the vector-meson dominance[8,9] or the $\gamma$GF[10] models. If one ignores any $\gamma$GF-model uncertainty, this result rules out the choice $|q_b| = \frac{2}{3}$ with 85% confidence. With 67% confidence, the data disfavor the existence of similar bound states of a second charge-$\frac{1}{3}$ quark in the $\Upsilon$ mass region.

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# Measuring Quantum-Chromodynamic Anomalies in Hadronic Transitions between Quarkonium States

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It is argued that the ratio $\Gamma((\bar{Q}Q)'\to(\bar{Q}Q)\eta)/\Gamma((\bar{Q}Q)'\to(\bar{Q}Q)\pi\pi)$ of hadronic transition rates between heavy quarkonium states is calculable within quantum chromodynamics in terms of triangle anomalies in the divergence of the axial current and in the trace of the energy-momentum tensor. In the case of transitions between $\psi'$ and $J/\psi$ the present analysis is consistent with the data. More reliable test can be provided by experimental study of the transitions between $\Upsilon''$ and $\Upsilon$.

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Hadronic transitions between quarkonium levels, like $\psi'\to(J/\psi)\pi\pi$ and $\psi'\to(J/\psi)\eta$ decays, can provide an insight into gluonic physics. Indeed, the transition can be viewed as a two-step process: first, emission of soft gluons by heavy quarks, and then conversion of the gluons into light hadrons.[1] As realized first by Gottfried,[2] the gluon emission can be described by the gluon multipole expansion.[2-4] The point is that heavy quarkonium is a rather compact object in the typical hadronic scale. On the other hand, gluon conversion which effectively measures the gluon admixture in ordinary hadrons is a large-distance process and is most difficult to trace theoretically.

 

Here we argue that at least in two particular cases the theory can be completed and gluon conversion in the $\eta$ meson and into a two-pion low-mass state is calculable in quantum chromodynamics (QCD). We show that for transitions between $S$ states of nonrelativistic heavy quarks the multipole expansion implies that the gluon conversion is governed by the matrix elements

$$\langle\eta| g^2 G_{\mu\nu}{}^a(x)\tilde{G}_{\mu\nu}{}^a(x)|0\rangle,$$

$$\langle\pi\pi| g^2 G_{\mu\nu}{}^a(x) G_{\mu\nu}{}^a(x)|0\rangle,$$

where $G_{\mu\nu}{}^a(x)$ is the gluon field strength operator, $\tilde{G}_{\mu\nu}=\frac{1}{2}\epsilon_{\mu\nu\gamma\sigma}G_{\gamma\sigma}$, and $g$ is the QCD coupling constant: $g^2=4\pi\alpha_s$. Moreover, the matrix elements turn out to be calculable through the use of triangle anomalies in the axial-vector current[5] and the trace of the energy-momentum tensor[6] as well as the use of low-energy theorems (the result for $\langle\eta|g^2G\tilde{G}|0\rangle$ is actually not new).[7] The quarkonium parts of the transition-matrix elements turn out to be proportional to each other so that the dependence on the exact form of the wave function is canceled in the ratio of the rates of the $\eta$ and $\pi\pi$ emissions.

We concentrate first on gluonic matrix elements for light mesons. In general the matrix element for conversion of gluons into a low-energy $S$-wave two-pion system has the form

$$\langle(\pi\pi)_{J=0}|g^2G_{\mu\nu}{}^aG_{\lambda\sigma}{}^a|0\rangle=A(g_{\mu\lambda}g_{\nu\sigma}-g_{\mu\sigma}g_{\nu\lambda})+B(g_{\mu\lambda}q_\nu q_\sigma-g_{\mu\sigma}q_\nu q_\lambda+g_{\nu\sigma}q_\mu q_\lambda-g_{\nu\lambda}q_\mu q_\sigma), \tag{1}$$

where $q=q_1+q_2$ is the total four-momentum of the pions and we omit higher powers in $q$. The Adler theorem[8] requires both $A$ and $B$ to be proportional to $q^2$ so that the $B$ term is actually of higher order in $q$ and is neglected henceforth. A further constraint comes from the triangle anomaly in the trace of the energy-momentum tensor[5] $\theta_{\mu\nu}$, which in the chiral limit is given by

