Consequences of Majorana and Dirac Mass Mixing for Neutrino Oscillations

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This Letter considers a second class of neutrino oscillations which can arise when both Majorana and Dirac neutrino mass terms exist. These oscillations mix neutrino members of weak current doublets with singlets of the same chirality. A depletion of a neutrino beam results, with apparent nonconservation of probability. Possible relevance to current oscillation experiments is discussed.

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The success and appeal of grand unified theories[1] have given a new theoretical impetus to the question of neutrino mass.[2] Moreover, recent analyses of reactor and beam-dump data have revealed the exciting possibility that neutrino oscillations exist.[3-5] The standard formalism[6,7] for neutrino oscillations is based on oscillations of the type $\nu_{eL} \leftrightarrow \nu_{\mu L} \leftrightarrow \nu_{\tau L}$, which mix flavors without change of chirality or lepton number; hereafter we refer to these as first-class oscillations. In this Letter we consider the possibility of a second class of neutrino oscillations,[8] involving transitions of the type $\nu_L \leftrightarrow \eta_L$ which mix neutral members of weak isospin doublets ν_L with singlets η_L. In the standard SU(2)⊗U(1) model, the usual right-handed singlets would be $\eta^c{}_R = C(\overline{\eta}_L)^T$ where C is the charge-conjugation matrix. To avoid confusion, we emphasize at the outset that second-class oscillations are not of the type $\nu_{eL} \leftrightarrow \nu^c_{eR}$, where ν^c_{eR} (usually denoted by $\overline{\nu}_e$) is the right-handed antineutrino produced in μ^- decay; ν_{eL} and ν^c_{eR} are related by charge conjugation, and ν^c_{eR} is an SU(2) doublet member along with e^+. In general, second-class oscillations can involve transitions among different doublet and singlet flavors.

Neutrino mass terms can be either of Majorana or Dirac type. At least some grand unified theories suggest[2] that both may be present simultaneously.[8] Dirac mass terms are of the form $\overline{\nu}_L \eta^c{}_R$ while Majorana mass terms are of the forms $\overline{\nu}_L \nu^c{}_R$ and $\overline{\eta}_L \eta^c{}_R$, which violate lepton-number conservation by two units.[9] Diagonalization of the mass matrix for a single lepton family yields two Majorana (i.e., self-conjugate) mass eigenstates. If we assume that both masses are small, neutrino oscillations will occur between the doublet and singlet gauge eigenstates of the same helicity. Since singlet fields are decoupled from gauge bosons, these second-class oscillations deplete neutrino beams, giving the appearance of probability nonconservation. In the general case of several lepton flavors, both first- and second-class oscillations can occur.

In the following we first develop the formalism for second-class oscillations involving a single lepton family. We then address possible phenomenological implications for neutrino oscillation experiments, comparing expectations from first- and second-class oscillations. Finally, we develop the formalism for the situation when both classes of oscillations are present. Our considerations are specifically based on a V-A structure for the charged weak current, in which case $\nu_L \leftrightarrow \nu^c{}_R$ oscillations[10] are suppressed by $(m_\nu/E)^2$ and are negligible.

For simplicity we first consider the consequence of having both Majorana and Dirac neutrino mass terms in a single-family version of the standard SU(2)⊗U(1) model. The left-handed leptons are $(\nu, e^-)_L$, $e^+{}_L$ and η_L. The associated charge-conjugate neutrino fields are defined as $\nu^c{}_R \equiv C(\overline{\nu}_L)^T$ and $\eta^c{}_R \equiv C(\overline{\eta}_L)^T$, where $C = i\gamma^2\gamma^0$ is the charge-conjugation matrix. The general form of the Lagrangian mass term is

$$\mathcal{L}_{\rm mass} = -\tfrac{1}{2}[a(\overline{\nu}_L \nu^c{}_R) + d(\overline{\nu}_L \eta^c{}_R + \overline{\eta}_L \nu^c{}_R) + s(\overline{\eta}_L \eta^c{}_R)] + \text{H.c.} \qquad (1)$$

In Eq. (1) we have made use of the identity $\overline{\nu}_L \eta^c{}_R = \overline{\eta}_L \nu^c{}_R$ to reduce the number of independent constants.

Defining the doublets $\omega^{\alpha}{}_L \equiv (\nu_L, \eta_L)$ and $\omega^{\alpha c}{}_R \equiv (\nu^c{}_R, \eta^c{}_R)$, we can write

$$\mathcal{L}_{\text{mass}} = -\tfrac{1}{2}\bar{\omega}^{\alpha}{}_L M^{\alpha\beta}\omega^{\beta c}{}_R + \text{H.c.}, \qquad (2)$$

with mass matrix

$$\underline{M} = \begin{pmatrix} a & d \\ d & s \end{pmatrix}. \qquad (3)$$

For symmetry breaking with the standard Higgs-doublet representation, the parameter d is nonzero but a vanishes; a nonzero value for a can be obtained by adding a Higgs triplet; s is due to a singlet Higgs or a bare mass term.

