# Evidence for Three-Gluon Coupling in $e^+e^-$ Annihilation

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(Received 28 March 1980)

The forward-backward asymmetry of the normal to the event plane for $e^+e^-$ annihilation into three jets in higher-order perturbation-theory ($\geqslant \alpha_s^2$) quantum chromodynamics is calculated. This could serve to establish the non-Abelian gluon self-couplings.

PACS numbers: 14.80.Kx, 12.40.Cc, 13.65.+i

Quantum chromodynamics (QCD) predicts a multijet structure in $e^+e^-$ annihilation into hadrons,[1-8] which has been nicely confirmed by the recent PETRA data.[9-11] The mere observation of three- (and higher-) jet final states, however, does not prove QCD. Almost any other field theory with a small fixed-point coupling will lead to a similar structure, and a quantitative test of QCD at this level might be opposed by nonperturbative and the unavoidable higher-twist contributions.[2,12]

In order to furnish unambiguous evidence of perturbative QCD, it is essential to construct observables that are sensitive to the distinctive elements of the theory. Since QCD shows its full gauge structure only in order $\alpha_s^2$, these must either involve four-jet final states or go beyond the tree-graph level. In this note we propose a *discriminative test of QCD* falling into the latter category, which has the unique feature of giving direct indication of the chromodynamic gluon self-couplings and which appears to be feasible experimentally.

The observables we discuss concern three-jet production in $e^+e^-$ annihilation. One requires longitudinally polarized beams, the other the presence of weak and electromagnetic interferences.

Let us first assume that the electrons and/or positrons are longitudinally polarized. Electrons and positrons naturally tend to polarize themselves in the direction of the magnetic field, i.e., transversally, and it seems feasible to rotate the transverse polarization into a longitudinal one.[13] The achievement of longitudinal polarization is also important in the exploration of weak-interaction contributions to the $e^+e^-$ annihilation process. It allows the extraction of physical information similar to that obtainable from helicity measurements of the final state. At DESY PETRA (~19 GeV/beam) and SLAC PEP (~18 GeV/beam) energies and higher we can hope for a large sample of cleanly separated three-jet events. The cross section for $e^+e^- \to \gamma^* \to q(p_1)\bar{q}(p_2)g(p_3)$ for longitudinally polarized beams is given by[5,14]

$$2\pi \frac{d^4\sigma}{d\cos\theta\, d\chi\, dx_1 dx_2} = \tfrac{3}{8}(1+Z)(1+\cos^2\theta)\frac{d^2\sigma_U}{dx_1 dx_2} + \tfrac{3}{4}(1+Z)\sin^2\theta \frac{d^2\sigma_L}{dx_1 dx_2}$$
$$+\tfrac{3}{4}(1+Z)\sin^2\theta\cos 2\chi \frac{d^2\sigma_T}{dx_1 dx_2} - \frac{3}{2\sqrt{2}}(1+Z)\sin 2\theta \cos\chi \frac{d^2\sigma_I}{dx_1 dx_2}$$
$$+\frac{3}{\sqrt{2}} L \sin\theta \sin\chi \frac{d^2\sigma_H}{dx_1 dx_2}, \quad Z = \xi^{(-)}\xi^{(+)}, \quad L = \xi^{(-)} + \xi^{(+)}, \tag{1}$$

where $x_{1,2} = 2E_{1,2}/(q^2)^{1/2}$ and $\xi^{(-)}$ and $\xi^{(+)}$ denote the longitudinal polarization of the electron and positron beam, respectively. For pure helicity states $\xi^{(\mp)} = \pm h^{(\mp)}$, where $h^{(\pm)}$ is the helicity of $e^\pm$, respectively. The first part of the cross section (1) being proportional to $(1+Z)$ [i.e., the remnant of (1) for $L=0$] is familiar.[5] The angle $\theta$ is the polar angle between the electron beam direction and the direction of the fastest of either quark or antiquark jet, while $\chi$ is the azimuthal angle defined in Fig. 1 which determines the orientation of the $q\bar{q}g$ production plane with respect to the scattering plane.