$$\theta_{\mu\mu}(x)=-(bg^2/32\pi^2)G_{\mu\nu}{}^a(x)G_{\mu\nu}{}^a(x), \tag{2}$$

where $b$ is the coefficient in the QCD $\beta$ function $[\beta(g)=bg^3/16\pi^2;\ b=11-\frac{2}{3}n_f]$. Note, that we have omitted the nonanomalous contribution of heavy quarks in Eq. (2). The reason is that in the low-energy matrix elements of $\theta_{\mu\mu}$ the contribution of heavy quarks to the anomaly is canceled by that of loops generated by the corresponding nonanomalous part of the $\theta_{\mu\mu}$.[9] Therefore in the problem discussed, only the term (2) is relevant with $b$ including the contribution of the light quarks only, so that $b=9$. Equation (2) and the low-energy pion Lagrangian imply that

$$-\langle\pi\pi|(bg^2/32\pi^2)G_{\mu\nu}{}^aG_{\mu\nu}{}^a|0\rangle =\langle\pi\pi|\theta_{\mu\mu}|0\rangle=q^2(\tfrac{1}{2}\varphi_\pi{}^\alpha\varphi_\pi{}^\alpha), \tag{3}$$

where $\varphi_\pi{}^\alpha$ is the pion isotopic amplitude ($\varphi^\alpha\varphi^\alpha=2\varphi^+\varphi^-+\varphi^0\varphi^0$) and we neglect for a while the pion mass $\mu_\pi$. Thus, comparing Eqs. (3) and (1) we arrive at the following low-energy theorem:

$$\langle(\pi\pi)_{J=0}|g^2G_{\mu\nu}{}^aG_{\lambda\sigma}{}^a|0\rangle =-(8\pi^2/3b)q^2(\tfrac{1}{2}\varphi_\pi{}^\alpha\varphi_\pi{}^\alpha)(g_{\mu\lambda}g_{\nu\sigma}-g_{\mu\sigma}g_{\nu\lambda}). \tag{4}$$

Moreover, the conversion of gluons into the pseudoscalar $\eta$ meson is described by single irreducible amplitude which is fixed[6] by the SU(3) symmetry and the axial current triangle anomaly:

$$\langle\eta|g^2G_{\mu\nu}{}^a\tilde{G}_{\lambda\sigma}{}^a|0\rangle =(8/27)^{1/2}\pi^2 f_\pi m_\eta{}^2(g_{\mu\lambda}g_{\nu\sigma}-g_{\mu\sigma}g_{\nu\lambda})\varphi_\eta, \tag{5}$$

where $f_\pi$ is the pion decay constant ($f_\pi\simeq 130$ MeV) and $\varphi_\eta(x)$ is the $\eta$ field.

We pause here to make some remarks concerning the matrix elements (4) and (5). First, we point out that the coupling constant $g^2$ is included into the definition of the gluonic operators, $g^2GG$ and $g^2G\tilde{G}$, to make them renormalization invariant so that there is no question at which point $g^2$ is normalized. Since gluons have to transform into light mesons, one would naively expect the gluonic matrix elements to be small. This is not the case, however, and the gluon operator, e.g., is "responsible" for the whole invariant mass of the pion system [Eq. (4)]. The same is true for the nucleon mass in the chiral limit.[9] Thus, there seem to be a few gluonic operators, related by the equations of motion to anomalies in quark operators, whose matrix elements over ordinary hadrons are not small (a similar remark concerning the matrix element $\langle 0|G\tilde{G}|\eta\rangle$ was recently made by Goldberg).[10] Second, we would like to mention that the matrix element in Eq. (4) which governs the form of the $\pi\pi$ spectrum does agree with the general current-algebra result.[11]

We next discuss the interaction of the soft-gluon fields with heavy quarkonium. Since quarkonium is a compact object the expansion in powers of its radius seems well justified[2] and, e.g., emission of the $0^+$ gluon system occurs in the second order in the electric-dipole–type interaction[2-4]

$$\mathcal{H}_d = -\tfrac{1}{2} g \xi^a \vec{r} \cdot \vec{E}^a(0), \tag{6}$$

where $E_i^a = G_{0i}^a$ and $\xi^a = t_1^a - t_{2}{}'^a t_{1,2}{}^a$ are the color SU(3) generators acting on the quark and antiquark indices, respectively (e.g., $t_1^a = \tfrac{1}{2}\lambda_1^a$). Note that the total color generator $t^a = t_1^a + t_2^a$ vanishes when applied to a colorless quarkonium state.

