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<sup>8</sup>Weighting includes corrections for pass rate, scanning efficiency, and a 1-cm neutral-track-length cut. The strange-particle events distribute themselves as 76 (40) events with one visible  $K_s^0$  ( $\Lambda$ ), and 6 (2) events with one visible  $K_s^0$  and  $\Lambda$  (two visible  $K_s^0$ ). For details, see R. Brock, Ref. 5.

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<sup>14</sup>We assume that for an isoscalar target ( $N$ ) the distributions are related to those of a proton target ( $p$ ) by  $x\bar{d}_N(x) = \frac{1}{2}[xu_p(x) + xd_p(x)]$  and  $xs_N(x) = xs_p(x)$ .

## Theory of Energy-Independent Nuclear Optical-Model Potentials

T. T. S. Kuo<sup>(a)</sup> and F. Osterfeld

*Institut für Kernphysik, Kernforschungsanlage Jülich, D-5170 Jülich, Federal Republic of Germany*

and

S. Y. Lee

*Physics Department, State University of New York at Stony Brook, Stony Brook, New York 11794*

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With use of an equation-of-motion approach, a new microscopic theory of the nuclear optical-model potential which is explicitly energy independent has been derived. This theory suggests the possibility of replacing a wide range of conventional energy-dependent optical-model potentials by a single, energy-independent but nonlocal, optical-model potential.

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Several authors<sup>1,2</sup> have already demonstrated quite generally that a generalized optical-model potential exists which produces the exact asymptotic wave function for elastic nucleon-nucleus scattering when used in solving the one-body Schrödinger equation. This potential is complex, nonlocal, and depends explicitly on the incident energy  $E$  of the full many-body wave function.<sup>3-5</sup> In the Green's-function formalism<sup>2</sup> (and also in Feshbach's formalism<sup>1</sup>) the optical-model wave function  $\rho_E(\vec{k})$  (in the  $k$  representation) is obtained by solving the nonlocal Schrödinger equation

$$T\rho_E(\vec{k}) + \int d^3k' V_G(\vec{k}, \vec{k}', E)\rho_E(\vec{k}') = E\rho_E(\vec{k}), \quad (1)$$

where  $T$  is the kinetic energy operator and  $V_G(\vec{k}, \vec{k}', E)$  is the mass operator of the one-particle Green's-function which depends on the independent variables  $\vec{k}$ ,  $\vec{k}'$ , and  $E$ , where  $E$  is the exact eigenenergy of the many-body wave function.

The purpose of this paper is to present a new microscopic derivation of the optical potential which leads to a formally energy-independent potential; i.e., we shall show that the optical-model wave function can be obtained from the following nonlocal Schrödinger equation:

$$T\rho_E(\vec{k}) + \int d^3k' V^{\text{eff}}(\vec{k}, \vec{k}', \epsilon_{\vec{k}})\rho_E(\vec{k}') = E\rho_E(\vec{k}). \quad (2)$$

The only but essential difference between Eqs. (1) and (2) is that  $V^{\text{eff}}(\vec{k}, \vec{k}', \epsilon_{\vec{k}})$  depends only on the single-particle energies  $\epsilon_{\vec{k}}$  of the unperturbed Hamiltonian  $T$  (or  $H_0 = T + \hat{V}$ , where  $\hat{V}$  is an arbitrary auxiliary potential) and *not* on the exact eigenenergy  $E$ . Since  $\epsilon_{\vec{k}}$  is a unique function of  $\vec{k}$ ,  $V^{\text{eff}}(\vec{k}, \vec{k}', \epsilon_{\vec{k}})$  has one variable less than  $V_G(\vec{k}, \vec{k}', E)$ . We call the energy dependence of  $V^{\text{eff}}(\vec{k}, \vec{k}', \epsilon_{\vec{k}})$  *implicit* (with respect to  $\epsilon_{\vec{k}}$ ) in contrast to that of  $V_G(\vec{k}, \vec{k}', E)$  which is explicitly energy dependent.