The diagonalized mass matrix is $\underline{M}_D = \underline{U}_L{}^{\dagger}\underline{M}\,\underline{U}_R$ where $\underline{U}_L$ and $\underline{U}_R$ are unitary transformations of the ω_L and $\omega^c{}_R$ fields. Since $\underline{M}$ is symmetric, $\underline{U}_R = \underline{U}_L{}^*\underline{K}^{\dagger}$ with $\underline{K}$ a symmetric unitary matrix. For nondegenerate mass eigenvalues, $\underline{K}$ is a diagonal matrix of phases, $K_{ij} = \exp(-i\varphi_i)\delta_{ij}$. By appropriate choice of the matrix $\underline{K}$, we can take $\underline{U}_L$ to be a real rotation matrix. The relation of mass eigenstates ν_{iL} to $\omega^{\alpha}{}_L$ is

$$\omega^{\alpha}{}_L = U^{\alpha i}{}_L \nu_{iL} \quad (i = 1, 2). \qquad (4)$$

The corresponding right-handed transformation is

$$\omega^{\alpha c}{}_R = C(\bar{\omega}^{\alpha}{}_L)^T = U^{\alpha i}{}_R K_{ij}\nu^c_{jR} \equiv U^{\alpha i}{}_R \nu^{\tilde{c}}_{iR}, \qquad (5)$$

where

$$\nu^{\tilde{c}}_{iR} \equiv K_{ij}\nu^c_{jR} = K_{ij}C(\bar{\nu}_{jL})^T. \qquad (6)$$

The free Lagrangian for the neutral leptons is diagonal in the basis $\nu_i = \nu_{iL} + \nu^{\tilde{c}}_{iR}$. From Eq. (6) we find $\nu^{\tilde{c}}{}_i = \nu_i$, where $\nu^{\tilde{c}}{}_i \equiv K_{ij}C(\bar{\nu}_j)^T$. Hence the ν_i are Majorana neutrino fields since they are self-conjugate.[11] The combined Dirac and Majorana mass terms in the Lagrangian produce two Majorana eigenstates which in general have different masses m_1 and m_2. When $m_1 \neq m_2$, there is no conserved lepton number.

From Eq. (4), the weak eigenstates ν_L and η_L are linear superpositions of the two Majorana mass eigenstates

$$\nu_L = (\cos\alpha)\nu_{1L} + (\sin\alpha)\nu_{2L}$$
$$\eta_L = -(\sin\alpha)\nu_{1L} + (\cos\alpha)\nu_{2L}, \qquad (7)$$

where $\cos\alpha = (\underline{U}_L)^{11}$, $\sin\alpha = (\underline{U}_L)^{12}$. The singlet state η_L does not couple to gauge bosons and interacts with fermions only via Higgs couplings. The doublet member ν_L has the usual charged- and neutral-current couplings. In the mass eigenstate basis, the neutral current is nondiagonal.

We mention two limiting cases of Eq. (3). If $a = 0$ and $s = 0$, the Lagrangian possesses an invariance $(\nu_L, e_L, \eta^c{}_R) \to e^{i\beta}(\nu_L, e_L, \eta^c{}_R)$, corresponding to lepton-number conservation; note that η_L is an antilepton in this case. The Majorana states are then degenerate ($m_1 = m_2$) and combine to form a single massive Dirac field. Another interesting limit is $a = 0$ and $|s| \gg |d|$, which occurs naturally in some grand unified theories.[2] In this case the mass eigenvalues are $m_1 = |d|^2/s$ and $m_2 = |s|$, and $\underline{U}_L$ is a unit matrix, to leading order in $|d|/|s|$. If $|d|$ is a typical fermion mass ~1 GeV and $|s|$ is the unification mass scale ~10^{14} GeV, the state m_2 cannot be produced and effectively decouples.