In the case of $0^-$ gluon-system emission, one looks for interference of the electric- and the magnetic-type interactions. To find the corresponding magnetic term in the Hamiltonian we use the Foldy-Wouthuysen expansion in the inverse heavy-quark mass $m_Q$. The relevant term has the form

$$\mathcal{H}_s = -(4m_Q)^{-1} g S_j \xi^a r_i D_i H_j^a(0), \tag{7}$$

where $\vec{S} = \tfrac{1}{2}(\vec{\sigma}_1 + \vec{\sigma}_2)$ is the total-spin operator. Note, that we do not keep here terms proportional to $\vec{\sigma}_1 - \vec{\sigma}_2$ or to $t^a$ since they drop out in the case considered.

A standard calculation of the transition amplitudes between $^3S_1$ quarkonium states arising in the second order in the Hamiltonian (6) and (7) results in

$$A_{\pi\pi} \equiv A(n\,^3S_1 \to m\,^3S_1 \pi\pi) = \langle \pi\pi | g^2 \vec{E}^a \vec{E}^a | 0 \rangle (\vec{\psi}' \vec{\psi}) A_0/4 \tag{8}$$

$$A_\eta \equiv A(n\,^3S_1 \to m\,^3S_1 \eta) = \langle \eta | g^2 E_k{}^a D_k H_j | 0 \rangle m_Q{}^{-1} i \epsilon_{jlm} \psi_l{}' \psi_m A_0/4. \tag{9}$$

Here $\vec{\psi}'$ and $\vec{\psi}$ stand for spin amplitudes of the initial and the final states of quarkonium, and

$$A_0 = (24)^{-1} \langle mS | \xi^a r_i G(\epsilon_\eta) r_i \xi^a | nS \rangle. \tag{10}$$

In the latter expression the bra and ket states are deprived of their spin variables and depend on $r$ only, while $G(\epsilon_n)$ denotes the nonrelativistic quarkonium Green's function[3] at the energy $\epsilon_n$ of the $n$th $S$ level. In derivation of Eqs. (8) and (9) it is taken into account that in the nonrelativistic limit the spin and the spatial parts of the wave functions factorize. Therefore for the transitions between $S$ levels the spatial part of the quarkonium matrix element adds no angular momentum to the hadronic system emitted (also the color dependence of the matrix element is reduced to $\delta^{ab}$ for trivial reasons). This matches well the phenomenological observation of the absence of the $\pi\pi$ $D$ wave in the $\psi' \to (J/\psi)\pi\pi$ decay.

The gluonic matrix element entering Eq. (8) is readily found from the relation (4). Thus the amplitude $A_{\pi\pi}$ can be written in the form

$$A_{\pi\pi} = 2\pi^2 b^{-1} (q^2 - \lambda \mu_\pi{}^2)(\vec{\psi}' \vec{\psi})(\tfrac{1}{2} \varphi_\pi{}^\alpha \varphi_\pi{}^\alpha) A_0. \tag{11}$$

Note that we have included here the term proportional to the pion mass squared which does not appear in the chiral limit, but can be determined from phenomenological analysis. In the chiral limit only the leading term proportional to $q^2$ is calculable so that, strictly speaking, we find the slope of the spectrum which is unaffected by corrections proportional to $\mu_\pi{}^2$.

As for the matrix element in the pseudoscalar case it is transformed as

$$\langle \eta | g^2 E_k{}^a D_k H_j{}^a | 0 \rangle = \langle \eta | g^2 \partial_k (E_k{}^a H_j{}^a) | 0 \rangle - \langle \eta | g^2 (D_k E_k{}^a) H_j{}^a | 0 \rangle.$$