The cross section $\sigma_H$ associated with longitudinally polarized beams involves the *imaginary*

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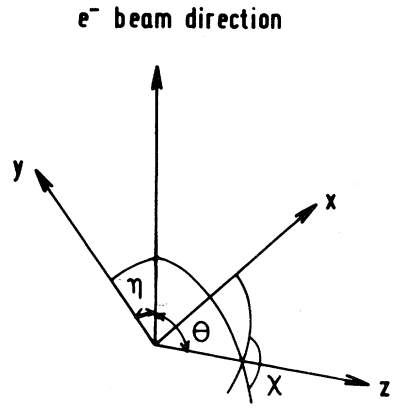


FIG. 1. Three-jet kinematics. The event lies in the $(x, z)$ plane. Our convention for the angles $\theta$, $\chi$, and $\eta$ is as follows: $\overrightarrow{Oz}$ is along the direction of the fastest of either quark or antiquark jet, and the second most energetic quark/antiquark jet lies in the half-plane $x > 0$. That means the normal $\overrightarrow{Oy}$ to the event plane is in the direction $\mathrm{sig}(x_1 - x_2)\vec{p}_1 \times \vec{p}_2$. The ranges of the angles $\theta$, $\chi$, and $\eta$ are $0 \leq \theta \leq \pi$, $0 \leq \chi \leq 2\pi$, and $0 \leq \eta \leq \pi$.

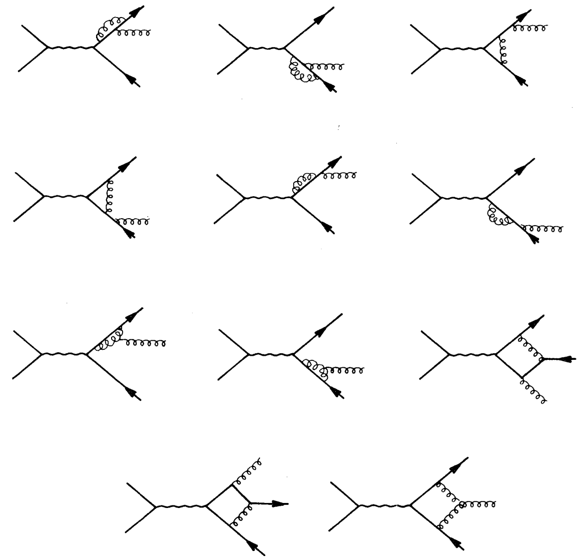

FIG. 2. Lowest-order QCD diagrams that contribute to the imaginary part of the hadronic tensor.

*part* of the helicity +1 and 0 interference amplitude (photon helicity axis $\overrightarrow{Oz}$, cf. Fig. 1), i.e.,

$$\frac{d^2\sigma}{dx_1 dx_2} \sim \mathrm{Im}\epsilon_\alpha(+) H_{\alpha\beta}\epsilon_\beta(0)^*, \quad \alpha,\beta = 1,2,3. \tag{2}$$

This projects onto the antisymetric part of the hadronic tensor. Noticing that $\sin\theta \sin\chi = \cos\eta$, being the polar angle between the normal to the event plane and the electron beam direction (Fig. 1), the polarization-dependent cross section is readily seen to be related to the $T$-odd observable[15]

$$L \sin\theta \sin\chi \frac{d^2\sigma_H}{dx_1 dx_2} \sim \vec{S} \cdot (\vec{p}_1 \times \vec{p}_2), \quad \vec{S} = L\vec{e}^{(-)} = (h^{(-)} - h^{(+)})\vec{e}^{(-)} \tag{3}$$

($\vec{e}^{(-)}$ unit vector in the direction of the electron beam), that will manifest itself in an asymmetry under $\eta \to \pi - \eta$.

Here we will be concerned with $\sigma_H$, which is particularly interesting for testing QCD since, being proportional to the imaginary part of the hadronic tensor, it only contributes to order $\alpha_s^2$ and higher. The pendant in heavy vector-meson production in $e^+e^-$ annihilation followed by its decay into three gluons has been investigated by De Rújula, Petronzio, and Lautrup.[17] Though $\sigma_H$ is zero for massless quarks,[18] we predict a sizable cross section for heavy quarks.

One now may ask how $\sigma_H$ can serve to establish QCD since it is not directly proportional to the gluon self-couplings but involves also other diagrams (Fig. 2). The answer is that, being an interference cross section, it can assume either sign, and it turns out that this is negative for a non-Abelian gauge theory while positive for an Abelian vector theory. Scalar and pseudoscalar theories, on the other hand, can already be excluded by measuring the angular distribution of the thrust axis.[12,19]