The optical potential  $V^{\text{eff}}(\vec{k}, \vec{k}')$  (we omit  $\epsilon_{\vec{k}}$  because it is not an independent variable) has the following two important properties:

(i) It is particularly suited for a microscopic calculation of the optical potential since the potential has to be calculated only *once* with respect to the basis of a solvable unperturbed Hamiltonian and *not* for various incident energies  $E$ .

(ii) A microscopically calculated optical potential which is only implicitly energy dependent may eventually uniquely replace a wide range of different potentials which are now in use in nuclear reaction theory.

In the following we shall prove that the potential  $V^{\text{eff}}(\vec{k}, \vec{k}')$  exists and we shall also present an exact diagrammatic expansion of it. This expansion is based on the Rayleigh-Schrödinger type of perturbation theory and, as we shall show, contains folded diagrams. The method that we use for deriving the optical potential is based on the equation-of-motion technique.<sup>6</sup>

We start from the many-body Hamiltonian  $H = T + V$ , where  $T$  is the kinetic energy and  $V$  the realistic nucleon-nucleon interaction. We consider the  $(A+1)$ -body-system Schrödinger equation for stationary state,

$$H|\psi_{A+1}^{(+)}(E)\rangle = E|\psi_{A+1}^{(+)}(E)\rangle. \quad (3)$$

The function  $\psi^{(+)}$  is the exact many-body wave function and obeys the standard asymptotic boundary conditions [superscript (+) indicates incident

plane wave in the elastic channel, radially outgoing waves in reaction channels]. In the case of elastic scattering, we are only interested in that part of  $|\psi_{A+1}^{(+)}(E)\rangle$  where  $A$  nucleons form the true ground state  $|\psi_A^0\rangle$  of the  $A$ -body system and where one nucleon is in a scattering state. Therefore we project out from  $|\psi_{A+1}^{(+)}(E)\rangle$  this particular part and define a model-space problem by

$$H_{\text{eff}}P|\psi_{A+1}^{(+)}(E)\rangle = (E - E_A^0)P|\psi_{A+1}^{(+)}(E)\rangle, \quad (4)$$

where  $E_A^0$  is the  $A$ -body ground-state energy. The projection operator  $P$  is defined by

$$P = \sum_{\vec{k}} |\tilde{\varphi}_{\vec{k}}\rangle \langle \varphi_{\vec{k}}|, \quad \langle \varphi_{\vec{k}}| = \langle \psi_A^0 | a_{\vec{k}}, \quad (5)$$

where the operator  $a_{\vec{k}}^+$  ( $a_{\vec{k}}$ ) creates (destroys) a particle in a momentum state  $\vec{k}$  and the wave functions  $\langle \varphi_{\vec{k}}|$  fulfill the biorthogonal relation  $\langle \varphi_{\vec{k}} | \tilde{\varphi}_{\vec{k}'} \rangle = \delta(\vec{k} - \vec{k}')$ . Now, the essential problem to be solved consists in the derivation of the effective Hamiltonian  $H_{\text{eff}}$ , since knowing  $H_{\text{eff}}$  we obtain readily from Eq. (4) a one-body Schrödinger equation for the quantity

$$\rho_E(\vec{k}) = \langle \psi_A^0 | a_{\vec{k}} | \psi_{A+1}^{(+)}(E) \rangle \quad (6)$$

which is just the optical-model wave function in the  $\vec{k}$  representation.

For the derivation of  $H_{\text{eff}}$  we start from the equation of motion for the operator  $a_{\vec{k}}$ :

$$\begin{aligned} \langle \psi_A^0 | [a_{\vec{k}}, H] | \psi_{A+1}^{(+)}(E) \rangle \\ = (E - E_A^0) \langle \psi_A^0 | a_{\vec{k}} | \psi_{A+1}^{(+)}(E) \rangle. \end{aligned} \quad (7)$$

