Grand Unification with the Exceptional Group E_8

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A truly unified model of the basic gauge interactions, except for gravity, based on the exceptional group E_8 is proposed. The fundamental fields belong to the smallest possible single representations for each spin. In addition to accounting for the three "observed" SU(5) families, this Letter predicts the existence of three more conjugate SU(5) families below 1 TeV.

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E_8 is the largest exceptional group. Many of its subgroups such as SO(10), E_6, etc., have been used in grand unified theories (GUT).[1-3] The relevant groups occur in one or more of the following subgroups of E_8[4]: SO(16), SU(9), $SU(8)\otimes SU_2$, $E_7 \otimes SU(2)$, $E_6\otimes SU(3)$, $SU(5)\otimes SU(5)$, $F_4\otimes G_2$, SO(14) $\otimes U_1$, $SO(10)\otimes SU(4)$, etc. In a GUT based on E_8 the low-energy physics depends on the symmetry-breaking chain used. For example, in some chains, such as SO(16), we find left-handed SU(5) families including a top-bottom quark doublet of $[SU(2)]_w$ while in other chains, such as $E_6\otimes SU(3)$, the left-handed bottom quark appears in a singlet of $[SU(2)]_w$. In addition, the number and types of families predicted depend on the chain of symmetry breaking. In this paper we discuss in detail some possibilities within the SO(16) line while other possibilities such as the $E_6\otimes SU(3)$ approach will be discussed elsewhere.

The smallest representation of E_8 is the 248-dimensional adjoint representation. Both the gauge bosons $A_\mu{}^a$ and the left-handed fermions $\psi^a{}_L$ are assigned to the adjoint representation. The Higgs bosons φ are chosen from among the multiplets that couple to the symmetric product of two fermionic representations, $\psi^a{}_L C\psi^b{}_L\varphi_{ab}$.

$$(\underline{248}\times\underline{248})_S=\underline{1}+\underline{3875}+\underline{27\,000}.$$

We take the smallest nontrivial multiplet $\underline{3875}$. A bare Majorana mass term is avoided by conserving a fermion number in the Yukawa coupling and thus taking a complex $\underline{3875}$. Actually, only a discrete subgroup of this phase invariance will be respected by the Higgs potential and thus the total Lagrangian is invariant only under a discrete symmetry. We consider two different assignments of discrete symmetry: In the first, ψ and φ get transformed with phases of $\pi/4$ and $-\pi/2$, respectively, and in the second, both ψ and φ get the same phase of $2\pi/3$. These choices lead to different Higgs couplings in the potential $V(\varphi)$. In the first case quartic couplings[5] of the type $\varphi\varphi\varphi\varphi$ and $\varphi^\dagger\varphi^\dagger\varphi^\dagger\varphi^\dagger$ are allowed in addition to $\varphi\varphi^\dagger\varphi\varphi^\dagger$ and $\varphi^\dagger\varphi^\dagger\varphi\varphi$, while in the second case cubic couplings $\varphi\varphi\varphi$ and $\varphi^\dagger\varphi^\dagger\varphi^\dagger$ occur in addition to the quartics $\varphi\varphi^\dagger\varphi\varphi^\dagger$ and $\varphi^\dagger\varphi^\dagger\varphi\varphi$. These different schemes are expected to lead to different symmetry-breaking chains. We shall assume that it is possible to arrange the potential so that the desired symmetry-breaking patterns that are considered below do arise.[6]

The model described here is anomaly-free. It is not asymptotically free since the quadratic Casimir operators for the various dimensions are $C_2(\underline{248})=480$, $C_2(\underline{3875})=768$.

The particle content of the representations becomes more transparent in a decomposition with respect to the maximal orthogonal subgroup SO(16):

$$\underline{248}=\underline{120}+\underline{128},$$
$$\underline{3875}=\underline{135}+\underline{1820}+\underline{1920},$$
$$\underline{27\,000}=\underline{1}+\underline{128}+\underline{1820}+\underline{5304}+\underline{6435}+\underline{13\,312},$$

where $\underline{120}$ is the adjoint, $\underline{128}$ is the spinor, $\underline{135}$ is the symmetric traceless tensor of rank 2, $\underline{1820}$ is the antisymmetric tensor of rank 4. The content of the Yukawa couplings can be seen in the following Kronecker products.

