Creating an Asymmetric Plasma Resistivity with Waves

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Preferential heating of electrons traveling in one direction can support a current even in the absence of a dc electric field. An immediate implication is that even waves which carry little toriodal momentum, such as electron cyclotron waves, may be attractive as a means for generating steady-state toroidal current in a tokamak. An analytical expression is derived for the current generated per power dissipated, which agrees remarkably well with numerical calculations.

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Various methods have been proposed for producing continuous toroidal currents in tokamak plasmas.[1-3] The common feature of these methods is that net toroidal momentum is delivered to one of the plasma species, usually the electrons, from an external source. The momentum absorption by a particular species leads to the production of current that is destroyed only by collisions with the opposite species.

A basically different scheme is proposed in which no net toroidal momentum is injected, but the collisionality of the plasma is somehow altered so that, for example, electrons moving to the left collide more frequently with the ions than do electrons moving to the right. There would result a net electric current with ions moving to the left and electrons moving, on average, to the right. The means of accomplishing this asymmetric resistivity could be selective heating of those electrons moving to the right. Being hotter, they naturally collide less.

The interest in driving currents in this manner arises from the possibility of operating tokamak reactors in the steady state by replacing the inherently pulsed Ohmic transformer current. The crucial quantity, by which the practicality of a reactor incorporating any of these schemes may be assessed, is J/P_d, the amount of current generated per power dissipated. This quantity which we determine presently for the new scheme is, of course, to be maximized.

The method employed in calculating this quantity is tantamount to finding the Green's function for the dynamical equations. Recommending this method is that the mathematics at each step has an easily understood physical interpretation. First we find the response to an impulse given to a select group of electrons. To sustain a steady-state current we envision the application of a continuous stream of impulses, or, equivalently, steady power input. The current at any time is then found as the cumulative response to all prior impulses, although only impulses within a finite time interval, say Δt, contribute significantly to the current.

Consider the displacement in velocity space of a small number, δf, of electrons from coordinates to be subscripted 1 to those to be subscripted 2. The energy expended to produce this displacement is given by

$$\Delta E = (E_2 - E_1)\delta f, \tag{1}$$

where E_i is the kinetic energy associated with velocity-space location i. Electrons at different coordinates will scatter at different rates; suppose the displaced electrons would have lost their momentum parallel to the magnetic field, which is in the z direction, at a rate ν_1, but now lose it at a rate ν_2. The z-directed current density is then given by

$$\mathcal{J}(t) = e\delta f[v_{z1}\exp(-\nu_1 t) - v_{z2}\exp(-\nu_2 t)], \tag{2}$$

where v_z is the velocity parallel to the magnetic field and e is the electron charge.

Consider the quantity J, the time-smoothed current over an interval Δt which is large compared both to the $1/\nu_1$ and $1/\nu_2$ so that

$$J \equiv \frac{1}{\Delta t}\int_0^{\Delta t} \mathcal{J}(t)dt \simeq \frac{e\delta f}{\Delta t}\left(\frac{v_{z1}}{\nu_1} - \frac{v_{z2}}{\nu_2}\right). \tag{3}$$

The term in the integral may be interpreted as the time-integrated current attributable to an energy input ΔE. Substituting now for δf from Eq. (1) and identifying $\Delta E/\Delta t$ as the dissipated power, we find the crucial parameter

$$\frac{J}{P_d} = -e\left(\frac{v_{z1}/\nu_1 - v_{z2}/\nu_2}{E_1 - E_2}\right)\xrightarrow[\lim \vec{v}_2 \to \vec{v}_1]{} \frac{-e\hat{s}\cdot\nabla(v_z/\nu)}{\hat{s}\cdot\nabla_E}, \tag{4}$$

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where in the limit the locations 1 and 2 are separated infinitesimally (which allows the dropping of the subscript) in the direction of the velocity displacement vector $\hat{s}$.

An accurate determination of the momentum destruction frequency ν is momentarily deferred; suppose, however, that $\nu \propto v^{-3}$, where v is the speed of the resonant electrons. It follows that for lower-hybrid waves, where $\hat{s} \parallel \hat{z}$, we have

$$J/P_d \propto v_z^{-1}(v_z^2+v_\perp^2)^{3/2}+3v_z(v_z^2+v_\perp^2)^{1/2}, \tag{5}$$

where v is the velocity perpendicular to the magnetic field and may be supposed for the resonant electrons to be far less than v_z. The surprise in Eq. (5) is that the term arising from the energy input is three times larger than the term arising from the momentum input. This gives a new interpretation of the success of the lower-hybrid scheme for current generation, and shatters the view that may now be labeled as a misconception—that the success relied on waves with net momentum.

In fact, since there is only marginal utility in employing waves with net momentum, we may envision supplying only energy to the resonant electrons by choosing $\hat{s}$ parallel to $v_\perp$. This may be accomplished, for example, by heating in the perpendicular direction with a wave that resonates with the selected electrons. An example of such a wave is the electron cyclotron wave. The associated J/P_d would be about $\frac{3}{4}$ that for the lower-hybrid scheme, so that the cyclotron-wave scheme may indeed be competitive in terms of reactor applications.