The commutator  $[a_{\vec{k}}, H]$  can be written as  $\epsilon_{\vec{k}} a_{\vec{k}} + A_{\vec{k}}$ , where  $\epsilon_{\vec{k}}$  is the single-particle kinetic energy and  $A_{\vec{k}}$  is defined by

$$A_{\vec{k}} = \sum_{\beta \gamma \delta} a_{\beta}^{\dagger} (\vec{k} \beta | V | \gamma \delta) a_{\delta} a_{\gamma}. \quad (8)$$

With use of the Gell-Mann-Low theorem,<sup>7</sup> the ground state  $\psi_A^0$  can be obtained by applying the time evolution operator to the unperturbed ground state  $\Phi_A^0$ . We shall prove also that the true scattering wave function  $|\psi_{A+1}^{(+)}(E)\rangle$ , satisfying Eq. (7), can be obtained in a similar procedure, i.e.,

$$\frac{|\psi_{A+1}^{(+)}(E)\rangle}{\langle \Phi_{A+1}(E) | \psi_{A+1}^{(+)}(E) \rangle} = \frac{U(0, -\infty) | \Phi_{A+1}(E) \rangle}{\langle \Phi_{A+1}(E) | U(0, -\infty) | \Phi_{A+1}(E) \rangle}, \quad (9)$$

with the parent state  $\Phi_{A+1}(E)$  given by

$$|\Phi_{A+1}(E)\rangle = \sum_{\vec{k}} c_{\vec{k}}^{-1}(E) a_{\vec{k}}^{\dagger} | \Phi_A^0 \rangle, \quad (10)$$

where a summation is carried over the discrete set together with an integration over the continuum set of states. The coefficients  $c_{\vec{k}}^{-1}(E)$  are to be determined. Substituting the above into Eq. (7), we ob-

tain

$$\begin{aligned} & \langle \Phi_A^0 | U(\infty, 0) A_{\vec{k}}^- U(0, -\infty) | \Phi_{A+1}(E) \rangle / (D_A D_{A+1}) \\ & = (E - E_A^0 - \epsilon_{\vec{k}}^-) \langle \Phi_A^0 | U(\infty, 0) a_{\vec{k}}^- U(0, -\infty) | \Phi_{A+1}(E) \rangle / (D_A D_{A+1}), \end{aligned} \quad (11)$$

where  $D_A$  and  $D_{A+1}$  stand for  $\langle \Phi_A^0 | U(\infty, 0) | \Phi_A^0 \rangle$  and  $\langle \Phi_{A+1}(E) | U(0, -\infty) | \Phi_{A+1}(E) \rangle$ , respectively.

We now analyze<sup>8,9</sup> the diagrammatic structure of this equation. The unlinked diagrams, denoted as  $S$ , of the right-hand-side and left-hand-side numerator are identical and can be factorized out as a whole, yielding

$$\begin{aligned} & (N_L)_{\vec{k}}^- S / (D_A D_{A+1}) \\ & = (E - E_A^0 - \epsilon_{\vec{k}}^-) (N_R)_{\vec{k}}^- S / (D_A D_{A+1}) \end{aligned} \quad (12)$$

with

$$(N_L)_{\vec{k}}^- \equiv \langle \Phi_A^0 | U(\infty, 0) A_{\vec{k}}^- U(0, -\infty) | \Phi_{A+1}(E) \rangle_{\mathcal{L}} \quad (12a)$$

$$(N_R)_{\vec{k}}^- \equiv \langle \Phi_A^0 | U(\infty, 0) a_{\vec{k}}^- U(0, -\infty) | \Phi_{A+1}(E) \rangle_{\mathcal{L}} \quad (12b)$$

where  $\mathcal{L}$  denotes the linked diagrams defined as those with all their vertices linked to the external particle line. Hence obviously  $N_L$  and  $N_R$  have the chain structure shown in Fig. 1. The boxes, the so-called  $Q$  boxes,<sup>8,9</sup> represent the sum of all irreducible diagrams beginning and terminating with one single-particle line. As usual an irreducible diagram is defined as a linked diagram which cannot be separated into