$$(\underline{120}\times\underline{120})_S=\underline{1}+\{\underline{135}\}+\{\underline{1820}\}+\underline{5304},$$
$$(\underline{128}\times\underline{128})_S=\underline{1}+\{\underline{1820}\}+\underline{6435},$$
$$\underline{120}\times\underline{128}=\underline{128}+\{\underline{1920}\}+\underline{13\,312},$$

where the representations occurring in $\underline{3875}$ that couple to the fermions are enclosed in curly brackets. We see that there is no singlet of SO(16) in $\underline{3875}$, therefore, we must break down to a subgroup of SO(16). A physically relevant subgroup is $SO(10)\otimes SO(6)$ with respect to which we decompose the SO(16) particle multiplets that occur in our model:

$$\underline{248} \to \begin{cases} \underline{120} = (\underline{45},\underline{1}) + (\underline{1},\underline{15}) + (\underline{10},\underline{6}), \\ \underline{128} = (\underline{16},\underline{4}^*) + (\underline{16}^*,\underline{4}) \end{cases}$$

$$\underline{3875} \to \begin{cases} \underline{135} = (\underline{1},\underline{1}) + (\underline{1},\underline{20}') + (\underline{54},\underline{1}) + (\underline{10},\underline{6}), \\ \underline{1820} = (\underline{10},\underline{10}) + (\underline{10},\underline{10}^*) + (\underline{1},\underline{15}) + (\underline{210},\underline{1}) + (\underline{45},\underline{15}) + (\underline{120},\underline{6}), \\ \underline{1920} = (\underline{16},\underline{4}^*) + (\underline{16}^*,\underline{4}) + (\underline{16},\underline{20}) + (\underline{16}^*,\underline{20}^*) + (\underline{144},\underline{4}^*) + (\underline{144}^*,\underline{4}). \end{cases}$$

The only singlet of $SO(10) \otimes SO(6)$ appears in $\underline{135}$ which is also the smallest SO(16) representation among the Higgs bosons. By giving it a large vacuum expectation value ($\gtrsim 10^{19}$ GeV) all fermions and gauge bosons become superheavy except for the adjoint representation of gauge bosons $(\underline{45},\underline{1}) + (\underline{1},\underline{15})$ and the spinor representations of fermions $(\underline{16},\underline{4}^*) + (\underline{16}^*,\underline{4})$. The fact that this occurs with a single parameter is remarkable. This degree of elegance seems possible only for the SO(16) and SU(9) chains. To further investigate the symmetry breaking, we give the Kronecker products appearing in the Yukawa couplings of the massless fermions:

$$[(\underline{16},\underline{4}^*) \times (\underline{16},\underline{4}^*)]_S = \{(\underline{10},\underline{10}^*)\} + \{(\underline{120},\underline{6})\} + (\underline{126},\underline{10}^*), \tag{1a}$$

$$[(\underline{16}^*,\underline{4}) \times (\underline{16},\underline{4})]_S = \{(\underline{10},\underline{10})\} + \{(\underline{120},\underline{6})\} + (\underline{126}^*,\underline{10}), \tag{1b}$$

$$[(\underline{16},\underline{4}) \times (\underline{16}^*,\underline{4})] = \{(\underline{1},\underline{15})\} + \{(\underline{210},\underline{1})\} + \{(\underline{45},\underline{15})\} + (\underline{1},\underline{1}) + (\underline{45},\underline{1}) + (\underline{210},\underline{15}), \tag{1c}$$

where the representations in the $\underline{3875}$ of Higgs bosons are enclosed in curly brackets.

From here on we identify the SU(5) subgroup of SO(10) as the Georgi-Glashow SU(5) containing $[SU(3)]_c \otimes [SU(2)]_w \otimes [U(1)]_w$. Other possibilities will be investigated elsewhere. Presently, known phenomenology has revealed the existence of quark and lepton multiplets belonging to the $\underline{16}$ of SO(10). However, up to 15 GeV there is no evidence for the $\underline{16}^*$ multiplets. Therefore, our strategy is to arrange symmetry breaking so as to make the $\underline{16}^*$'s and the SU(5) singlets in the $\underline{16}$'s heavy. First, we show how the singlets of SU(5) become heavy. The right-hand side of Eq. (1c) contains three SU(5) singlet Higgs fields $\varphi_1 = (\underline{1},\underline{15})$, $\varphi_2 = (\underline{1},\underline{1})$, and $\varphi_3 = (\underline{1},\underline{15})$ coming from the $(\underline{1},\underline{15})$, $(\underline{210},\underline{1})$, and $(\underline{45},\underline{15})$ representations, respectively. Their coupling to the fermions can be written symbolically as follows:

$$(\underline{10}^a \times \underline{10}^*{}_b)(\alpha_1\varphi_1 + \alpha_2\varphi_2 + \alpha_3\varphi_3)_a{}^b$$

$$(\underline{5}^{*a} \times 5_b)(\beta_1\varphi_1 + \beta_2\varphi_2 + \beta_3\varphi_3)_a{}^b$$

$$(\underline{1}^a \times \underline{1}^*{}_b)(\gamma_1\varphi_1 + \gamma_2\varphi_2 + \gamma_3\varphi_3)_a{}^b,$$

where α_i, β_i, and γ_i are Clebsch-Gordan coefficients and the φ_i are 4×4 matrices with indices a,b. φ_1 and φ_3 are traceless and φ_2 is proportional to 1. Taking the vacuum expectation values (VEV) $\langle\varphi_1\rangle = v_1\, \mathrm{diag}(1,1,1,-3)$, $\langle\varphi_2\rangle = v_2$, and $\langle\varphi_3\rangle = v_3\, \mathrm{diag}(1,1,1,-3)$ and satisfying the constraints

$$\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 = 0 = \beta_1 v_1 + \beta_2 v_2 + \beta_3 v_3$$