Our primary considerations are for another logical possibility in which both m_1 and m_2 are small compared with the electron mass. This possibility has interesting implications for neutrino oscillations. Since the mass eigenstates propagate differently in time, $\nu_{eL} \to \eta_{eL}$ oscillations occur. These "second-class" oscillations conserve helicity. At a distance L from a source of ν_{eL}, the probability (for energy $E \gg m_1, m_2$) of finding ν_{eL} is

$$P(\nu_{eL} \to \nu_{eL}) = 1 - \sin^2(2\alpha)\sin^2(\tfrac{1}{2}\Delta), \qquad (8)$$

where the oscillation argument is $\frac{1}{2}\Delta = 1.27\delta(m^2)L/E$, with $\delta(m^2) = m_1{}^2 - m_2{}^2$ in electronvolts squared units and L/E in meters per megaelectronvolt units. The oscillations result in a depletion of an electron neutrino beam, or equivalently a deviation from a $1/r^2$ law for a point ν_{eL} source. Moreover, since η_{eL} is effectively noninteracting, probability conservation would appear to be experimentally violated by an amount $P(\nu_{eL} \to \eta_{eL}) = 1 - P(\nu_{eL} \to \nu_{eL})$, in contrast to first-class oscillations where a depletion in $\nu_{eL} \to \nu_{eL}$ coincides with $\nu_{eL} \to \nu_{\mu L}, \nu_{\tau L}, \ldots$ transitions which are in principle observable.

In second-class oscillations, both the charged-current (CC) $\nu_{eL}p \to e^- X$ and neutral-current (NC) $\nu_{eL}p \to \nu_{eL}X$ cross sections oscillate,

$$\sigma(L)/\sigma(L=0) = P(\nu_{eL} \to \nu_{eL};\ L/E), \qquad (9)$$

and the ratio $\sigma_{\text{NC}}/\sigma_{\text{CC}}$ is unaffected in the one family case. This contrasts with first-class oscillations where σ_{CC} and $\sigma_{\text{NC}}/\sigma_{\text{CC}}$ oscillate, but σ_{NC} does not. Corresponding statements apply to ν^c_{eR} cross sections.

We now turn to possible phenomenological implications of second-class oscillations for current experiments.

Solar experiments.—Lepton-number-nonconserving oscillations have the capability of explaining the deficiency in the ratio of observed to expected solar neutrinos.[12] With first- and second-class oscillations among three families, the minimum probability for $\nu_e \to \nu_e$ transitions is $\frac{1}{6}$.

Reactor experiments.—The cross sections for an initial ν_{eR}^c beam scattering on proton and deuteron targets indicate depletions[3,4] in $\sigma_{CC}(p)$, $\sigma_{CC}(d)$, and $\sigma_{CC}(d)/\sigma_{NC}(d)$ but not (at the $\simeq 20\%$ uncertainty level) in $\sigma_{NC}(d)$. To explain both the σ_{CC} and σ_{CC}/σ_{NC} results, first-class oscillations are required with $\delta(m^2) \simeq 1\ \text{eV}^2$.

Beam-dump experiments.—Charged- and neutral-current events are produced by prompt neutrinos created in the dump. Since the prompt neutrinos originate from decays of charmed particles, identical ν_e and ν_μ spectra and numbers are generated. The charged- and neutral-current interactions of the prompt neutrinos are measured in bubble-chamber and counter experiments[13] at CERN at a distance $L \simeq 800$–900 m downstream.

In the bubble-chamber experiment, the measured e/μ ratio[13] is $R(e/\mu) = 0.48_{-0.16}^{+0.24}$. Such deviations of the e/μ ratio from unity may indicate a $P(\nu_e \to \nu_e)$ depletion arising from oscillations.[3,5] For the CERN beam dump, $L/E \simeq 0.01$ m/MeV; so the mass scale of the oscillations would be $\delta(m^2) \simeq 100\ \text{eV}^2$. To discuss such oscillations we assume a prompt-neutrino beam with equal parts of ν_{eL} and $\nu_{\mu L}$, neglecting any ν_{eR}^c and $\nu_{\mu R}^c$ contributions for simplicity.

For second-class oscillations of the ν_e family alone, the e/μ ratio is given by

$$R(e/\mu) = \langle P(\nu_e \to \nu_e)\sigma_{CC}\rangle / \langle \sigma_{CC}\rangle , \tag{10}$$

where σ_{CC} is the inclusive production cross section for e or μ and the angular brackets denote a spectrum average. For first-class oscillations $\nu_e \to \nu_e$, $\nu_e \to \nu_\tau$ (stringent experimental limits exist on $\nu_\mu \to \nu_e$ and $\nu_\mu \to \nu_\tau$ oscillations in this L/E range), the corresponding prediction is

$$R(e/\mu) = \frac{\langle P(\nu_e \to \nu_e)\sigma_{CC}\rangle + 0.17\langle P(\nu_e \to \nu_\tau)\sigma_{CC}{}^\tau\rangle}{\langle \sigma_{CC}\rangle + 0.17\langle P(\nu_e \to \nu_\tau)\sigma_{CC}{}^\tau\rangle} , \tag{11}$$

where $\sigma_{CC}{}^\tau$ is the inclusive τ cross section. For comparable mixing in the two classes, the predictions in Eqs. (10) and (11) are similar. One can discriminate experimentally between the classes of oscillations by ascertaining whether τ is produced and whether σ_{NC}/σ_{CC} changes.