To isolate $\sigma_H$ we need to keep only the $\eta$ dependence in (1). We define the normal to the event plane to point into the direction[20] $\vec{p}_1 \times \vec{p}_2$ if $|\vec{p}_1| > |\vec{p}_2|$, and $\vec{p}_2 \times \vec{p}_1$ if $|\vec{p}_2| > |\vec{p}_1|$. This convention has been chosen since it should be relatively easy to locate the heavy-quark (-antiquark) jets. Equation (1) then reduces to

$$\frac{d^3\sigma}{d\cos\eta\, dx_1 dx_2} = \tfrac{3}{8}(1+Z)(1+\tfrac{1}{2}\sin^2\eta)\frac{d^2\sigma_U}{dx_1 dx_2} + \tfrac{3}{4}(1+Z)(1-\tfrac{1}{2}\sin^2\eta)\frac{d^2\sigma_L}{dx_1 dx_2} + \tfrac{3}{4}(1+Z)(-1+\tfrac{3}{2}\sin^2\eta)\frac{d^2\sigma_T}{dx_1 dx_2} + \frac{3}{\sqrt{2}} L\cos\eta \frac{d^2\sigma_H}{dx_1 dx_2}, \tag{4}$$

where $\sigma_I$ drops out. This leads us to define the asymmetry

$$A \equiv \left[\frac{d^3\sigma(\cos\eta = |\cos\eta|)}{d\cos\eta\, dx_1 dx_2} - \frac{d^3\sigma(\cos\eta = -|\cos\eta|)}{d\cos\eta\, dx_1 dx_2}\right]\left[\frac{d^3\sigma(\cos\eta = |\cos\eta|)}{d\cos\eta\, dx_1 dx_2} + \frac{d^3\sigma(\cos\eta = -|\cos\eta|)}{d\cos\eta\, dx_1 dx_2}\right]^{-1}$$

$$= \frac{8}{\sqrt{2}}\left(\frac{L}{1+Z}\right)\cos\eta \frac{d^2\sigma_H}{dx_1 dx_2}\left[(1+\tfrac{1}{2}\sin^2\eta)\frac{d^2\sigma}{dx_1 dx_2} + (1-\tfrac{3}{2}\sin^2\eta)\left(\frac{d^2\sigma_L}{dx_1 dx_2} - 2\frac{d^2\sigma_T}{dx_1 dx_2}\right)\right]^{-1}, \tag{5}$$

where $d^2\sigma/dx_1 dx_2 = d^2(\sigma_U + \sigma_L)/dx_1 dx_2$. In lowest order and for massless quarks we have $\sigma_L = 2\sigma_T$, which makes the second term in the denominator of (5) vanish. For massive quarks this is found to be negligibly small relative to the first term so that we may write

$$A \approx \frac{8}{\sqrt{3}}\left(\frac{L}{1+Z}\right)\frac{\cos\eta}{1+\frac{1}{2}\sin^2\eta}\frac{d^2\sigma_H}{dx_1 dx_2}\left[\frac{d^2\sigma}{dx_1 dx_2}\right]^{-1}$$

$$\equiv \left(\frac{L}{1+Z}\right)\frac{\cos\eta}{1+\frac{1}{2}\sin^2\eta}R. \tag{6}$$

The order-$\alpha_s^2$ diagrams that contribute to $\sigma_H$ are listed in Fig. 2. The denominator, on the other hand, may well be approximated by the dominant lowest-order cross section. We have calculated[19] the ratio $R$ for $m/(q^2)^{1/2} = 0.125$ and $m/(q^2)^{1/2} = 0.25$, where $m$ is the quark mass. The result is shown in Fig. 3 for $\alpha_s = 0.2$. Instead of the quark energies we have chosen thrust $T$ and the angle $\theta_{12}$ between the quark and antiquark momentum. The predicted asymmetry depends quite strongly on the value of $m/(q^2)^{1/2}$ as was to be expected. In favorable cases we may expect an asymmetry of more than ten percent which we find very encouraging. For the bottom quark ($m \approx 5$ GeV) and 40 GeV center-of-mass energy the effect will still be on the percent level (Fig. 3, upper curve). Integrated over $\theta_{12}$ the three-jet cross section normalized by the point cross section $\sigma_0$ has for $m/(q^2)^{1/2} = 0.25$ and $T = 0.7$, 0.8, and 0.9 the following values: $(1/\sigma_0)(d\sigma/dT) = 0.07$, 0.4, and 1.6. For $m/(q^2)^{1/2} = 0.125$ the relative cross section is even larger. So for $T \geq 0.8$ the cross section and also the asymmetry should be measurable.

