In summary, we find that dephasing in an ensemble of dipolar coupled electron spins can be studied successfully in wide temperature and magnetic field ranges by using as a probe excited triplet spins of which the coherence decay is monitored optically. Furthermore, our results demonstrate that the dephasing mechanism in the B-spin system is consistent with a phonon-assisted random modulation of dipolar interactions in the fast exchange limit.

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Dielectric Response of a Thin-Layer Zero-Gap Semiconductor

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It is shown that the dielectric response of a thin layer of α-Sn–type semiconductor is radically different than that of the bulk material. The q^{-1} and $\omega^{-1/2}$ singularities of the bulk polarizability are absent in the layer. Instead, the dielectric response and optical absorption of the layer are highly anisotropic, and the static dielectric constant is linearly dependent on layer thickness. In the random-phase approximation, the thin-layer energy-band structure is unconditionally stable against exciton formation.

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Materials of the α-Sn type (α-Sn, HgTe, HgSe, Cd_3As_2) are of considerable interest because of their anomalous dielectric response. Their band structures are characterized by a symmetry-induced degeneracy of the extrema of the highest-lying valence band and lowest-lying conduction band.[1] At absolute zero, the zero-energy excitation at the point of contact produces a $\omega^{-1/2}$ singularity[2,3] in the frequency dependence and a q^{-1} singularity[4] in the momentum-transfer dependence of the dielectric function. This unusual behavior[5] prevents an excitonic instability,[3,6] and its effects have been seen in the transport[7-11] and optical[12,13] properties of the materials.

However, the situation in a very thin layer of zero–band-gap semiconductor will be quite different. Consider an infinite layer of zero–band-gap semiconductor, of thickness small in comparison with the electron mean free path and de Broglie wavelength, imbedded in a high–band-gap material. The electrons in the thin zero–band-gap layer will appear to be in a one-dimensional potential well in the direction normal to the layer,[14] and the conduction and valence bands will split into two-dimensional subbands. In particular, an energy gap will appear between the

extrema of the highest valence and lowest conduction subbands, thus removing the zero-energy excitation possible in the bulk material. In this note, I investigate the dielectric response and stability against exciton formation of such a system.

I assume that the confining potential is a square well, which, for layers many atomic spacings thick, has been found to be highly accurate[14] for thin layers of GaAs. For simplicity, I also assume that the well is of infinite depth, which should be accurate for energies small in comparison with the band gap of the high-band-gap material. Let the plane of the layer be the x-y plane. The energy levels are then given by

$$E_{c,v}(\vec{K}_n) = \frac{\hbar^2}{2\mu_{c,v}m_0} K_n^2 \tag{1}$$

where $\vec{K}_n = (k_x, k_y, k_n)$, $k_n = n\pi/a$ $(n = \pm 1, \ldots, \pm\infty)$, a is the layer thickness, and μ_c and μ_v are the conduction-band and valence-band effective-mass ratios, respectively.

At absolute zero, the longitudinal frequency-dependent dielectric function in the random-phase approximation[15] then takes the form for the principal directions:

$$\epsilon_p(\omega) = \epsilon_b + \lim_{\tau_n \to \infty} \frac{4\pi e^2}{\Omega} \sum_{\vec{K}_n} \lim_{q \to 0} \frac{1}{q^2} \left[\frac{|\langle \vec{K}_n, v | e^{-i\vec{q}\cdot\vec{r}} | \vec{K}_n + \vec{q}, c \rangle|^2}{E_c(\vec{K}_n) - E_v(\vec{K}_n) - \hbar\omega - i\hbar/\tau_n} + \frac{|\langle \vec{K}_n, c | e^{-i\vec{q}\cdot\vec{r}} | \vec{K}_n + \vec{q}, v \rangle|^2}{E_c(\vec{K}_n) - E_v(\vec{K}_n) + \hbar\omega + i\hbar/\tau_n} \right], \tag{2}$$

