[4]S. R. Amendolia *et al.*, Nuovo Cimento A17, 735 (1973).
[5]R. Kephart *et al.*, Phys. Rev. D 14, 2909 (1976).
[6]F. W. Büsser *et al.*, Phys. Lett. 55B, 232 (1975); C. Kourkoumelis *et al.*, Phys. Lett. 84B, 277 (1979). At low p_T, both π^0 and η invariant cross sections were assumed to vary as $\exp^{(-bm_T)}$, where $m_T=(p_T^2+m^2)^{1/2}$ and $b=5.07\ \mathrm{GeV}^{-1}$.
[7]G. W. Abshire *et al.*, Nucl. Instrum. Methods 164, 67 (1979).
[8]D. Hitlin *et al.*, Nucl. Instrum. Methods 137, 225 (1976).
[9]Complete tabulation of our results is available in David Lloyd Owen, PhD thesis, State University of New York, Stony Brook, 1980 (unpublished).
[10]F. W. Büsser *et al.*, Phys. Lett. 46B, 471 (1973).
[11]A. L. S. Angelis *et al.*, Phys. Lett. 79B, 505 (1978).
[12]E. W. Beier *et al.*, Phys. Rev. D 18, 2235 (1978).
[13]G. Donaldson *et al.*, Phys. Lett. 73B, 375 (1978).
[14]D. C. Carey *et al.*, Phys. Rev. D 14, 1196 (1976).
[15]F. E. Taylor *et al.*, Phys. Rev. D 14, 1217 (1976).
[16]R. M. Baltrusaitis *et al.*, Phys. Rev. Lett. 44, 122 (1980).
[17]K. Eggert *et al.*, Nucl. Phys. B98, 49 (1975).

Neutron Oscillations and the Existence of Massive Neutral Leptons

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The neutron-antineutron transition which changes baryon number by two units is considered under some general assumptions. The resultant neutron oscillation time scale is found to vary as M^4, with M the unification mass scale, if there exists a neutral, massive, Majorana lepton. In this case, the oscillation time scale is comparable to the proton lifetime. However, in the presence of matter or external magnetic fields, the detection of such oscillations, at the present time, seems improbable.

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Baryon number (B) nonconservation is a natural consequence of grand unified theories[1] of strong, weak, and electromagnetic interactions. The most spectacular process of this kind is the decay of the proton, which has been amply discussed in the literature.[2] Recently, Weinberg[3] and Wilczek and Zee[4] discussed a general method of dealing with B nonconserving processes. They pointed out that such processes are governed by a local effective Lagrangian containing only ordinary particles. The Lagrangian is invariant under $\mathrm{SU}(3)_c\otimes\mathrm{SU}(2)\otimes\mathrm{U}(1)$, and has an effective coupling proportional to M^{4-d}, where M is the unification mass and d is the dimension of the operator Lagrangian. Thus they showed that the dominant B-nonconserving process varies as $(M^{-2})^2$ and satisfies the selection rules $\Delta B=1$, $\Delta(B-L)=0$, where L is the lepton number.

It is straightforward to generalize their method and construct the next most important B-nonconserving effective Lagrangian involving only fermions, which necessarily takes the form[5]

$$\mathcal{L}_{\mathrm{eff}}\sim\epsilon_{\alpha\beta\gamma}\epsilon_{\xi\eta\sigma}[\bar{q}_\alpha{}^c q_\beta][\bar{q}_\gamma{}^c q_\xi][\bar{q}_\eta{}^c q_\sigma], \tag{1}$$

so that $d=9$. Thus, $\mathcal{L}_{\mathrm{eff}}$ describes a $\Delta B=2$ process and has an effective coupling of M^{-5}. A particular example of such a process is the transition of a neutron (N) into an antineutron ($\bar{N}$). The mixing of N and $\bar{N}$ arising from $\mathcal{L}_{\mathrm{eff}}(\Delta B=2)$ can be treated in complete analogy to the K^0-$\bar{K}^0$ system, where the weak interaction produces a $\Delta S=2$ effective Lagrangian which mixes K^0 and $\bar{K}^0$. Thus, just as the K^0-$\bar{K}^0$ transition generates "kaon (strangeness) oscillations", the N-$\bar{N}$ transition generates "neutron (baryon) oscillations". This oscillation is characterized by the period $2\pi\tau_{N\bar{N}}$, where

$$\tau_{N\bar{N}}=|(2\delta m)^{-1}|. \tag{2}$$

Here δm is the matrix element governing the $N\bar{N}$ mixing in the neutron mass matrix,

$$\delta m=\langle\bar{N}|-\int d^3x\,\mathcal{L}_{\mathrm{eff}}(x)|N\rangle. \tag{3}$$

