are a couple of inaccuracies in this paper: The Jastrow function is obtained by solving the *un*reduced $s$-wave equation and thus differs slightly from that of Ref. 9 of the paper. In addition the numbers which label the vertical axis of Fig. 1 should be multiplied by a factor of 2.

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# Convection in Dilute Solutions of $^3$He in Superfluid $^4$He

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Thermal convection in dilute solutions of $^3$He in superfluid $^4$He has been studied in a novel cell of unity aspect ratio which permits inhomogeneity in the heat flux to be measured. At least three different convecting states have been observed, including a time-dependent state with diminished thermal conductance which can exist very close to the onset of convection.

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Dilute solutions of $^3$He in superfluid $^4$He constitute an important model system for studying nonlinear thermohydrodynamics[1-3] in general and thermal convection in particular, because of the advantages both of the cryogenic environment for measurement[4] and of the superfluid for its unique properties. In this work we have studied thermal convection in a unity aspect ratio, vertical-axis, cylindrical cell with insulating side walls and constant-temperature horizontal bounding planes, with the novel feature that horizontal inhomogeneity in the heat flux out of one bounding plane can be measured in addition to the usual total thermal conductance. Using this arrangement, we have observed a convective instability in a cell heated from above and have found several convective structures. These include two steady-state structures, probably corresponding to toroidal flow with center either falling or rising, and time-dependent flow structures. The latter includes a spectacular structure with a characteristic frequency which displays a critical–slowing-down effect, with a diminished overall thermal conductance, and with a horizontal inhomogeneity in the heat flux which oscillates between that for center fluid rising and that for center fluid falling.

The experimental fluid was confined to a cylindrical volume with axis within 0.5 degree of vertical and bounded radially by a fused quartz annulus ground to 4.14 cm i.d. and 2.07 cm height, with faces parallel and perpendicular to the axis, and typical tolerances of 1–2 $\mu$m. Against each face of the quartz annulus was held, by means of springs, a 2-cm-thick cylinder of copper. These copper cylinders bounded the fluid above and below. The faces in contact with the quartz and the fluid were mechanically polished until flat to the same tolerance. This entire assembly was contained in a close-fitting vessel with stainless-steel walls and low longitudinal conductance.

The bottom copper slab was placed in thermal contact with a $^3$He refrigerator. The temperature of this slab, indicated by a GRT-200A-100 germanium thermometer calibrated by Lakeshore Cryotronics, was maintained constant electronically. The upper copper slab was constructed out of two pieces, a ring-shaped piece with a central tapered hole of approximately 1.4 cm smallest diameter and a plug-shaped tapered cylinder. These were pressed together with a 12-$\mu$m sheet of Mylar plastic carefully fitted between them so as to provide some thermal isolation of the plug from the ring. The surfaces of this assembly were then machined and polished. A germanium thermometer and a four-wire Manganin heater were slid into close-fitting holes in each of the plug and the ring. These two heaters and two thermometers were used to maintain the temperature of the ring ($T_R$) *equal* to the temperature of the plug ($T_P$) and *greater* than that of the lower slab by a known amount ($\Delta T$). The total heat flow out of the plug ($\dot{Q}_P$) was measured separately from the total heat flow out of the ring ($\dot{Q}_R$), thus giving a measure of the horizontal homogeneity of the heat flux into the fluid. This feature of the measurement geometry is indispensable for providing a deeper understanding of the convection motion.

Measurements of the effective thermal conductance, defined as $K_{\rm eff} \equiv (\dot{Q}_P + \dot{Q}_R)/\Delta T$, and the power ratio $\dot{Q}_P/\dot{Q}_R$ were made at several temperatures between 0.8 and 1.0 K, on solutions with $^3$He mole fraction $x = 0.013$, 0.0047, and 0.0024. As in the $^3$He-$^4$He solutions above the $\lambda$ point,[5,6] the fluid allows considerable flexibility in the choice of the onset temperature difference for thermal convection, ranging from tens of millidegrees Kelvin at low concentrations and temperatures to unmeasurably small temperature differences at larger concentrations and temperatures. The qualitative features of the convecting system, however, were essentially unchanged over this entire range of temperature and concentration. For this reason, the data presented here are all chosen from a particular temperature, 0.925 K, and concentration, 0.24%.

In Fig. 1 both $K_{\rm eff}$ and the power ratio are plotted against the temperature difference across the cell for the time-independent data. The conductance varies continuously with temperature difference with a slight rounding at the transition to a convecting state. A critical temperature difference for convection, $\Delta T_c$, was defined experimentally by fitting $K_{\rm eff}$ above and below the transition to straight lines whose intersection was taken to be at $\Delta T_c$. At low temperature differences the heat flow is diffusive. The effective conductance is independent of temperature difference, as is the ratio of plug to ring power. The power ratio measurements, in this regime, are scattered about the approximate ratio of the plug area to the ring area in contact with the fluid. At larger temperature differences, as the thermal conductance of the cell increases because of convection, the relative power required to maintain the plug temperature usually drops continuously to about 9% of the ring power. The measurements were not taken in any particular order, nor was it found to matter. All of the early measurements on this convecting system were found to have power ratios below that of the nonconvecting state. By special preparation of a given temperature difference, however, a similar state could be formed with approximately the same thermal conductance but a power ratio near 14%, corresponding to the upper branch in Fig. 1(b).