(this equation probably has a group-theoretical explanation), we see that all singlets of SU(5) and the fourth families of $\underline{5}^* + \underline{10}$ and $\underline{5} + \underline{10}^*$ become heavy. For consistency with limits on neutrino masses this VEV should be larger than 10^{13} GeV. Thus we see that at this stage the scheme requires three pairs of conjugate families. So far the remaining unbroken gauge symmetry is the product between the SU(5) of Georgi and Glashow and a family group of U(3). This symmetry must be broken down to $[SU(3)]_c \otimes [SU(2)]_w \otimes [U(1)]_w$ in order to give large masses to the unwanted gauge bosons. This can be done with the Higgses in the $\underline{135}$ of SO(16) without affecting the three remaining massless families and their conjugates. Note that we have avoided mass mixings between the remaining families and their conjugates in order to be consistent with known phenomenology.

It is known that the unwanted $(\underline{5} + \underline{10}^*)$ families cannot develop mass terms by themselves unless $[SU(2)]_w$ is broken. Since $W^\pm$ and Z bosons get their masses from this breaking, this puts an upper bound on the masses of the conjugate families. Such VEV's come from the Higgs bosons appearing on the right-hand side of Eqs. (1a) and (1b). The $(\underline{10},\underline{10}) + (\underline{10},\underline{10}^*) + (\underline{120},\underline{6})$ contain $20 + 20 + 12 = 52$ doublets of $[SU(2)]_w$ all coming from $\underline{5}$'s and $\underline{5}^*$'s of SU(5). In accordance with the gauge hierarchy problem, we assume that only one linear combination of all these doublets remains at a low mass[7] and the others become very heavy. This is precisely the Weinberg-Salam doublet of Higgs bosons. It will have effective coupling constants to all quark and lepton families (mixing angles among 52 doublets). We assume that the Higgs potential can be arranged so that the effective coupling constants exhibit a hierarchy for various families. In particular, the unwanted conjugate families will be arranged to have larger effective coupling constants. The existence of this freedom can be seen in Eqs. (1a) and (1b) where the $(\underline{10},\underline{10})$ and $(\underline{10},\underline{10}^*)$ are independent Higgses provided the $\underline{3875}$ is complex. With all this it is clear that when the Weinberg-Salam doublet develops a VEV all observed fermions and $W^\pm, Z$ bosons get their usual masses and mixing angles. The conjugate families are heavy. However, on the basis of the analysis of Chanowitz, Furman, and Hinchliffe[8] there should be a 1-TeV limit on the masses of the extra fermions. Here there seems to be an opportunity to understand the family hierarchy problem on a group-theoretical basis à la Michel and Radicati if the mixing angles among the 52 doublets come out quantized and exhibit a hierarchy. In order to make our statements more precise, it is clear that a detailed study of the Higgs potential is necessary. This is a very interesting and formidable task in the present model[5,6] and will be deferred to future investigations.

This model truly unifies the basic gauge interactions except for gravity, with single representations of E_8 for the fundamental particles. The representations used are the lowest possible for each spin. In addition to accounting for the three observed families (with as yet unobserved top quark) it predicts three conjugate families which should be discovered below 1 TeV. If we take the $E_6 \otimes SU(3)$ chain instead of the SO(16), we predict families without the top quark as in the E_6 model.[2] Such other chains will be investigated elsewhere.

The fundamental fermions are forced to be in the adjoint representation of E_8, which is a "real" representation. It is this fact which led us to predict the additional conjugate families. Real representations of other groups have remained unexplored. They could be analyzed in the spirit of this paper. Recently other approaches were suggested to unify families.[9]

It is well known that a gauge theory in which the left-handed fermions are in the adjoint representation is automatically supersymmetric in the absence of scalars. If the scalars of $\underline{3875}$ are assumed to arise dynamically, the theory would be entirely supersymmetric. It is not clear then how supersymmetry will break. It would be interesting to explore the consequences of such a supersymmetry even if it is broken by the presence of elementary scalars. For example, extended supersymmetric models can be constructed by starting in higher dimensions and doing dimensional reduction. Such an approach may lead us to unification with gravity.