Unlike the proton decay, whose rate is second order in $\mathcal{L}_{\mathrm{eff}}(\Delta B=1)$, the $N\bar{N}$ transition rate is linear in $\mathcal{L}_{\mathrm{eff}}(\Delta B=2)$. It is thus conceivable that $\tau_{N\bar{N}}$ could be comparable to τ_p, the proton lifetime. However, when $\mathcal{L}_{\mathrm{eff}}$ is given by an $\mathrm{SU}(3)_c\otimes\mathrm{SU}(2)\otimes\mathrm{U}(1)$-invariant six-quark local operator, dimensional arguments suggest that $\delta m\sim m_N{}^6/M^5$, where m_N is the nucleon mass. This translates into $\tau_{N\bar{N}}\sim\tau_p\,(M/m_N)$. Such time scales are still completely inaccessible to experimental observation.

The above argument is predicated on the assumption that there is only one mass scale, the unification mass M. It is possible to change the above result provided another mass scale becomes relevant. This can happen in processes which do not conserve weak isospin. For instance, a neutral lepton, $l_L{}^0$, might acquire a Majorana mass, m_{l^0}, via the $\Delta I_{\text{weak}} = 1$ transition $l_L{}^0 \to (l_L{}^0)^c$. Such a coupling can be instrumental in allowing for $\Delta B = 2$ processes. A prototype of such a process is given by the diagram of Fig. 1. Note that, because of $B - L$ conservation for $\Delta B = 1$ processes, it is necessary to have the transition $l_L{}^0 \to (l_L{}^0)^c$ in order to mediate the $\Delta B = 2$ transition.

If all such neutral leptons acquire their masses through the superheavy Higgs sector, then m_{l^0} is necessarily of order[3] $\alpha_{GU} G_F{}^{-1} M^{-1}$, where α_{GU} is the fine structure constant of the grand unification group and $G_F \simeq 10^{-5} m_N{}^{-2}$ is the Fermi constant. In this case, the effective couplings for $\mathcal{L}_{\text{eff}}(\Delta B = 2)$ are again of order M^{-5}, and we are back where we started. On the other hand, if massive leptons exist with a mass m_{l^0} such that $\alpha_{GU} G_F{}^{-1} M^{-1} \ll m_{l^0} \ll M$, then $\mathcal{L}_{\text{eff}}(\Delta B = 2)$ has an effective coupling of order M^{-4}. This leads to a $\tau_{N\bar{N}}$ comparable to τ_p.

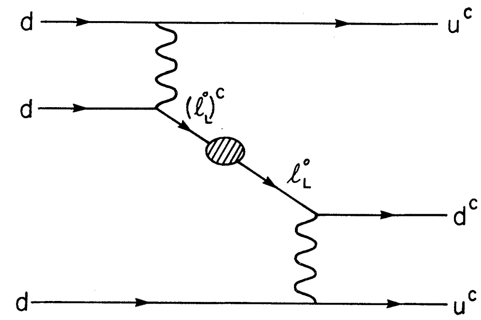

FIG. 1. Feynman diagram contributing to the neutron-antineutron transition. The transition is mediated by the exchange of two superheavy vector bosons and involves the mixing of a massive neutral lepton with its charge conjugate. This mixing is represented by the blob.

To make things more definite, we now give a crude estimate of δm by approximately evaluating the contribution from the superheavy vector exchange diagram of Fig. 1 with use of SU(5) as the underlying grand-unification group. This is done by collapsing the vector lines and using the effective four-Fermi interaction

$$(\alpha_{GU}/M^2)\epsilon_B 2\pi \epsilon_{\alpha\beta\gamma}[\bar{u}_{\gamma R}{}^{c}\gamma_\mu d_{\beta L}][\bar{l}_L{}^{0c}\gamma^\mu d_{\alpha R}] + \text{H.c.}. \tag{4}$$

