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Do Black Holes Really Evaporate Thermally?

Frank J. Tipler
Center for Theoretical Physics, University of Texas, Austin, Texas 78712
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The Raychaudhuri equation is used to analyze the effect of the Hawking radiation back reaction upon a black-hole event horizon. It is found that if the effective stress-energy tensor of the Hawking radiation has negative energy density as expected, then an evaporating black hole initially a solar mass in size must disappear in less than a second. This implies that either the evaporation process, if it occurs at all, must be quite different from what is commonly supposed, or else black-hole event horizons—and hence black holes—do not exist.

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The most exciting development in general-relativity theory during the past decade was Hawking's proof[1,2] that an approximately stationary black hole would necessarily emit thermal radiation with a temperature $1/8\pi M$. The great interest of this result lies in the fact that it indicates a deep connection between the apparently disparate disciplines of general relativity, quantum mechanics, and thermodynamics.[3] However, since no quantized theory of gravitation yet exists, Hawking's calculation was necessarily based on a semiclassical approximation in which the quantized fields were assumed to be acting in a classical Schwarzschild geometry that was itself unaffected to first order by the presence of these quantized fields. A large number of authors[3-8] have attempted to carry Hawking's analysis to a higher order by first calculating the expectation value $\langle T_{ab}\rangle$ of the regularized stress-energy tensor of these quantized fields at the black-hole event horizon, and then studying the effect of this $\langle T_{ab}\rangle$ upon the time evolution of the event horizon. Because of the limitations of present techniques, no definite expression for $\langle T_{ab}\rangle$ has become generally accepted, but a consensus has been reached about certain properties of $\langle T_{ab}\rangle$: First, $\langle T_{ab}\rangle l^a l^b$ is *negative* at the horizon, where l^a is the tangent vector to a null geodesic generator of the horizon; second, the stationary approximation is valid except for times near the final stages of black-hole evaporation. I shall show in this paper that in fact these two properties are inconsistent: An evaporating black hole for which the Hawking static analysis is valid for even a brief period and which has $\langle T_{ab}\rangle l^a l^b < 0$ during its future evolution must violate the approximately static assumption almost immediately. For a solar-mass black hole, the static approximation would break down in less than a second, and smaller black holes would violate it even more rapidly. If one really believed the semiclassical equations of evolution for the horizon, one would be forced to conclude that a solar-mass black hole would completely evaporate in less than a second. I will end the paper with a brief discussion of the implications of this result for black-hole evaporation.

The semiclassical approximation assumes that a black-hole event horizon exists, and that its future evolution is governed by the Raychaudhuri equation[9]

$$d\rho/dv = \rho^2 + \sigma\bar{\sigma} + 2\epsilon\rho + 4\pi\langle T_{ab}\rangle l^a l^b. \qquad (1)$$

The relation between ρ and the area A of the black-hole horizon is

$$\rho = -(1/2A)dA/dv. \qquad (2)$$

For simplicity I will consider only spherically symmetric black holes, and so $\sigma\bar{\sigma} \equiv 0$. If the time parameter along the horizon is chosen in the usual way (e.g., see Ref. 9, p. 47) to be nonaffine but to coincide with the Schwarzschild time t at infinity, then $v = t$ and

$$\epsilon = 1/8M. \qquad (3)$$

(This will be the value of ϵ if the black hole is roughly static.) If the evaporating black hole were approximately static, we could ignore $d\rho/dt$ and ρ^2 in Eq. (1), and we would have[7]

$$\rho = -(2\pi/\epsilon)\langle T_{ab}\rangle l^a l^b. \qquad (4)$$

This implies that $\langle T_{ab}\rangle l^a l^b$ is negative by Eq. (2), since it is assumed that $dA/dt < 0$ for an evaporating black hole. In fact, we must have $dA/dt < 0$ by conservation of energy if the black hole can be approximated by the Schwarzschild solution to first order. Furthermore, if this approximation were valid, one would have

$$\rho = -L/M, \text{ or } dA/dt = -32\pi LM, \qquad (5)$$

where

$$L = -dM/dt \cong 10^{-71}(M_\odot/M)^2 M_\odot \text{ sec}^{-1}. \qquad (6)$$

Since $A = 16\pi M^2$, the conventional picture of black-hole evaporation is illustrated in Fig. 1(a).