two connected pieces, each with at least one vertex, by cutting one particle line. Notice the similarity in structure between  $N_L$  and  $N_R$  of Fig. 1. The only difference between them is that  $N_L$  is terminated at time  $t=0$  by the operator  $A_{\vec{k}}^-$  of Eq. (8) while  $N_R$  terminates at time  $t=0$  with the operator  $a_{\vec{k}}^-$ . Therefore the last box in  $N_L$  is a slashed  $Q$  box indicating that the operator  $A_{\vec{k}}^-$  acts at time  $t=0$ , and as a result  $N_L$  does not have the free propagator term contained in  $N_R$ . The chain structure of  $N_L$  and  $N_R$  suggests clearly that we can write  $N_L$  as a product of  $N_R$  and a quantity containing the slashed  $Q$  boxes. This can indeed be done by using a folded-diagram factorization procedure<sup>8,9</sup> leading to the basic result

$$(N_L)_{\vec{k}}^- = \int d^3k' V_{\vec{k}\vec{k}'}^- \text{eff} (N_R)_{\vec{k}'}^-, \quad (13)$$

where the effective interaction  $V^{\text{eff}}$  is given by the linked-diagram expansion shown in the bottom part of Fig. 1. Combining Eqs. (6), (7), (11), (12), and (13), we obtain as a final result the one-body Schrödinger equation for the optical-model wave function  $\rho_E(\vec{k})$ :

$$\begin{aligned} & \epsilon_{\vec{k}}^- \rho_E(\vec{k}) + \int d^3k' V_{\vec{k}\vec{k}'}^- \text{eff} \rho_E(\vec{k}') \\ & = (E - E_A^0) \rho_E(\vec{k}). \end{aligned} \quad (14)$$

Clearly  $V^{\text{eff}}$  is the energy-independent optical-model potential of Eq. (2) we have been looking for. Equation (14) serves to determine the optical-model wave function  $\rho_E(\vec{k})$ . From Eqs. (6), (7), and (12) we see that  $\rho_E(\vec{k})$  is just  $(N_R)_{\vec{k}}^- S / (D_A D_{A+1})$ , where  $(N_R)_{\vec{k}}^-$  is related to the coefficients  $c_{\vec{k}}^-(E)$  through Eqs. (12b) and (10). Thus Eq. (14), which determines  $\rho_E(\vec{k})$ , also formally determines  $c_{\vec{k}}^-(E)$ . This completes our proof that starting from the equation of motion (7) we derive the optical-model potential of Eq. (14) and Fig. 1. And this derivation is indeed quite straightforward.

A special feature of our optical-model potential is that it contains folded diagrams, as shown by its diagrammatic expansion in Fig. 1. For example, the third term is twice folded. The symbols  $f$  indicate folding operations.<sup>8,9</sup> The folded diagrams can be conveniently calculated and should not add much computational difficulty to the present theory. This is based on some bound-state folded-diagram calculations<sup>10</sup> with use of a

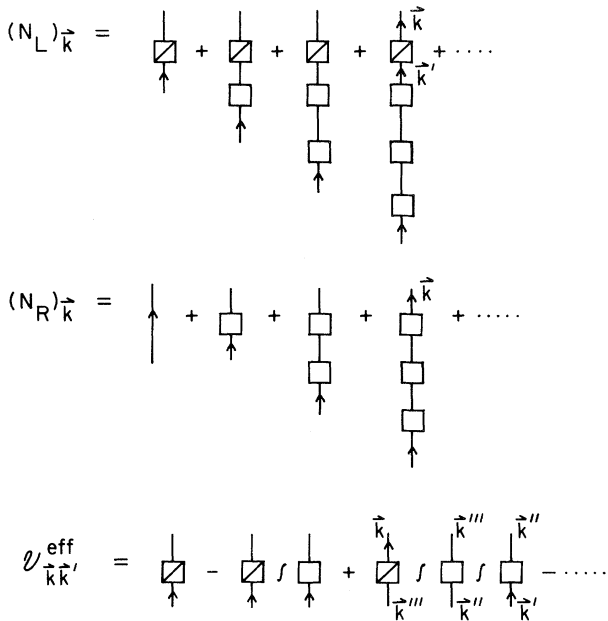


FIG. 1. Diagrammatic structures of  $N_L$ ,  $N_R$ , and  $V^{\text{eff}}$ .