Here ϵ_B is a Cabibbolike mixing factor for the lepton sector. Note that, under our hypothesis of a massive neutral lepton, ϵ_B cannot be "rotated away." We also emphasize that[6] ϵ_B need not be related to the Cabibbo mixing factors appearing in the couplings of the usual W boson to the leptons. Since the effective couplings of Eq. (4) are implicitly defined at the unification scale M and we eventually want to take matrix elements between neutron and antineutron states, we must include an enhancement factor due to quantum-chromodynamics (QCD) renormalizations occurring between the unification scale and the neutron mass scale μ (≈ 1 GeV). This enhancement factor is given by[7]

$$A = [\alpha_s(\mu)/\alpha_{GU}]^E, \tag{5}$$

where the exponent E is $4/(11 - \frac{2}{3}f)$, and where $\alpha_s(\mu)$ is the QCD coupling at the scale μ and f is the number of quark flavors. We now make the drastic approximation of collapsing the lepton lines and inserting a factor of $m_{l^0}/m_N{}^2$. While this may be a very crude procedure, we do not expect it to change the order of magnitude of our estimate. After making a Fierz transformation and including all the factors, the contribution of Fig. 1 may be written as

$$\mathcal{L}_{\text{eff}} = \frac{\alpha_{GU}{}^2}{M^4}\left(\frac{m_{l^0}}{m_N{}^2}\right)\epsilon_B 8\pi^2 A(\epsilon_{\alpha\gamma\xi}\epsilon_{\beta\eta\sigma} + \epsilon_{\beta\gamma\xi}\epsilon_{\alpha\eta\sigma})[\bar{d}_{\alpha L}{}^c d_{\beta L}][\bar{d}_{\gamma R}{}^c u_{\xi R}][\bar{d}_{\eta R}{}^c u_{\sigma R}]. \tag{6}$$

The matrix element $\langle \bar{N} | -\int d^3x\, \mathcal{L}_{\text{eff}}(x) | N\rangle$ can now be evaluated using nonrelativistic SU(6) wave functions for N and $\bar{N}$ and by applying nonrelativistic limits to the field operators in Eq. (6). We thus obtain

$$\delta m = \frac{\alpha_{GU}{}^2}{M^4}\,\frac{m_{l^0}}{m_N{}^2}\,\epsilon_B 480\pi^2 A\,|\psi(0)|^4. \tag{7}$$

Here $|\psi(0)|^2$ is the probability of finding two quarks at the same point in the neutron. By use of[6]

$$|\psi(0)|^2 \simeq 1/\pi R^3, \tag{8}$$

where R is the neutron radius, the neutron oscillation period is then given by

$$2\pi\tau_{N\bar{N}} \simeq \frac{2\pi}{960}\frac{R^6 m_N{}^2}{m_{t0}\epsilon_B A}\frac{M^4}{\alpha_{GU}{}^2}. \tag{9}$$

It is useful to compare this with the estimate for the proton lifetime,[7]

$$\tau_p \simeq \frac{3}{10}\frac{R^3}{m_N{}^2 A}\frac{M^4}{\alpha_{GU}{}^2}. \tag{10}$$

The ratio of these two expressions gives

$$\frac{\tau_{N\bar{N}}}{\tau_p} \simeq \frac{1}{288}\frac{R^3 m_N{}^4}{m_{t0}\epsilon_B} \simeq \frac{3}{20}\frac{m_N}{m_{t0}\epsilon_B}, \tag{11}$$

where in obtaining the second estimate we have used $R \simeq \frac{3}{4}$ fm. Finally, choosing the Cabibbo factor $\epsilon_B \simeq 5\times10^{-2}$ and $m_{t0} \simeq 300$ MeV, we find

$$\tau_{N\bar{N}} \approx 10\tau_p. \tag{12}$$

Thus we see that $\tau_{N\bar{N}}$ is comparable to the proton lifetime τ_p, especially in light of the sensitivity of the above result to the value of $|\psi(0)|^2$ (or R) which is rather uncertain. The numerical value of $\tau_{N\bar{N}}$ is secured from Eq. (9) by use of $\alpha_s(\mu \simeq 1\ \mathrm{GeV}) \approx \frac{1}{2}$, $\alpha_{GU} \simeq \frac{1}{50}$, $f=6$, and[2] $M \simeq 3\times10^{14}$ GeV in addition to the above chosen values for R, m_{t0}, and ϵ_B. This yields

$$\tau_{N\bar{N}} \simeq 10^{61}(\epsilon_B m_{t0})^{-1} \simeq 10^{31}\ \mathrm{yr}, \tag{13}$$

and corresponds to $\delta m \simeq 10^{-54}$ eV.

Although the neutron oscillation time scale in vacuum is comparable to the estimated proton lifetime, the presently proposed experimental disigns to measure the proton lifetime are incapable of detecting these oscillations. The origin of the difficulty is a suppression of the N-$\bar{N}$ transition amplitude due to the difference in the effective mass of the N and $\bar{N}$ in their interactions with ordinary matter and external magnetic fields.