where Ω is the volume of normalization, p denotes the direction of $\vec{q}$ (i.e., in the x-y plane or along z), and ϵ_b is the contribution from all excitations other than those between the subbands arising from the highest valence and lowest conduction bands of the bulk material. ϵ_b is assumed to be real, isotropic, and frequency independent. We also assume that the cell-periodic part of the wave function in the thin layer is the same as in a bulk zero-band-gap material, which is reasonable for layers which are many atomic spacings in thickness. The matrix elements can then be found by purely group-theoretic methods.[7,16] For $\vec{q}$ in the x-y plane one finds to leading order in q (for pure Γ_8 bulk symmetry)

$$|\langle \vec{K}_n, c | e^{-i\vec{q}\cdot\vec{r}} | \vec{K}_n + \vec{q}, v \rangle|^2 = |\langle \vec{K}_n, v | e^{-i\vec{q}\cdot\vec{r}} | \vec{K}_n + \vec{q}, c \rangle|^2 = \frac{3}{4} \frac{q^2(k^2 \sin^2\theta + k_n^2)}{(k_n^2 + k^2)^2}, \tag{3}$$

where θ is the angle between $\vec{k}$ $[= (k_x, k_y, 0)]$ and $\vec{q}$. For $\vec{q}$ in the z direction, one finds

$$|\langle \vec{K}_n, c | e^{-i\vec{q}\cdot\vec{r}} | \vec{K}_n + \vec{q}, v \rangle|^2 = |\langle \vec{K}_n, v | e^{-i\vec{q}\cdot\vec{r}} | \vec{K}_n + \vec{q}, c \rangle|^2 = \frac{3}{4} \frac{q^2k^2}{(k_n^2 + k^2)^2}. \tag{4}$$

The dielectric tensor is then found to be given by

$$\epsilon_x = \epsilon_y = \epsilon^{\parallel} = \epsilon_b + \frac{3e^2}{\pi\hbar} \left(\frac{\mu m_0}{2E_0} \right)^{1/2} [f_1(\hbar\omega/E_0) + i f_2(\hbar\omega/E_0)] \tag{5}$$

and

$$\epsilon^z = \epsilon_b + \frac{3e^2}{\pi\hbar} \left(\frac{\mu m_0}{2E_0} \right)^{1/2} [g_1(\hbar\omega/E_0) + i g_2(\hbar\omega/E_0)], \tag{6}$$

where μ is the reduced mass and E_0 is the energy splitting between the highest valence and lowest conduction subbands ($\hbar^2\pi^2/2\mu m_0 a^2$).

These functions are shown in Fig. 1. The left ordinate refers to the values of the universal f_i and g_i functions. The right ordinate refers to ϵ values for a 12.86-nm layer of HgTe ($\mu_c = 0.029$, $\mu_v = -0.53$) with an E_0 value of 0.083 eV. In the x-y plane, the electric field strongly couples the nonzero joint density of states at the excitation threshold (the two-dimensional density of states is a sum of step functions). $\epsilon_2^{\parallel}$ thus has a finite discontinuity, and therefore $\epsilon_1^{\parallel}$ must have a singularity which, for this coupling matrix element [Eq. (3)], is logarithmic. This behavior contrasts to that of a normal direct-band-gap semiconductor where the joint density of states is proportional to $(E - E_g)^{1/2}$, and ϵ_2 has only a discontinuity in slope which produces a cusp in ϵ_1, rather than a singularity.

In the z direction, the coupling at the excitation threshold ($k = 0$) vanishes by wave-function symmetry [see Eq. (4)]. Thus, ϵ_2^z has only a discontinuity in slope. For this coupling matrix element, the discontinuity in the slope of ϵ_2^z leads to a singularity in the slope of ϵ_1^z and a peak displaced to the high-energy side of the excitation

