Complete Relativistic Faddeev Calculations of Pion-Deuteron Scattering

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Pion-deuteron elastic scattering in the region of the (3, 3) resonance is calculated with two covariant reductions of the relativistic Faddeev equations. The two S-wave nucleon-nucleon channels and the six S- and P-wave pion-nucleon channels are included by means of separable T matrices. Both of the space and the spin variables are treated relativistically, by use of the three-body helicity formalism of Wick. The results agree very well with the data at 142 and 182 MeV, but do not resolve the discrepancy at 256 MeV.

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Pion-deuteron elastic scattering has been calculated by several groups using three-body theories which incorporate the relativistic aspects of the problem in various degrees of approximation. The simplest calculations are those in which one solves the standard nonrelativistic Faddeev equations with use of relativistic kinematics only for the pion[1,2] [relativistic pion kinematics (RPK) approximation]. A different approach,[3-6] which is more appealing from a theoretical point of view, is that in which one starts with a relativistic version of the Faddeev equations which are then covariantly reduced such as to eliminate the fourth components of the relative momenta, while preserving relativistical and two- and three-body unitarity.

In the case of the RPK calculations, the most complete ones are those of Giraud *et al.*,[2] in which the nucleon-nucleon $^2S_1$-$^3D_1$ channel was included as well as the six S and P pion-nucleon channels for incident pions with an energy of 142 MeV. In the case of the fully relativistic approach, almost all calculations have been restricted to take into account only the pion-nucleon resonant channel and the nucleon-nucleon deuteron channel.[3-5] However, recently Rinat *et al.*[6] have performed a calculation in which they have taken into account approximately the effect of other two-body channels by means of perturbation theory. In this work, I have solved exactly the relativistic equations, including for the first time the two S-wave nucleon-nucleon channels and the six S- and P-wave pion-nucleon channels as well as the relativistic kinematical effects due to the spin of the nucleons.

One can obtain the relativistic analog of the Faddeev equations for three particles, by summing all possible diagrams in which only two particles interact while the third particle acts as spectator, which leads to integral equations identical in form to the nonrelativistic Faddeev equations[7] but depending now on two four-component variables.[8,9] In order to eliminate in a covariant way the fourth components of the relative momenta, two reductions have been proposed in the literature. The first reduction which was proposed by Blankenbecler and Sugar[10] (BS reduction) consists of replacing the propagators for three particles by their δ-function parts and performing a dispersion integral in the total energy of the system. This leads to the relativistic Faddeev propagator

$$G_0(s_0)=\frac{1}{\omega_i\,\omega_j\,\omega_k}\,\frac{\omega_i+\omega_j+\omega_k}{(\omega_i+\omega_j+\omega_k)^2-s_0-i\epsilon}\,. \tag{1}$$

In the second reduction which was proposed by Aaron, Amado, and Young[11] (AAY reduction) when one goes from a state in which particles j and k interact while particle i acts as spectator to a state in which particles i and k interact while particle j acts as spectator, one requires that the two spectator particles be on their mass shell. The corresponding relativistic Faddeev propagator in this case is

$$G_0(s_0^{1/2})=\frac{1}{\omega_i\,\omega_j}\,\frac{1}{\omega_k{}^2-[(s_0)^{1/2}-\omega_i-\omega_j]^2-i\epsilon}\,. \tag{2}$$

The propagators (1) and (2) have the same residues and imaginary parts, so that the two sets of equations obey the same unitarity relations. The next step with either of the two reductions is to project out the angular momentum components of the equations and apply the separable or isobar assumption, so as to be left with integral equations in only one continuous variable.

All angular momentum reductions used in the pion-deuteron problem up to now have concentrated on treating relativistically only the space variables, while leaving out any relativistic effects for the spin.[4,5] I treat relativistically both the spin and the space variables, by performing the angular momentum reduction with the three-body helicity states constructed by Wick.[12] In particular, I use Wick's three-body recoupling coefficients to go from one configuration of the three-body system to another, and perform the various Lorentz transformations between different reference frames including the transformation of the spin.