If an E_8 theory proves to be successful, then it is tempting to explore the possibility that octonions play a fundamental role in nature.[10] We may believe then that the large number of fields appearing in our theory are all fundamental.

It could be that GUT's are phenomenological theories which give a correct description of Nature up to a few teraelectronvolts. In this case the fields appearing in the theory will be composites of more fundamental constituents whose interactions at low energies will be described by an effective gauge theory. We have already indicated one such possible scheme based on ternary algebras.[11] It is interesting that in our E_8 model the low-energy fermions sit in the coset space of E_8. Coset spaces are directly related to ternary algebras. In the case of exceptional groups some of these ternary algebras are octonionic. In fact, it was through ternary algebraic considerations that we were led to the E_8 model.

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Note added.—Since the fermions are in a real representation, there is a possibility that radiative corrections may generate a mass of the order of Planck's scale for all fermions and mix the $(\underline{16},\underline{4}^*)$ with the $(\underline{16}^*,\underline{4})$ families. If so, we could tune the vacuum expectation values to cancel this large contribution and leave the three $(\underline{5}+\underline{10}^*)$ and $(\underline{5}^*+\underline{10})$ families at a low mass as required by phenomenology. There is freedom within the $\underline{3875}$ to do this by appropriately choosing the VEV's of the $(\underline{1},\underline{15})+(\underline{210},\underline{1})+(\underline{45},\underline{15})$ that appear in Eq. (1c). Large cancellations of this type, between the choice of VEV's and radiative corrections, are not "natural." However, this is not

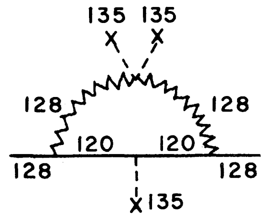

FIG. 1. Radiative correction which vanishes.

unphysical, since it is no worse of canceling infinities by doing renormalization and *choosing* the physical value of a parameter. Such fine tuning already appears in any unified gauge theory (gauge hierarchy problem), and here it may happen in the fermion mass matrix. At present it is not known whether this fine tuning is necessary in our model since it may be possible to find a group-theoretical argument to prove the absence of the problem "naturally," as hinted below.

E_8 has some very special properties. To see some of these at work, consider the diagram of Fig. 1 which ordinarily would contribute the large unwanted mass to the 128 of fermions. In lowest order in one loop and at the SO(10) $\otimes$ SU(4) level of symmetry this is the only such diagram. (We consider only the 135 of Higgses since it is the only one that develops a VEV at this symmetry level.) This diagram could generate an effective coupling $\psi_{\underline{128}}\psi_{\underline{128}}\langle\varphi_{\underline{135}}\rangle\langle\varphi_{\underline{135}}^{\dagger}\rangle\langle\varphi_{\underline{135}}\rangle$, and thus may contain a singlet mass term $\psi_{\underline{128}}\psi_{\underline{128}}$ which is unwanted. However, this diagram is zero from the group properties of E_8: i.e., in the vertex $(A_{\underline{128}}{}^{\mu}A_{\underline{128}}{}^{\mu})(\varphi_{\underline{135}}\varphi_{\underline{135}}{}^{\dagger})$ each parenthesis must form a singlet of SO(16) since there is no common channel other than a singlet in the products $(\underline{128}\otimes\underline{128})_S=\underline{1}+\underline{1820}+\underline{6435}$ and $\underline{135}\times\underline{135}=\{\underline{1}+\underline{135}+\underline{3740}+\underline{5304}\}_S+[\underline{120}+\underline{8925}]_A$. Therefore, in Fig. 1 the three 135's must form an effective 135. However, 135 does not couple to the symmetric product of two 128's of fermions. Thus, the diagram vanishes. This example shows that there may be an underlying group property (discrete subgroups of E_8?) which is responsible for such natural cancellations.

We become aware of a paper on E_8 by N. S. Baaklini,[12] which appeared after we submitted our manuscript for publication. This E_8 model is quite different and considerably incomplete relative to our model in many respects, and has little overlap with our paper.

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[5]The quartics shown here are symbolic. There are several E_8-invariant quartic couplings which can be obtained from the Kronecker product of two 3875's: $(\underline{3875}\times\underline{3875})_S=\underline{1}+\underline{3875}+\underline{27\,000}+\underline{147\,250}+\underline{2\,450\,240}+\underline{4\,881\,384}$; $(\underline{3875}\times\underline{3875})_A=\underline{248}+\underline{30\,380}+\underline{779\,247}+\underline{6\,696\,000}$.

[6]Here we are assuming for simplicity that the fermions get their masses only from the 3875. If the desired breaking patterns cannot be achieved by the self-couplings of the 3875 alone, it is always possible to induce such breaking with additional Higgses in the potential such as 248, etc., which do not couple to fermions.

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