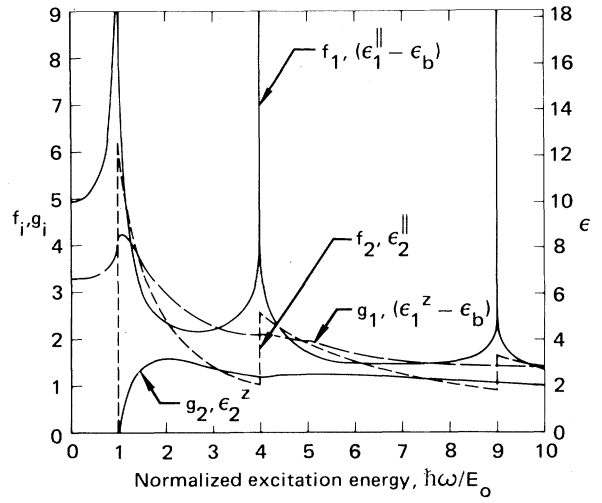

FIG. 1. The frequency-dependent dielectric tensor of a thin layer of zero-band-gap semiconductor.

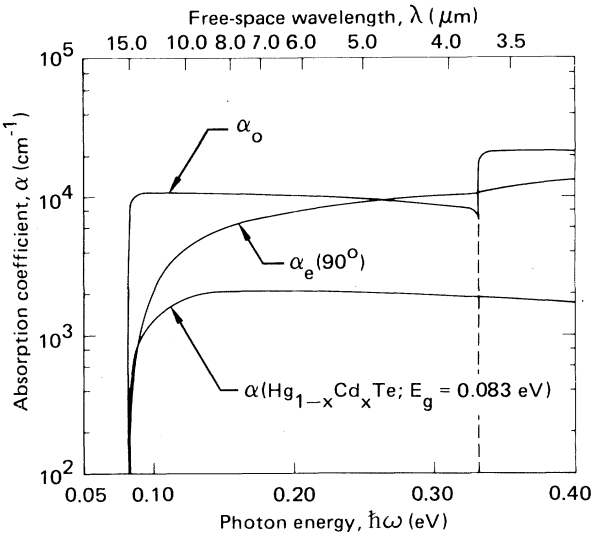

FIG. 2. Ordinary- (α_0) and extraordinary-wave [$\alpha_e(\theta_e)$] absorption coefficients for a 12.86-nm HgTe layer. Also shown is the absorption coefficent for a bulk $Hg_{1-x}Cd_xTe$ alloy with the same effective gap (0.083 eV).

threshold. These structures are repeated at every excitation threshold between pairs of conduction and valence subbands with the same well quantum number.

The zero-band-gap semiconductor layer is thus a highly anisotropic dielectric medium, as will be observable in its optical absorption. A medium with this dielectric tensor ($\epsilon_x = \epsilon_y = \epsilon^{\parallel} \neq \epsilon^z$) can support two transverse waves.[17] The ordinary wave ($\vec{E}$ in the plane of the layer) is attenuated with an absorption coefficient, α_0, which is independent of the direction of propagation. The extraordinary wave ($\vec{E}$ in the plane of the wave vector and the z axis) is attenuated with an absorption coefficient, $\alpha_e(\theta_e)$, which is strongly dependent on θ_e (the angle between the wave vector and the z axis). These absorption coefficients are shown in Fig. 2 for the same 12.86-nm layer of HgTe to which the right ordinate of Fig. 1 applies, with $\epsilon_b = 10.4$. The values for α_e are for $\theta_e = 90°$, and are the lowest α_e attains. For propagation normal to the plane of the layer ($\theta_e = 0$), the two waves coincide and $\alpha_e = \alpha_0$. Also shown in Fig. 2 is α for a bulk $Hg_{1-x}Cd_xTe$ alloy with $E_g = 0.083$ eV, calculated with the simplified expression of Burstein *et al.*,[18] assuming $\epsilon_b = 10.4$ and a Kane momentum matrix element (Ref. 16) $P = 8 \times 10^{-8}$ eV cm.