Unfortunately, this conventional picture is inconsistent with Eq. (1); it is not possible to neglect the terms $d\rho/dt$ and ρ^2 for any significant length of time. This can be clearly seen by transforming the Riccati equation (1) into a second-order linear equation via the variable change $x = A^{1/2}$, or $\rho = -x^{-1}dx/dt$. This gives

$$d^2x/dt^2 = 2\epsilon\, dx/dt - 4\pi\langle T_{ab}\rangle l^a l^b x. \qquad (7)$$

If (4) holds initially, then the two terms on the right-hand side of (7) cancel. This means dx/dt = const, and so $2\epsilon\, dx/dt \sim 1/x$ and $-4\pi\langle T_{ab}\rangle l^a l^b x \sim 1/x^3$. Thus d^2x/dt^2 would become positive, and would remain positive, since dx/dt becomes smaller in magnitude if d^2x/dt^2 is positive. The true evolution of the horizon must therefore be pictured in Fig. 1(b): dA/dt must initially be very negative, but its magnitude must *decrease* with time until the singularity is reached.[10] The negative energy density $\langle T\rangle_{ab} l^a l^b$ gives rise to a *repulsive* gravitational force which tends to *slow down* the rate of horizon-area decrease. A singularity must be reached before dA/dt turns positive, since otherwise $A \to +\infty$ as $t \to +\infty$. (With spherical symmetry a singularity can occur only if $A = 0$ in the maximal extension.)

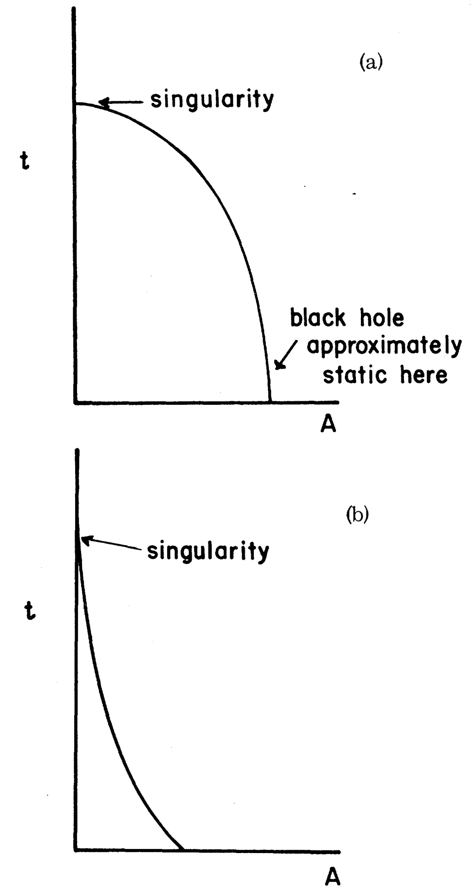

FIG. 1. (a) The conventional picture of black hole evaporation. (b) The actual behavior of a black hole event horizon during the evaporation.

Since dA/dt is decreasing in magnitude, we can ignore the ρ^2 term in (1) except near the singularity. Thus (1) is approximately a linear equation in ρ with the general solution

$$\rho = \exp\left(2\int_0^t \epsilon\, dt'\right)\int_0^t \exp\left(-2\int_0^{t''} \epsilon\, dt'''\right)(4\pi\langle T_{ab}\rangle l^a l^b)\,dt'' + \rho(0)\exp\int_0^t \epsilon\, dt'. \qquad (8)$$

Since $\rho(0) > 0$, the singularity is reached (or rather the static approximation breaks down) at or before $\rho(t)$ becomes negative. One can obtain an upper bound on this time by assuming

$$4\pi\langle T_{ab}\rangle l^a l^b = -2\epsilon\rho(0) \text{ in } (0, t_1)$$

and $4\pi\langle T_{ab}\rangle l^a l^b = -2\epsilon\rho(0) - \delta$ $(\delta > 0)$ thereafter, where δ is given by assuming the static approximation holds at t_1. We have, if $\epsilon \sim \text{const} = \epsilon_0$,

$$\rho \leq -(\delta/2\epsilon)\exp[2\epsilon_0(t - t_1)] + \rho(0) + (\delta/2\epsilon_0), \qquad \text{for } t \geq t_1. \quad (9)$$