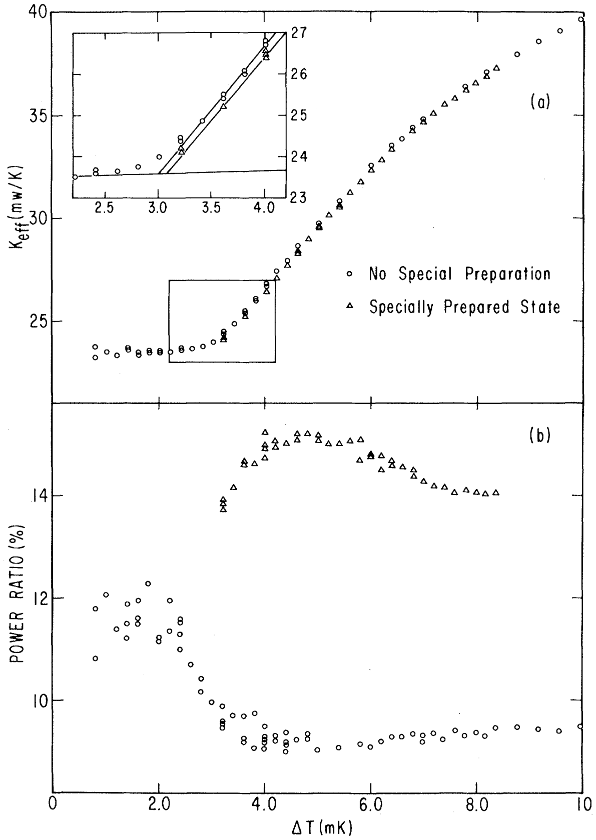

FIG. 1. (a) $K_{\rm eff}$ as a function of the temperature difference across the cell, for $x = 0.0024$ and $T = 0.925$ K. The inset shows the region near $\Delta T_c$ in greater detail, including the small difference in $\Delta T_c$ between the state with center fluid rising and center fluid falling. (b) The power ratio of the two time-independent states for the same data as shown in (a).

The experimental fluid in our apparatus has an aspect ratio, defined as the ratio of cell radius to cell height, of very nearly unity. For a cell of this shape a normal, one-component fluid has been observed[7] to convect in a single toroidal roll. From this knowledge and our data we concluded that we most commonly produced a single toroidal roll convecting such that the center of the cell had its convective velocity directed downward, and that by initially driving the convection we could produce the nearly symmetric state with the velocity in the center directed upward. This less stable state was found to have a slightly higher $\Delta T_c$ than the state with the center falling. [Note inset to Fig. 1(a).] In addition, there existed a regime near $\Delta T_c$ where the center-rising state would decay in time to the more stable state. These qualitative features appeared at all temperatures and concentrations studied.

As the temperature difference was further increased, in the early weeks of the measurements, the fluid would usually make a discontinuous transition to a time-dependent state. Once time dependence had occurred it could be eliminated only by reducing the temperature difference to well below $\Delta T_c$. The transition to time dependence was more likely to occur at larger temperature differences, but in the early days of the experiment would occur at $\Delta T_c \lesssim 2\Delta T_c$. After the apparatus had been cold for several months, the transition to time dependence became impossible for us to induce, although many attempts were made. These included temperature-difference step changes of both signs, heat pulses, inhomogeneous heating and heat pulses, and square-wave modulation of heat flow, both homogeneously and inhomogeneously, at the appropriate frequency for the time-dependent state. Warming the apparatus to liquid-nitrogen temperature and then reestablishing the usual conditions rendered the fluid susceptible to a time-dependent state as before.

In the time-dependent state the average thermal conductance was well below the corresponding time-independent value. This usually made the transition to time dependence unmistakable, if not dramatic. For reduced temperature difference $\Delta t \equiv (\Delta T - \Delta T_c)/\Delta T_c$ less than 0.5, the time dependence was nonperiodic with low amplitude and low characteristic frequency. At slightly larger $\Delta t$, a large-amplitude feature would occasionally be visible in the power ratio but not easily observed in $K_{\text{eff}}$. At even larger $\Delta t$ we observed a state characterized by intermittent, large-amplitude oscillations in the power ratio. For $\Delta t \gtrsim 1$, these large-amplitude oscillations were well developed and of sufficiently high frequency to be measured. In all cases the time dependence of the power ratio was much more regular and well defined than that of $K_{\text{eff}}$.