To ascertain the impact of the different effective masses on N-$\bar{N}$ transitions, we consider the N-$\bar{N}$ mass matrix

$$\begin{bmatrix} (m_N)_{\mathrm{eff}} & \delta m \\ \delta m & (m_{\bar{N}})_{\mathrm{eff}} \end{bmatrix}, \tag{14}$$

where $(m_N)_{\mathrm{eff}} \neq (m_{\bar{N}})_{\mathrm{eff}}$ and δm is given by Eqs. (3) and (7). Diagonalizing this mass matrix yields the eigenvalues

$$m_{1,2} = \tfrac{1}{2}\{[(m_N)_{\mathrm{eff}} + (m_{\bar{N}})_{\mathrm{eff}}] \pm [((m_N)_{\mathrm{eff}} - (m_{\bar{N}})_{\mathrm{eff}})^2 + 4\delta m^2]^{1/2}\} \tag{15}$$

and the corresponding eigenvectors

$$|N_1\rangle = \cos|N\rangle + \sin\theta|\bar{N}\rangle,$$
$$|N_2\rangle = -\sin\theta|N\rangle + \cos\theta|\bar{N}\rangle, \tag{16}$$

where

$$\tan\theta = \frac{2\delta m}{(m_N)_{\mathrm{eff}} - (m_{\bar{N}})_{\mathrm{eff}} + \{[(m_N)_{\mathrm{eff}} - (m_{\bar{N}})_{\mathrm{eff}}]^2 + 4\delta m^2\}^{1/2}}. \tag{17}$$

A neutron state $|N\rangle$ at $t=0$ evolves into a linear combination of $|N\rangle$ and $|\bar{N}\rangle$ given by

$$(\cos^2\theta + \sin^2\theta e^{-i\Delta m t})\exp(im_1 t)|N\rangle + \cos\theta\sin\theta(1-e^{-i\Delta m t})\exp(im_1 t)|\bar{N}\rangle, \tag{18}$$

where

$$\Delta m = m_1 - m_2 = \{[(m_N)_{\mathrm{eff}} - (m_{\bar{N}})_{\mathrm{eff}}]^2 + 4\delta m^2\}^{1/2}. \tag{19}$$

In an external magnetic field, $|(m_N)_{\mathrm{eff}} - (m_{\bar{N}})_{\mathrm{eff}}| = 2\mu_N B \simeq 10^{-11}$ eV, for $B \simeq 1$ G. An even larger effective mass difference arises from the additional annihilation channels open to the $\bar{N}$ when present in nuclear matter. This difference[8] is given by

$$(m_N)_{\mathrm{eff}} - (m_{\bar{N}})_{\mathrm{eff}} \simeq (2\pi n/m_N)(f_N - f_{\bar{N}}), \tag{20}$$

where n is the particle density in nuclear matter and f_N ($f_{\bar{N}}$) is the forward scattering amplitude of N ($\bar{N}$). We estimate $|(m_N)_{\mathrm{eff}} - (m_{\bar{N}})_{\mathrm{eff}}| \simeq 1$ MeV. Thus, the N-$\bar{N}$ oscillation amplitude has a relative suppression factor

$$\tan\delta \simeq \delta m/[(m_N)_{\mathrm{eff}} - (m_{\bar{N}})_{\mathrm{eff}}]$$

which renders the oscillations unlikely to be seen.

In conclusion, we find that, theoretically, the most promising $\Delta B = 2$ process seems to be neutron oscillations. Experimentally, however, they cannot be seen until the tremendous difficulties presented by the existence of external magnetic fields and nuclear matter can be surmounted.

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Note added.—After this work was submitted for publication, we received a preprint by Marshak and Mohapatra,[9] which also discusses neutron oscillations, albeit in a somewhat different framework. It has also come to our attention that neutron oscillations have been considered by Glashow.[10]

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[1]J. C. Pati and A. Salam, Phys. Rev. D 8, 1240 (1973); H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974).

[2]There has been a succession of estimates for the unification mass M and the proton lifetime starting with H. Georgi, H. R. Quinn, and S. Weinberg, Phys. Rev. Lett. 33, 451 (1974). Subsequently, these estimates have been refined and improved by A. Buras, J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. B135, 66 (1978); D. Ross, Nucl. Phys. B140, 1 (1978); T. J. Goldman and D. A. Ross, Phys. Lett. 84B, 208 (1979); W. Marciano, Rockefeller University Report No. COO-2232B-173, 1979 (to be published); M. Machacek, Harvard University Report No. HUTP-79/A021, 1979 (to be published).