It remains to show that ν, to be used in Eq. (4), may be analytically determined with remarkable accuracy. From the Fokker-Planck equation written in the high-velocity limit, two collisional scattering rates may be distinguished: a slowing down rate $\nu_E = \nu_0/2u^3$ and a momentum destruction rate $\nu_M = (2+Z_i)\nu_E$, where $\nu_0 = \omega_p^4 \ln\Lambda/2\pi n_0 v_{\rm the}^3$, $u = v/v_{\rm the}$, Z_i is the ion charge state, and $v_{\rm the}$ is the electron thermal velocity. Consider a test electron that slows down energetically as it loses its parallel momentum. Diffusion in energy is ignored, but the slowing down in energy causes both collision rates to vary. The slowing down equation is given by

$$du/dt = -\nu_E u. \tag{6}$$

The current carried by the electron may be written as

$$j(t) = j(t=0)\exp[-\textstyle\int_0^t \nu_M(t)dt] = j(t=0)[u(t)/u_0]^{2+Z_i}, \tag{7}$$

where u_0 is the initial normalized electron speed and the second equality was written by changing variables in the integral with use of Eq. (6). Note that the background electrons, as well as the ions, contribute to the current destruction. This is because, as discussed in Ref. 3, any current absorbed by the thermal electrons, which are far more collisional than the test electron, is in any case quickly destroyed by the ions.

Using Eq. (6) again, we may now write the time-integrated current as

$$\int_0^\infty j(t)dt = -j(t=0)\int_{u_0}^0 \left(\frac{u}{u_0}\right)^{2+Z_i} \frac{du}{u\nu_E} = \frac{j(t=0)}{\nu_0}\frac{2u_0^3}{5+Z_i}. \tag{8}$$

It follows that the correct ν to be employed in Eqs. (2)–(4) is given by

$$\nu = \nu_0(5+Z_i)/2u^3. \tag{9}$$

Adopting now conventions characteristic of previous work,[3-5] we normalize velocities to $v_{\rm the}$, J to $-en_0v_{\rm the}$, and P_d to $m_e n_0 v_{\rm the}^2\nu_0$. Upon using Eq. (9) in Eq. (4) we can write the normalized J/P_d as

$$\frac{J}{P_d} = \frac{\hat{s}\cdot\nabla(wu^3)}{\hat{s}\cdot\nabla u^2}\frac{4}{5+Z_i}, \tag{10}$$

where $w \equiv v_z/v_{\rm the}$. A comparison with previous numerical work is now possible; in particular for large w, for $Z_i = 1$, and for $\hat{s}$ parallel to $\hat{w}$, it was determined numerically in Ref. 5 that $J/P_d = 1.4w^2$. In such a case, which characterizes current drive with lower-hybrid waves, Eq. (10) predicts $J/P_d = 1.33w^2$, which is accurate agreement indeed.

An interesting observation is that the one-dimensional (in velocity space) Fokker-Planck equation previously employed in examining current drive by lower-hybrid waves[3] does in fact distinguish, albeit only in the parallel direction, the momentum input contributions to the current from the energy input contributions. The failure of the one-dimensional theory to obtain the correct numerical factor in determining J/P_d is due to the lack of differentiation, in one dimension, between ν_E and ν_M. Note that were $\nu_E = \nu_M$, then the one-dimensional theory would have agreed with Eq. (10), correctly predicting J/P_d.

The range of application of Eq. (10) is limited to situations where the location in velocity space of the electrons which absorb energy from the external source is known. When the external source is a wave, the parallel velocity of the absorbing electrons is known from the resonance condition. The precise perpendicular velocity of the electrons is immaterial to Eq. (10), unless it becomes comparable to the parallel velocity. Thus, Eq. (10) may be accurately employed in the case of fast lower-hybrid waves, such as examined in Ref. 5, that are too weak to significantly perturb the distribution function from a Maxwellian. It is only when the spectrum becomes broad and intense, or $Z_i \gg 1$, that significant perturbations from the Maxwellian distribution occur, in particular, in the form of a flattening in the perpendicular direction.[4,6] The waves may then interact with a significant number of high-$v_\perp$ electrons. Such is the case for the numerical examples computed in Ref. 4, in which $J/P_d = 1.7w^2$ was found. The perpendicular flattening is a nonlinear effect in, for example, the spectral width, and cannot be treated by the analysis offered in this Letter. Nevertheless, although a strict comparison is not appropriate, it may be observed that the result in Ref. 4 does not widely disagree with the prediction of Eq. (10). Of particular interest is that the scaling of J/P_d with Z_i as predicted by Eq. (10) agrees quite closely with the numerical cases offered in Ref. 4, and is far superior to the scaling predicted by a purely one-dimensional analysis.[3] This close agreement must be considered somewhat fortuitous; for Z_i large enough the agreement will be lost. It should be appreciated, however, that were the wave spectrum narrow in v_z and were its effect truncated at low $v_\perp$, then Eq. (10) would be accurate no matter what Z_i.

Although it is not within the scope of this paper to speculate on the attractiveness of driving currents with electron cyclotron waves, a host of new considerations that require further study may be identified. Electron cyclotron waves induce velocity-space diffusion primarily in the perperpendicular direction, so that the perpendicular flattening of the electron distribution is expected to be more pronounced than when excited by lower-hybrid waves. The flattening may be expected to enhance J/P_d. On the other hand, too much flattening will also enhance synchrotron radiation. The propagation characteristics of the electron cyclotron waves are also cause for comment: Accessibility constraints, especially in high-β plasmas, may be milder than for lower-hybrid waves, but launching from the high-field region may be necessary to control the absorption. Further considerations of this scheme in the context of a survey of current-drive mechanisms may be found elsewhere.[7]

In conclusion, what has been discovered is a new way of understanding current generation. An accurate formula has been derived for the efficiency of current generation by a general acceleration mechanism. What emerges is the recognition that a variety of plasma waves, hitherto considered inappropriate for driving currents, may be worthy of exploration for just such a task.

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