In Fig. 2 the time-averaged frequency, as averaged by hand over 10–20 periods of the power-ratio oscillations, is plotted against reduced-temperature difference, $\Delta t$, over a wide range. The characteristic frequency of oscillation depends linearly on $\Delta t$ for small values of this quantity. This illustrates the critical slowing down of this time-dependent state.[8] At larger $\Delta t$ the measured frequencies fall below those predicted by the linear dependence. The maximum frequencies meas-

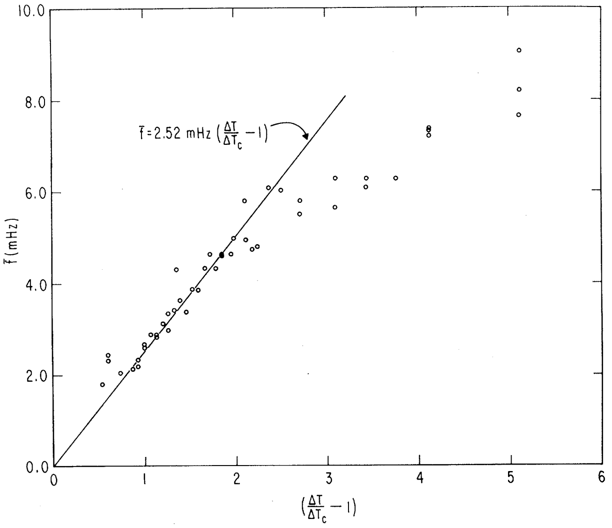

FIG. 2. Time-averaged frequency of the power-ratio oscillations in the time-dependent state as a function of $\Delta t$, the reduced-temperature difference. The averages were taken by hand over 10 to 20 periods.

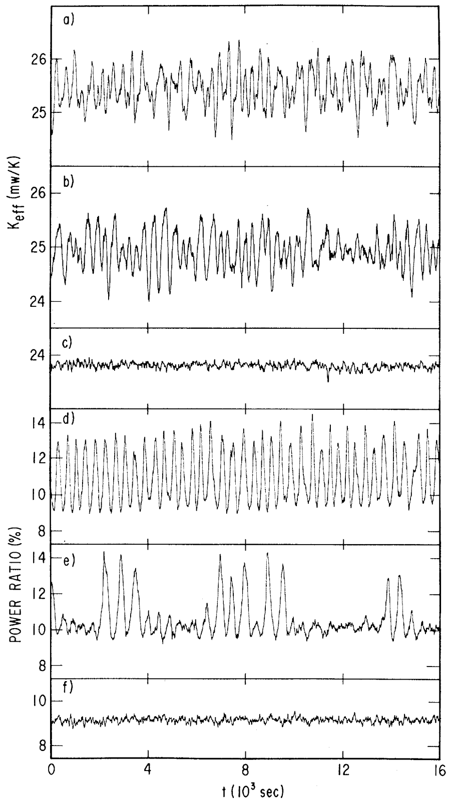

FIG. 3. Time dependence of, respectively, $K_{\text{eff}}$ and power ratio for $\Delta t = 0.998$ [(a) and (d)]; $\Delta t = 0.735$ [(b) and (e)], illustrating the intermittency in the power ratio; and $\Delta t = 0.72$ [(c) and (f)], plotted to show the noise level for a time-independent datum near $\Delta T_c$.

ured roughly coincide with the inverse thermal time constant of the fluid. It is reasonable to assume that some maximum limiting frequency would exist. Finally, it is important to point out that both frequency and temperature differences could be measured accurately with significantly greater precision than is indicated by the scatter in the data. This undoubtedly indicates that the spectral features corresponding to these frequencies are not sharp. Work is in progress to measure the spectra of both $K_{\mathrm{eff}}$ and the power ratio.

A plot of both the power ratio and $K_{\mathrm{eff}}$ versus time at three temperature differences is shown in Fig. 3. In traces (a) and (d) the oscillations are well developed. At a smaller temperature difference intermittent oscillations may be visible, as in (b) and (e). Traces (c) and (f) correspond to a time-independent state at $\Delta t = 0.072$ and illustrate the noise level. Several qualitative features are immediately clear from these plots. First, referring to Fig. 3(d) the power ratio oscillates roughly between its value, Fig. 1(b), near 9% for steady-state center fluid falling and near 14% for steady-state center fluid rising. Second, the oscillations in the power ratio are asymmetric, with the ratio more often near 9% than 14%. Third, the maxima in the power ratio are not nearly as consistent as the minima. Fourth, there is a characteristic frequency of $K_{\mathrm{eff}}$ which is approximately twice the frequency observed in the power ratio. These observations are consistent with the possibility that in this oscillatory state the fluid spends most of the time in a center-falling state but regularly makes excursions to a center-rising state. That is, this oscillatory state may be a toroidal roll structure which regularly slows, stops, and reverses briefly.

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