal element of the B_0-$\bar{B}_0$ mass matrix, $M_{12}-\frac{1}{2}i\Gamma_{12}$, we have $p/q=M_{12}/|M_{12}|[1+O(\mathrm{Im}(\Gamma_{12}/2M_{12}))]$. It can be shown that[3] $\mathrm{Im}\Gamma_{12}/2M_{12}=O((m_c^2-m_u^2)/m_b^2)$ and $M_{12}/|M_{12}|=U_{bt}^{*2}/|U_{bt}|^2$. ($U_{ij}$ denotes an element of the KM matrix.) M describes $b\to c\to s$ cascade decay and it is proportional to $U_{bc}U_{sc}^*$. With this, we obtain

$$(\Gamma-\bar{\Gamma})/(\Gamma+\bar{\Gamma})\cong -x\alpha\sin2\varphi/(1+y\cos2\varphi),\qquad(4)$$

where $\varphi=\arg(U_{bt}U_{sc}U_{bc}^*)$.

Instead of considering a single B_0 or $\bar{B}_0$ produced at $t=0$, a somewhat more practical situation imagines a B_0-$\bar{B}_0$ pair produced in an e^+e^- colliding beam. For instance, the processes

$$e^+e^-\to\text{“}\Upsilon\text{”}\to B_d^0\bar{B}_d^0+X_1\to K^\pm K_S+X^\mp\qquad(5)$$

can occur frequently since they represent one of the major modes of B mesons. We further restrict $B_d^0\bar{B}_d^0$ to be in the S wave in their rest frame. Note that “Υ”$\to\bar{B}^0B^{0*}\to\bar{B}^0B^0\gamma$ produces an S-wave $\bar{B}^0B^0$ pair. Decay of one of the mesons to K^- or K^+ at some reference time t specifies that meson as either B_d^0 or $\bar{B}_d^0$. This in turn determines the other meson as a particular linear combination of B_d^0 and $\bar{B}_d^0$ at that time. The wave function of the second meson is then known for all previous and subsequent times, and we may ask for the probability that it decays during this history to K_S+X+Y. Integrating over the reference time t then gives the probability for final states $K^\pm K_SX^\mp$. Denoting the cross sections to these states by $\sigma^\pm$, we obtain

$$a\equiv\frac{\sigma^+-\sigma^-}{\sigma^++\sigma^-}=-\frac{2x\alpha^2\sin2\varphi}{1+y^2+2y\cos2\varphi}.\qquad(6)$$

($a=0$ if $B_d^0\bar{B}_d^0$ are in the odd-angular-momentum state in their rest frame.)

Method (*B*).—The quark amplitudes of Fig. 1 correspond to decays of the type[5] $B^-\to DK_SX_n$ and $B^-\to\bar{D}K_SX_n$. The two amplitudes can be made to interfere if a common decay channel $K_S+K_S+X_n'$ is detected. Let $A_ne^{i\eta_n}$ and $\bar{A}_ne^{i\bar{\eta}_n}$ be the magnitudes and phases associated with the strong interactions which organize the final states. Since the isospin structures of the two amplitudes are different, in general $\eta_n\neq\bar{\eta}_n$. If we denote by $\Gamma^\pm$ the widths for $B^\pm\to K_SK_S+$anything, $f=U_{bc}U_{su}$, $g=U_{bu}U_{sc}$,

$$\frac{\Gamma^+-\Gamma^-}{\Gamma^++\Gamma^-}=\frac{-A\,\mathrm{Im}(f^*g)}{B|f|^2+C|g|^2+D\,\mathrm{Re}(f^*g)},\qquad(7)$$

where $A=\sum_n2A_n\bar{A}_n\sin(\eta_n-\bar{\eta}_n)$, $B=\sum A_n^2$, $C=\sum\bar{A}_n^2$, and $D=\sum2A_n\bar{A}_n\cos(\eta_n-\bar{\eta}_n)$.