realistic nucleon-nucleon potential. Some typical diagrams of our optical-model potential are given in Fig. 2. With use of standard methods,<sup>8,9</sup> the value for diagram  $\alpha$  is obtained as

$$\sum \frac{1}{2}vv/(\epsilon_{\vec{k}'} - \epsilon_a - \epsilon_b + \epsilon_c + i\eta), \quad (15a)$$

and that of diagram  $\beta$  as

$$-\sum \frac{1}{2}vvv/[(\epsilon_{\vec{k}''} - \epsilon_a - \epsilon_c + \epsilon_b + i\eta)(\epsilon_{\vec{k}'} - \epsilon_a - \epsilon_c + \epsilon_b + i\eta)], \quad (15b)$$

where the summations run over all intermediate indices and for brevity we have omitted the indices associated with the antisymmetric  $v$  vertices. Note that diagram  $\beta$  is a folded diagram. In the above, we take the limit of  $\eta \rightarrow 0+$ . This comes from the boundary condition of  $\psi_{A+1}^{(+)}(E)$  that, at time  $t=0$ , it contains a component where we have one particle in an outgoing spherical wave. By making a diagrammatic expansion of  $\psi_{A+1}^{(+)}(E)$  by way of Eq. (9), we see readily that this boundary condition is assured by the above limiting procedure. These  $i\eta$  factors will make our optical-model potential generally complex. Several observations are now in order:

(i) The central result of this paper is the derivation of an optical-model potential which is explicitly energy independent. As shown by Eqs. (15a) and (15b), individual diagrams of  $V^{\text{eff}}$  depend energetically only on the single-particle energies  $\epsilon$ 's defined by the unperturbed Hamiltonian. This is in contrast to the optical-model potential  $V_G(\omega_E)$  of the Green's-function formalism<sup>2,5,11,12</sup> which is explicitly dependent on the exact energy  $\omega_E = E - E_A^0$ .

For the energy-dependent potentials we expect threshold effects since with increasing projectile energy new inelastic or reaction channels open up energetically suddenly. At least these threshold effects appear in the imaginary part of the potential. Below the inelastic threshold the energy-dependent potential is Hermitian (no imaginary part). Now the question arises whether there is a contradiction between the energy-dependent and energy-independent optical potentials. The answer is, no! Indeed, by using a partial summation method<sup>8,13</sup> for summing up the folded-diagram series one can obtain an exact mathemati-

cal relation between the energy-independent  $V^{\text{eff}}$  and the Green's-function optical potential, namely

$$(T + V^{\text{eff}}) = \int d\omega_E |\rho_E\rangle \langle \tilde{\rho}_E| [T + V_G(\omega_E)] |\rho_E\rangle \langle \tilde{\rho}_E|, \quad (16)$$

where the optical-model wave function  $\rho_E$  obeys the biorthogonal relation  $\langle \tilde{\rho}_E | \rho_{E'} \rangle = \delta(E - E')$ . From Eq. (16) one can see that  $V^{\text{eff}}$  is a particular energy-averaged potential obtained by averaging over  $V_G(\omega_E)$  with respect to energy. Hence if we know  $\rho_E$  and  $V_G(\omega_E)$  for all energies, we can construct, in principle, an energy-independent  $V^{\text{eff}}$  as shown by Eq. (16). In reality, this is of course not practical. What we have done in this paper is to have attained a systematic method for calculating  $V^{\text{eff}}$ , as indicated by Fig. 1, without prior knowledge of  $\rho_E$  and  $V_G(\omega_E)$ .