I apply the isobar assumption by using separable T matrices which are normalized to the experimental phase shifts. In the case of the pion-nucleon interaction, I use the phase shifts of Ref. 13, together with the P_{33} vertex function of Woloshyn, Moniz, and Aaron,[14] for all πN channels. I extend the T matrices to the unphysical region, by using the linear function $t(p,p';s)=t(p,p';s_0)s/s_0$, where s is the pion-nucleon invariant mass squared, and s_0 its threshold value $(M+\mu)^2$. I checked that the results are rather insensitive to the assumed behavior of the T matrix in the unphysical region, by replacing s/s_0 with $(s/s_0)^{1/2}$ and $(s/s_0)^{3/2}$, and seeing that the differential cross sections differed in all cases by less than half of a percent. I do not include in the present calculations the effect of real pion absorption and emission, since this P_{11} T matrix is constructed without the nucleon pole at $s=M^2$. In the case of the nucleon-nucleon interaction, I use ${}^1S_0$ and ${}^3S_1$ separable T matrices normalized to the phase shifts and mixing parameters of Arndt, Hackman, and Roper.[15] The calculation of vertex functions and the extension to the unphysical region are done using separable potentials which reproduce the scattering lengths, effective ranges and deuteron pole, as described in Ref. 4. I do not include the coupling between the ${}^3S_1$ and ${}^3D_1$ channels in the intermediate nucleon-nucleon rescatterings, since we only keep the ${}^3S_1 \to {}^3S_1$ part of the two-channel T matrix, although we include it in the initial and final states by using the deuteron wave function of Moravcsik[16] which has a D-state admixture of 6.7%.

We solve the integral equations of the BS and AAY reductions, by the method of the Padé approximants,[17] for values of the total angular momentum $J<6$ and use the impulse approximation for the remaining values up to $J=14$. I found that the Padé approximants converge so fast, that a [2/2] Padé approximant already gives the solution to within 0.01%. I have performed internal consistency tests with the numerical solution, by substituting it back into the integral equations to see that it is indeed a solution. By varying the mesh size and the distribution of mesh points, I estimated the accuracy of the calculated differential cross sections to be about 2%.

I show the results in Fig. 1 together with the experimental data.[18] The agreement, one can see, is very good at 142 and 182 MeV (however, a recent measurement by Holt *et al.*[19] produced a value of 1.57 mb/sr for the differential cross section at an energy of 140 MeV and an angle of 180°, which is about 25–30% higher than the present results at 142 MeV), but I find that the discrepancy at large angles at 256 MeV cannot be resolved by using a fully relativistic angular momentum reduction and including exactly all the relevant two-body channels, so that these data represent a very serious challenge to the presently accepted formulations of the relativistic three-body problem. We also see from Fig. 1 that the results of the BS and AAY reductions are identical in the forward direction and differ by about 10% at backward angles, so that it does not make much difference which reduction is used to calculate the results.

Another question that I would like to answer

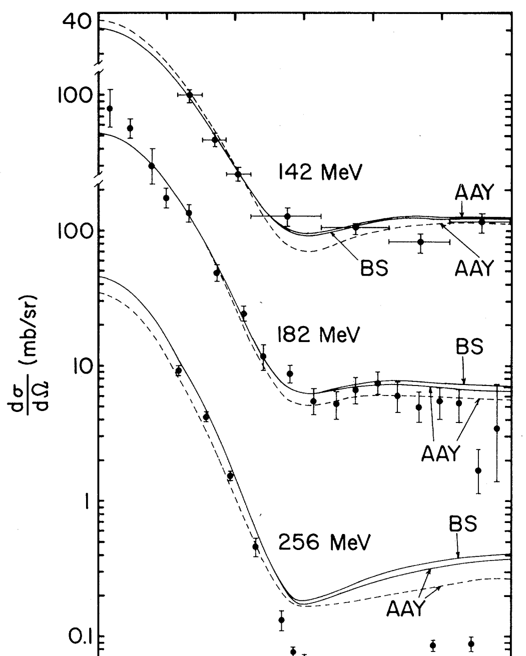

FIG. 1. Center-of-mass differential cross sections calculated with the Blankenbecler-Sugar reduction (BS) and with the Aaron-Amado-Young reduction (AAY) for three different laboratory kinetic energies of the pion. The solid lines are the results of the full calculations and the dashed lines the results considering only the P_{33} channel for the pion-nucleon interaction.

next concerns the effect of the small pion-nucleon partial waves. These were calculated exactly for the RPK theory by Giraud *et al.*,[2] and approximately for the relativistic theory by Rinat *et al.*,[6] with the two groups obtaining different answers, so that while Giraud *et al.* suggest that this difference may be due to the use of perturbation theory by Rinat *et al.*, these last authors argue that it is the use of the RPK theory by Giraud *et al.* which is inadequate in the resonance region. The result of Giraud *et al.* at 142 MeV is that the small partial waves lower the cross section at all angles and in particular in the backward direction by as much as 40%. The result of Rinat *et al.* is that they lower the cross section only in the forward direction and raise it for angles larger than about 50°. Our results as we see in Fig. 1 are in good agreement with those of Rinat *et al.*, so that this strongly suggests that the RPK approximation is not adequate in the resonance region.

I summarize the present results as follows: Complete relativistic Faddeev calculations of pion-deuteron scattering provide an excellent description of the data at 142 and 182 MeV but do not resolve the discrepancy at 256 MeV. The calculated results do not depend strongly on whether one uses the BS or AAY reduction. The effect of the small pion-nucleon partial waves lends support to the conclusion that the RPK approximation is not adequate in the resonance region.

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