We shall briefly discuss the magnitude of these CP asymmetries taking a [Eq. (6)] as an example. Present knowledge of the KM matrix[6] is crude, and we can expect no more than a rough estimate. For this purpose, we take $c_i=1$, $y\ll x$, and $x=(M_t^2/700\ \mathrm{GeV}^2)/[(1+\xi\cos\delta)^2(1+\tau^2)]$ (see Ref. 7), where $\xi=s_2/s_3\tau=\xi\sin\delta/(1+\xi\cos\delta)$. In terms of τ, we have $a=4x\tau/[(1+x^2)^2(1+\tau^2)]$. We first obtain $\sin\delta$ for each M_t^2, s_2, and ξ by requiring[8] $|\mathrm{Re}\epsilon_K|=1.6\times10^{-3}$ and then compute a. Figure 2 shows a for $\cos\delta>0$. We have also examined a for the case $\cos\delta<0$ as well as the asymmetries given in Eqs. (4) and (7). From these studies, we conclude that if nature chooses $s_2\lesssim0.1$ and $s_3/s_2\lesssim1$, there is a good possibility that a is large enough ($\gtrsim$ 2%–20%) to be observed in e^+e^-

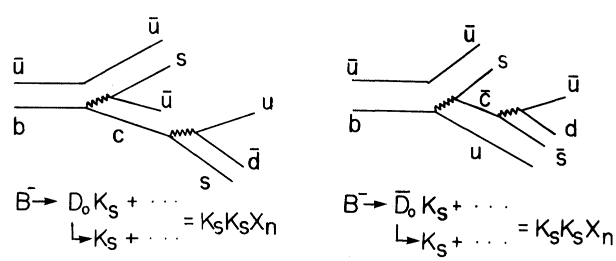

FIG. 1. Two amplitudes which can be made to interfere by selecting out the final state containing K_S. The CP nonconserving phase, in this case $\arg(U_{bc}U_{bu}^*)$, can be measured by observing asymmetries in cascade decays of B mesons involving on-shell intermediate states.

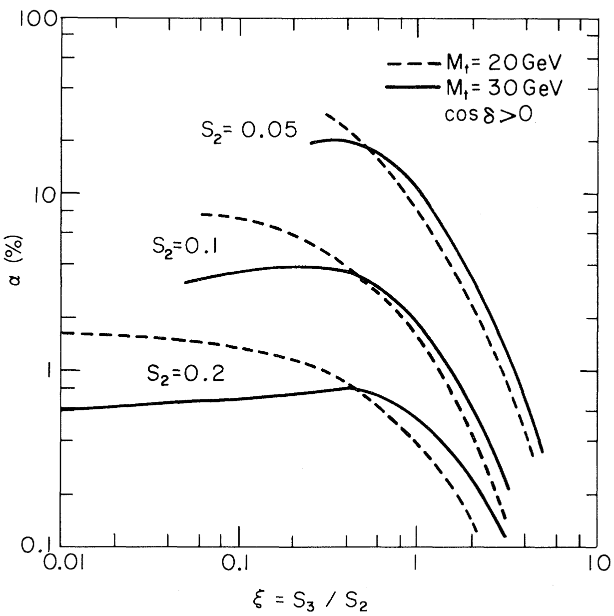

FIG. 2. The asymmetry a for $\cos\delta > 0$, and $M_t = 20$ and 30 GeV. We have also studied the $\cos\delta < 0$ case. While for $s_2 > 0.2$ or for $\xi > 3$, a is too small to be measured, if $s_2 \lesssim 0.1$ and $\xi < 1$, a should be large enough to be measured in e^+e^- colliding-beam experiments.

colliding-beam experiments. If nature chooses[6,8] $s_2 \gtrsim 0.2$, $a < 1\%$ and it is too small to be observed in the near future.

These examples demonstrate the possibility of truly dramatic CP-nonconserving effects in B-meson decays. The important theme is to focus upon symmetry nonconservation in cascade decays involving on-shell intermediate states as opposed to mixing effects involving virtual intermediate states. This implies observation of hadronic as opposed to semileptonic final states. We also stress the importance of observing final states containing K_S, since these can arise from subprocesses possessing CP-nonconserving relative phases.

We would like to emphasize that these asymmetries, of $O(\sin\delta)$, are the maximum obtainable within the KM model. Thus, though our selection of examples is illustrative rather than exhaustive, we believe that the techniques we describe offer a new possibility for exposing CP nonconservation in B-meson decays.

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[8]Information on the allowed range for s_2, s_3, δ, and M_t can be obtained from a comparison of the theoretical computations of $\mathrm{Re}\,\epsilon_K$ and $M_{K_L-K_S}$ with experimental measurements. The bag-model computation (Ref. 6) leads to a result $s_2 \gtrsim 0.02$ while the free-quark–model computation allows for a much smaller s_2. For our purpose of showing a possibility of large asymmetry, we have used the free-quark model. Note that experimental information on x, the $B^0\bar{B}^0$ mixing parameter discussed in Ref. 1, will be available independently from measurement of a. The detailed analysis of a, including the model dependence mentioned above, should await the measurement of x.