The first term in the right-hand side of this equation is determined by Eq. (5), and we neglect the second one. The reason for the second term to be comparatively small is twofold. First, the matrix element (5) is enhanced by one power of $N_c$ = (number of colors) with respect to the naive counting. A nice explanation of this enhancement is given by Veneziano[12] in terms of mixing with the ghost gluonic state. Second, equations of motion imply that $D_k E_k{}^a$ is proportional to the coupling constant which is small inasmuch quarkonium is a compact object. Thus, we get finally

$$A_\eta = (\pi^2/9)(3/2)^{1/2} f_\pi m_\eta{}^2 m_Q{}^{-1} [\epsilon_{ijk} \psi_i \psi_j{}' (p_\eta)_k] A_0, \tag{12}$$

where $(p_\eta)_k$ is the $\eta$ three momentum. To eliminate the quarkonium matrix element $A_0$, which cannot be calculated reliably at the present state of the art, we consider the ratio

$$\frac{\Gamma(n\,^3S_1 \to m\,^3S_1 \eta)}{d\Gamma(n\,^3S_1 \to m\,^3S_1 \pi^+\pi^-)/dq^2} = 16\pi^2 \left(\frac{b}{9}\right)^2 f_\pi{}^2 \left(\frac{P_n}{M}\right)^2 \frac{P_n}{|\vec{q}|} \left(\frac{m_\eta{}^2}{q^2 - \lambda \mu_\pi{}^2}\right)^2 \left(1 - \frac{4\mu_\pi{}^2}{q^2}\right)^{-1/2}, \tag{13}$$

where $M \simeq 2m_Q$ is the quarkonium mass and $q^2 \equiv m_{\pi\pi}{}^2$. Since the derivation of this relation rests on the low-energy theorem (4) for the $\pi\pi$ matrix element, it cannot be applied at too high $q^2$. For large $|\vec{q}|$ the multipole expansion for the gluon-quarkonium interaction also breaks down since the gluons are no longer "soft." Therefore, Eq. (13) is most reliable when the kinematics in the two decays is similar, i.e., when $m_{\pi\pi}{}^2 = m_\eta{}^2$, and to a good approximation it can be also used in other parts of the $\pi\pi$ spectrum where the linear $q^2$ dependence of the matrix element persists.

In conclusion let us give a few applications of Eq. (13). At present only experimental data on charmonium transitions are available and they can be confronted with the following theoretical estimate obtained by integrating Eq. (13) over the phase space:

$$\frac{\Gamma(\psi' \to (J/\psi)\eta)}{\Gamma(\psi' \to (J/\psi)\pi^+\pi^-)} \simeq 16\pi^2 \frac{f_\pi^2 p_\eta^3 m_\eta^4}{M^2(M_{\psi'} - M_{J/\psi})^7} \times 19.6.$$

This gives numerically 0.10 if $M = M_{\psi'}$ and 0.14 if $M = M_{J/\psi}$ which also shows the theoretical uncertainty arising from application of the nonrelativistic picture to charmonium. Experimentally the ratio is equal to $0.12 \pm 0.02$ according to the Tables[13] or $0.09 \pm 0.01$ as is extracted from more recent data.[14] In any case we find the agreement seems to resolve the long-standing puzzle of the charmonium phenomenology, that is, the relatively large rate of the $\psi' \to (J/\psi)\eta$ decay. Indeed, the amplitude $A_\eta$ [see Eq. (12)] contains the dynamical damping factor $p_\eta/M$ and is also proportional to $m_\eta^2$ which vanishes in the SU(3) limit. Still numerically it fits the data well. The field-theoretical reason behind this is the above mentioned $N_c$ enhancement of $A_\eta$. Theoretical uncertainty dies away for heavier quarks. In particular we predict

$$\Gamma(\Upsilon'' \to \Upsilon\eta)/\Gamma(\Upsilon'' \to \Upsilon\pi^+\pi^-) \simeq 0.020, \qquad (14)$$

where $\lambda = 4$ is assumed[14] and $M_{\Upsilon''} - M_\Upsilon = 891$ MeV[15] is used. The $\Upsilon' \to \Upsilon_\eta$ decay rate depends rather crucially on the mass difference $\Delta = M_{\Upsilon'} - M_\Upsilon$ which is about 555–560 MeV[15,16] and falls close to the $\eta$ mass. With $\Delta - m_\eta \lesssim 11$ MeV we find that the ratio of the $\eta$ and $\pi^+\pi^-$ transition rates is $\lesssim 4.5 \times 10^{-3}$.

Although experimental verification of the predictions (13) and (14) might be difficult, we believe it is worth efforts since the ratios considered measure the triangle anomalies associated with quark and gluon color charges in much the same way as the famous $\pi^0 \to 2\gamma$ decay measures the triangle anomaly associated with electric quark charges.

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