If we act with the optical-model operator in Eq. (14) onto a wave function  $|\rho_E\rangle$  with  $E_0$  below threshold, then, of course, only the Hermitian part of  $V^{\text{eff}}$  will be active. In this sense  $V^{\text{eff}}$  also includes threshold effects implicitly. This one can also easily understand from a physical point of view. The optical-model wave function  $\rho_E(\vec{k})$  of Eq. (6) is expected to be a smooth function of  $k$ , peaked around  $k_0 \simeq (2m\omega_E/\hbar^2)^{1/2}$ . Then from Eq. (14) we see that  $\rho_E(\vec{k})$  depends primarily on the portion of  $V_{\vec{k}\vec{k}'}^{\text{eff}}$  with  $k \simeq k' \simeq k_0$ . Hence for  $\rho_E(\vec{k})$  the important contribution to  $V^{\text{eff}}$  comes from the diagrams with starting energies  $\epsilon_{\vec{k}}$  [see Eqs. (15a) and (15b)] in the vicinity of  $\epsilon_{\vec{k}'} = \omega_E$ . If  $\omega_E$  is below inelastic threshold then the incoming nucleon is essentially sensitive to only those matrix elements  $V_{\vec{k}\vec{k}'}$  which are real and therefore Hermitian. To give a further support of our theory, let us mention the work of Johnson<sup>14</sup> in which he showed that by the inclusion of folded diagrams one can obtain an energy-independent nucleon-nucleon potential.

(ii) In order to support our theory by numerical findings we refer to the analyses of neutron scattering data which Perey and Buck<sup>15</sup> have performed using different sets of nonlocal optical potentials. They found by variation of the nonlocal-

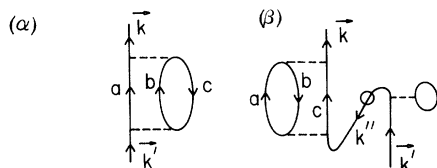


FIG. 2. Typical diagrams contained in the energy-independent optical-model potential  $V_{\vec{k}\vec{k}'}^{\text{eff}}$ .

ity a nonlocal potential which could fit neutron elastic scattering data in the energy range from 0.4 to 24 MeV and where the optical-model parameters were energy independent. This is, of course, a remarkable result since we know that the absorption, for instance, is changing quite dramatically by going from 0.4 to 24 MeV incident projectile energy. We believe that these findings of Perey and Buck strongly support our result that there exists a nonlocal potential, which is not explicitly energy-dependent, which describes elastic nucleon scattering in a wide energy range. The theory presented in this paper may serve as a convenient tool in deriving such a potential. Actual calculation of an energy-independent optical-model potential as outlined in this paper is in progress.

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<sup>(a)</sup>Permanent address: State University of New York at Stony Brook, Stony Brook, N.Y. 11794.

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## Noncanonical Hamiltonian Density Formulation of Hydrodynamics and Ideal Magnetohydrodynamics

Philip J. Morrison and John M. Greene

*Plasma Physics Laboratory, Princeton University, Princeton, New Jersey 08544*

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A new Hamiltonian density formulation of a perfect fluid with or without a magnetic field is presented. Contrary to previous work the dynamical variables are the physical variables,  $\rho$ ,  $\vec{v}$ ,  $\vec{E}$ , and  $\vec{s}$ , which form a noncanonical set. A Poisson bracket which satisfies the Jacobi identity is defined. This formulation is transformed to a Hamiltonian system where the dynamical variables are the spatial Fourier coefficients of the fluid variables.

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Several advantages may be gained from expressing a set of equations in Hamiltonian form. In addition to their formal elegance, Hamiltonian systems possess Poincaré invariants that influence the dispersion of an ensemble of systems with clustered initial conditions. A manifestly Hamiltonian formulation of a given problem makes it easier to find those approximations that preserve the Hamiltonian character. Here we present such a formulation of hydrodynamics and magnetohydrodynamics.

Hamiltonian systems are most elegant when expressed in canonical coordinates. Hydrodynamics is most usefully expressed in Eulerian variables. These two desiderata conflict. In practice, the penalty paid for adopting noncanonical coordinates is not severe, so that branch of the dichotomy is pursued here.

Previously, the equations of hydrodynamics<sup>1</sup> and magnetohydrodynamics,<sup>2</sup> in both Eulerian and Lagrangian form, have been shown to arise from a suitable Hamilton's principle. Such a Lagrangian