According to the classes of diagrams contributing to $\sigma_H$, those involving the triple-gluon coupling and the QED-type diagrams, $R$ can be written ($N$ number of colors)

$$R = \frac{1}{N}\mathrm{Tr}\left(if_{\alpha\beta\gamma}\frac{\lambda_\beta}{2}\frac{\lambda_\gamma}{2}\frac{\lambda_\alpha}{2}\right)r_C + \frac{1}{N}\mathrm{Tr}\left(\frac{\lambda_\beta}{2}\frac{\lambda_\alpha}{2}\frac{\lambda_\beta}{2}\frac{\lambda_\alpha}{2}\right)r_E. \tag{7}$$

We found that, apart from the edges of phase space where $R$ gets very small, $r_C \approx r_E$. Noticing that $(1/N)\mathrm{Tr}[if_{\alpha\beta\gamma}(\lambda_\beta/2)(\lambda_\gamma/2)(\lambda_\alpha/2)] = -2$ and $(1/N)\,\mathrm{Tr}[(\lambda_\beta/2)(\lambda_\alpha/2)(\lambda_\beta/2)(\lambda_\alpha/2)] = -(2/g)$ this gives ($\alpha = \alpha_s$)

$$R_{\mathrm{QED}} \approx -\tfrac{1}{2}R_{\mathrm{QCD}} \tag{8}$$

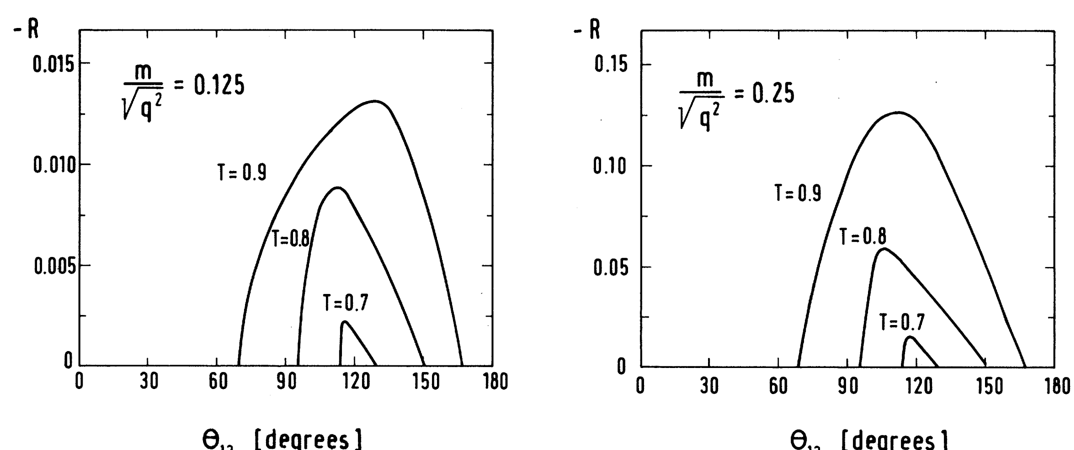


FIG. 3. The asymmetry parameter $R$ for $m/(q^2)^{1/2} = 0.125$, $m/(q^2)^{1/2} = 0.25$ and various thrust values as a function of $\cos\theta_{12}$.

substantiating our claim that $R$ directly measures the gluonic self-interactions. In order to prove (or disprove) QCD it would be sufficient to establish the sign of $R$.

At higher energies $[(q^2)^{1/2} \gtrsim 40\ \mathrm{GeV}]$ there will be an increasing admixture of the weak neutral current which gives rise to the same kind of asymmetry and has the advantage of not requiring longitudinally polarized beams. Below the $Z$ pole where only the $\gamma$-$Z$ interference is of importance we find

$$A \approx \left(\frac{2Q_f a v_f \mathrm{Re}\beta}{Q_f^2 - 2Q_f v v_f \mathrm{Re}\beta}\right)\frac{\cos\eta}{1+\frac{1}{2}\sin^2\eta}R, \qquad (9)$$

where $Q_f$ is the quark electric charge, and the neutral-current parameters are defined in Ref. 21 ($R$ is the same as before). The asymmetry $A$ now is related to the $T$-odd and parity-odd observable $\vec{\epsilon}^{(-)} \cdot (\vec{p}_1 \times \vec{p}_2)$. It appears that the factor in brackets is positive for any flavor, which makes it relatively easy to establish the sign of $R$.

If QCD proves to be right, the asymmetry $A$ can be looked at as a measure of the heavy quark masses. Since (6) decreases very fast with decreasing quark mass, only the highest-mass quark will contribute. Other proposals to measure the three-gluon vertex have been discussed.[17,22] These proposals, as well as the one presented here, will involve very difficult experiments.

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