[3]S. Weinberg, Phys. Rev. Lett. 43, 1566 (1979).

[4]F. Wilczek and A. Zee, Phys. Rev. Lett. 43, 1571 (1979).

[5]The most general $SU(3)_c \otimes SU(2) \otimes U(1)$-invariant Lagrangian, containing u and d quarks only, which changes baryon number by two units can be written in terms of the six local operators

$$\{[\bar{d}_{\alpha L}{}^c u_{\beta L}][\bar{d}_{\gamma L}{}^c u_{\xi L}][\bar{d}_{\eta R}{}^c d_{\sigma R}] - [\bar{d}_{\alpha L}{}^c d_{\beta L}][\bar{u}_{\gamma L}{}^c u_{\xi R}][\bar{d}_{\eta R}{}^c d_{\sigma R}]\}\Gamma_{\alpha\beta\gamma\xi\eta\sigma}{}^{(1)}, \qquad \text{(i)}$$

$$[\bar{d}_{\alpha L}{}^c u_{\beta L}][\bar{d}_{\gamma L}{}^c u_{\xi L}][\bar{d}_{\eta R}{}^c d_{\sigma R}]\Gamma_{\alpha\beta\gamma\xi\eta\sigma}{}^{(2)}, \qquad \text{(ii)}$$

$$[\bar{d}_{\alpha L}{}^c u_{\beta L}][\bar{d}_{\gamma R}{}^c u_{\xi R}][\bar{d}_{\eta R}{}^c d_{\sigma R}]\Gamma_{\alpha\beta\gamma\xi\eta\sigma}{}^{(2)}, \qquad \text{(iii)}$$

$$[\bar{u}_{\alpha R}{}^c u_{\beta R}][\bar{d}_{\gamma R}{}^c d_{\xi R}][\bar{d}_{\eta R}{}^c d_{\sigma R}]\Gamma_{\alpha\beta\gamma\xi\eta\sigma}{}^{(1)}, \qquad \text{(iv)}$$

$$[\bar{d}_{\alpha R}{}^c u_{\beta R}][\bar{d}_{\gamma R}{}^c u_{\xi R}][\bar{d}_{\eta R}{}^c d_{\sigma R}]\Gamma_{\alpha\beta\gamma\xi\eta\sigma}{}^{(1)}, \qquad \text{(v)}$$

$$[\bar{d}_{\alpha R}{}^c u_{\beta R}][\bar{d}_{\gamma R}{}^c u_{\xi R}][\bar{d}_{\eta R}{}^c d_{\sigma R}]\Gamma_{\alpha\beta\gamma\xi\eta\sigma}{}^{(2)}, \qquad \text{(vi)}$$

where

$$\Gamma_{\alpha\beta\gamma\xi\eta\sigma}{}^{(1)} = \epsilon_{\alpha\gamma\eta}\epsilon_{\beta\xi\sigma} + \epsilon_{\beta\gamma\eta}\epsilon_{\alpha\xi\sigma} + \epsilon_{\alpha\xi\eta}\epsilon_{\beta\gamma\sigma} + \epsilon_{\beta\xi\eta}\epsilon_{\alpha\gamma\sigma}$$

and

$$\Gamma_{\alpha\beta\gamma\xi\eta\sigma}{}^{(2)} = \epsilon_{\alpha\gamma\xi}\epsilon_{\beta\eta\sigma} + \epsilon_{\beta\gamma\xi}\epsilon_{\alpha\eta\sigma}$$

are $SU(3)_c$ tensors which insure that the operators are $SU(3)_c$ singlets. Here and elsewhere in the text, all left-handed fermion fields transform as SU(2) doublets, while right-hand fermion fields are SU(2) singlets. Fierz transformations have been used to eliminate vector and tensor couplings.

[6]C. Jarlskog, Phys. Lett. 82B, 401 (1979); R. N. Mohapatra, Phys. Rev. Lett. 43, 893 (1979).

[7]A. Buras, J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. B135, 66 (1978).

[8]See, for example, P. K. Kabir, *The CP Puzzle; Strange Decays of the Neutral Kaon* (Academic, New York, 1968), p. 67.

[9]R. E. Marshak and R. N. Mohapatra, Phys. Rev. Lett. 44, 1316 (1980).

[10]S. Glashow, in *Proceedings of Neutrino '79, International Conference on Neutrinos, Weak Interactions, and Cosmology, Bergen, Norway, 1979,* edited by A. Haatuft and C. Jarlskog (Fysik Institutt, Bergen, 1980).