If the static approximation held in the interval $(0, t_1)$, Eq. (5) would give $\rho(0)/\rho(t_1) = (M_{t_1}/M_0)^3$. Equation (4) applied at $t = 0$ and $t = t_1$ then implies

$$\frac{4\pi\langle T_{ab}\rangle l^a l^b|_{t=t_1}}{4\pi\langle T_{ab}\rangle l^a l^b|_{t=t_0}} = \frac{\epsilon_{t_1}\rho_{t_1}}{\epsilon_0\rho(0)} = \left(\frac{M_0}{M_{t_1}}\right)^4.$$

Thus $\delta/2\epsilon_0 \cong [(M_0/M_{t_1})^4 - 1]\rho(0)$. If (6) is integrated between $t = 0$ and t we get

$$t_1 = 10^{71}(M_t/M_\odot)^3[(M_0/M_t)^3 - 1]$$

which allows us to use M_t as a time parameter by setting $t_1(M_\odot/M_{t_1})^3 = 1$ sec. Since $(M_0/M_t)^4 - 1 > (M_0/M_t)^3 - 1$, we have from (9)

$$\rho(t) \leq \rho(0)[-10^{-71}\exp 2\epsilon(t - t_1) + 1]. \quad (10)$$

Since $2\epsilon = (5\times 10^4 \text{ sec}^{-1})M_\odot/M$, it follows that $\rho(t)$ must go negative in less than a second; the black hole must either come to an end at a singularity, or else the static approximation must break down.

It is illustrative to compare the above analysis with the standard perturbation analysis for black holes (e.g., see Ref. 9) which also uses the solution (8). In the latter case the boundary condition $\rho(\infty) = 0$ is imposed at $t = +\infty$ rather than $t = 0$, and this kills the homogeneous part of (8) which is the source of the instability in the black-hole evaporation. Such a future boundary condition cannot be imposed in the evaporation case, for it would be inconsistent with the initially static approximation, the negative energy density, and the fact that an evaporating black hole does not settle down, but instead hits a singularity.

Three ways to avoid the conclusions of the above analysis spring to mind. First, we could accept the earlier contention of Boulware[11] and Gerlach[12] that quantum effects prevent horizons from ever forming.[13] In this case there would be no objection to $A \to +\infty$ for spherical null congruences, since A would no longer be the area of an event horizon. However, this would mean that black holes in the usual sense do not exist, since a black hole is defined to be $M - J^-(\mathscr{I}^+)$. Second, we could abandon the belief that $\langle T_{ab}\rangle l^a l^b$ is everywhere negative. It is conceivable that $\langle T_{ab}\rangle l^a l^b$ could be negative in some average sense, but locally $\langle T_{ab}\rangle l^a l^b$ might fluctuate to positive values, which would invalidate the above analysis.[14] However, it is not obvious that such violent fluctuations at the event horizon would preserve the thermal nature of the Hawking radiation. Third, black-hole evaporation may not occur at all. In any case, the effect of the quantum back reaction must be much more complex than is generally believed.

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[5] S. M. Christensen, Phys. Rev. D 14, 2490 (1976).

[6] P. Candelas and D. W. Sciama, Phys. Rev. Lett. 38, 1372 (1977).

[7] P. Candelas, Phys. Rev. D 21, 2185 (1980). Equation (4) can also be derived not by making the assumption that $d\rho/dt$ and ρ^2 are second order, but by reversing the order of the argument in section VI of Candelas' paper.

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[9] S. W. Hawking, in *Black Holes*, edited by C. DeWitt and B. S. DeWitt (Gordon and Breach, New York, 1973), p. 46.

[10] If the horizon geodesic generators are affinely parametrized which implies $\epsilon = 0$, then $\rho \equiv -x^{-1}dx/dv$ gives $d^2x/dv^2 = -4\pi\langle T_{ab}\rangle l^a l^b x$, which obviously must be positive.

[11] D. G. Boulware, Phys. Rev. D 11, 1404 (1976).

[12] U. Gerlach, Phys. Rev. D 14, 1479 (1976).

[13] Thus the Boulware vacuum would have to be regarded as more physical than the other vacua which have been considered. See Ref. 7 for a discussion of these.

[14] I am grateful to Professor W. G. Unruh for suggesting this possibility.