The ordinary-wave absorption coefficient, α_0, rises at the absorption threshold with nearly a step-function behavior, reflecting the discontinuity in $\epsilon_2^{\parallel}$. It remains nearly constant with increasing photon energy until the $n = 2$ threshold is reached. It then goes to zero, because of the singularity in $\epsilon_1^{\parallel}$, and rises to a higher plateau because of excitation between the second set of subbands. This behavior is repeated for every pair of subbands. The absorption coefficient of the alloy with the same gap rises more slowly, because the alloy joint density of states is zero at the absorption threshold, and is also much smaller, because the alloy effective mass necessarily decreases with decreasing energy gap, while that of the zero-band-gap layer is independent of effective gap. The extraordinary-wave absorption coefficient for propagation in the plane of the layer, $\alpha_e(90°)$, also rises more slowly than α_0, a consequence of wave-function symmetry forbidding the transition at the absorption threshold for $\vec{E}$ polarized normal to the layer. The dependence of α_e on propagation angle for wavelengths near the absorption threshold is illustrated in Fig. 3. The effective or observed absorption coefficient of a stack of quantum-mechanically isolated zero-band-gap semiconductor layers will be slightly different than those shown in Figs. 2 and 3 because the effective fields depend on the dielectric constant and thickness of the intervening high-band-gap regions. This problem is too lengthy to be considered here.

In a similar manner, one can find the static

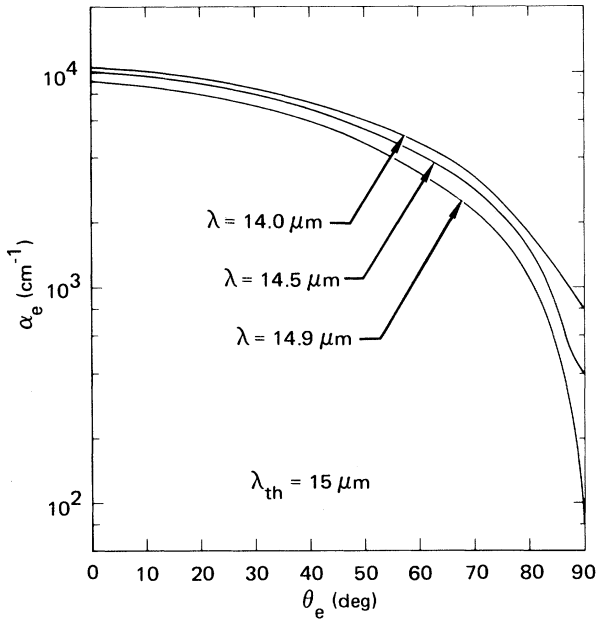

FIG. 3. Angular dependence (referred to the z axis) of the extraordinary-wave absorption coefficient of a 12.86-nm HgTe layer, for three wavelengths near the threshold absorption wavelength (15 μm).

momentum-transfer–dependent dielectric function. It is given in the plane of the layer by

$$\epsilon^{\parallel}(q) = \epsilon_b + \epsilon_0 + \sum_{n,m>0} O((q/k_n)^m), \tag{7}$$

where

$$\epsilon_0 = \frac{3\pi e^2}{2\hbar}\left(\frac{\mu m_0}{2E_0}\right)^{1/2} = \frac{3\mu m_0 e^2}{2\hbar^2}\, a. \tag{8}$$

The q-dependent terms are small in comparison with ϵ_0 for $q \ll k_1$. The q^{-1} singularity of the bulk static dielectric response is not present in the thin layer, and the layer dielectric constant increases linearly with layer thickness. This dependence of screening on layer thickness will have an important effect on the thickness dependence of electron mobility.

Is the thin-layer band structure stable against exciton formation? The binding energy of a truly two-dimensional exciton[19] is four times that of a three-dimensional exciton, and the increase of exciton binding energy with decreasing layer thickness has been observed[14] in thin GaAs layers. Since the layer thickness is much less than the exciton radius, I assume, for the sake of argument, the two-dimensional limit. One then finds for the exciton energy (referred to the minimum of the lowest conduction subband):

$$E_{ex} = -\frac{2\mu m_0 e^4}{\hbar^2 \epsilon_0^{\parallel 2}} = \frac{-(4/3\pi)^2 E_0}{[1 + 2\hbar^2 \epsilon_b / 3\mu m_0 e^2 a]^2}$$
$$\geq -(4/3\pi)^2 E_0 > -E_0. \tag{9}$$

Thus, in the random-phase approximation, the exciton always lies in the gap, and the band structure is stable against exciton formation for all values of effective mass, background dielectric constant, and layer thickness.

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