The beam-dump counter experiments measure the ratio $N(0\mu)/N(1\mu)$ of muonless to single-muon events. With second-class oscillations of the ν_e family, the prediction is

$$N(0\mu)/N(1\mu) = \{\langle [1 + P(\nu_e \to \nu_e)]\sigma_{NC}\rangle + \langle P(\nu_e \to \nu_e)\sigma_{CC}\rangle\} / \langle \sigma_{CC}\rangle \tag{12}$$

in the limit of perfect acceptance. The corresponding prediction for first-class oscillations is

$$\frac{N(0\mu)}{N(1\mu)} = \frac{2\langle \sigma_{NC}\rangle + \langle P(\nu_e \to \nu_e)\sigma_{CC}\rangle + 0.83\langle P(\nu_e \to \nu_\tau)\sigma_{CC}{}^\tau\rangle}{\langle \sigma_{CC}\rangle + 0.17\langle P(\nu_e \to \nu_\tau)\sigma_{CC}{}^\tau\rangle} . \tag{13}$$

Taking comparable mixing in the two classes [and hence similar $R(e/\mu)$ predictions], the value of $N(0\mu)/N(1\mu)$ is significantly lower for second-class oscillations. A detailed analysis with experimental cuts could thereby differentiate between first- and second-class oscillations in this L/E range on the basis of measured $R(e/\mu)$ and $N(0\mu)/N(1\mu)$ values. Still other alternatives are simultaneous first- and second-class oscillations or first-class oscillations involving additional families.

We next turn to the general case of first- and second-class oscillations involving three families of leptons. The neutral members of the weak doublets are ν_{eL}, $\nu_{\mu L}$, and $\nu_{\tau L}$. We assume an equal number of singlets η_{eL}, $\eta_{\mu L}$, and $\eta_{\tau L}$ (though there could be a different number).[14] As in the single-family case, we define vectors $\omega^\alpha{}_L = (\nu_{eL}, \nu_{\mu L}, \nu_{\tau L}, \eta_{eL}, \eta_{\mu L}, \eta_{\tau L})$ and $\omega^{\alpha c}{}_R = (\nu_{eR}^c, \nu_{\mu R}^c, \nu_{\tau R}^c, \eta_{eR}^c, \eta_{\mu R}^c, \eta_{\tau R}^c)$ with $\alpha = 1, \ldots, 6$. The mass term can then be written as in Eq. (2) with

$$\underline{M} = \begin{pmatrix} \underline{A} & \underline{D} \\ \underline{D}^T & \underline{S} \end{pmatrix}, \tag{14}$$

where $\underline{A}$, $\underline{S}$, and $\underline{D}$ are 3×3 matrices. $\underline{A}$ and $\underline{S}$ are symmetric matrices, which implies $\underline{M}$ is symmetric also. To diagonalize $\underline{M}$, we make the transformations analogous to Eqs. (4) and (5) with $i = 1, \ldots, 6$. The Majorana fields $\nu_i = \nu_{iL} + \nu_{iR}^{\tilde{c}}$ are the physical eigenstates with masses m_i, $i = 1, \ldots, 6$.

The unitary matrix $\underline{U}_L$ can be written in 3×3 matrix block form as

$$\underline{U}_L = \begin{pmatrix} \underline{W} & \underline{X} \\ \underline{Y} & \underline{Z} \end{pmatrix} = \underline{U}_R{}^* \underline{K}^\dagger . \tag{15}$$

The matrix $\underline{W}$ ($\underline{Z}$) describes first-class oscillations among the doublet (singlet) members; $\underline{X}$

and $\underline{Y}$ describe second-class oscillations connecting doublets and singlets.

In the special cases of only Dirac mass terms ($\underline{A} = \underline{S} = \underline{0}$) or of only Majorana mass terms ($\underline{D} = \underline{0}$), the freedom of choice of $\underline{K}$ can be used to set $\underline{X} = \underline{Y} = \underline{0}$. Hence only first-class oscillations occur and the unitarity of $\underline{U}_L$ implies that $\underline{W}$ and $\underline{Z}$ are unitary. $\underline{W}$ describes the conventional flavor-changing oscillations; $\underline{Z}$ is essentially unobservable.

In the general case in which both first- and second-class oscillations are present simultaneously, the unitarity of $\underline{U}_L$ no longer implies that $\underline{W}$ is unitary. This corresponds to the depletion effect of doublets oscillating into singlets, such as $\nu_{eL} \to \eta_{eL}, \eta_{\mu L}, \eta_{\tau L}$. The crucial test of second-class oscillations is the direct measurement of all flavors of produced neutrino doublet members to test for apparent probability nonconservation.

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