mass. Again discrete levels at the bottom of the well should be resolved. The ultimate resolution of the 1^3S-2^3S transition will be 1.3 MHz, limited by 3γ annihilation. In both the H and positronium cases, spectroscopy at the ultimate resolution would provide a new level of tests of atomic theory.

Detailed calculations to support the estimates given above will be presented elsewhere. An experiment to check certain points is also under way. I have benefitted greatly from stimulating conversations with A. Ashkin, P. L. Bender, G. H. Dunn, J. W. Farley, J. P. Gordon, J. L. Hall, D. Kleppner, W. E. Lamb, A. P. Mills, D. Pritchard, W. P. Reinhardt, D. J. Wineland, C. Woods, and numerous others while preparing this note. C. V. Kunasz assisted with graphics. The work has been supported in part by grants from the U. S. National Science Foundation to the University of Arizona. Acknowledgment is made also to the donors of the Petroleum Research Fund, administered by the American Chemical Society, for partial support of this research.

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Forced Phase Diffusion in Rayleigh-Bénard Convection

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The first experimental evidence of the diffusive character of the phase variable which describes the position of the convective rolls is presented. To produce a phase modulation a small alternating flow in a slightly supercritical fluid layer is injected. With use of this method, the diffusion coefficient $D_{\parallel}$ concerning the propagation of phase disturbances parallel to the roll axis is measured.

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During the last years there has been a growing interest in Rayleigh-Bénard (RB) convection.[1] The vicinity of the convection threshold R_c has been compared with a second-order phase transition and found to be well described by the Landau model.[2] Concerning the transition to turbulence, in RB convection, two cases can be distinguished depending on the aspect ratio A (A = horizontal dimension/vertical dimension). In the case of small A, turbulence occurs after several bifurcations far from the instability threshold.[3] The behavior of this transition to turbulence is consistent with modern works on dynamical systems with few degrees of freedom and with the concept of strange attractors.[4] In the case of large A, experimental evidence shows that chaos may occur very soon after the onset of convection, R_c.[5] No complete explanation has been given yet for this behavior. However, it has been suggested that this kind of turbulence resulted from slow erratic motions of large parts of the roll system, i.e., from a spontaneous instability of the phase of the roll pattern.[6] To get a better understanding of the transition to turbulence in containers with large A, a problem to tackle seems to be that of phase dynamics.

In this paper we describe an experimental study of a forced phase modulation in RB convection. For this purpose we have generated a localized phase disturbance in a RB convection experiment and measured, using laser Doppler anemometry (LDA), the propagation of this disturbance. Its diffusive behavior confirms recent theoretical results.[6]

The convective motions were confined in a rectangular Plexiglass frame sandwiched between two massive horizontal copper plates, the upper one being uniformly cooled and the lower one uniformly heated. The physical dimensions of the apparatus are $L_x = 30d$, $L_y = 5d$, with $L_z = d = 0.6$ cm, as described previously.[7] We have performed our experiments on a layer of oil with a Prandtl number ($P = \nu/\kappa$) equal to 492 (ν being the kinematic viscosity and κ the thermal diffusivity). The vertical thermal diffusion time of this layer d^2/κ was 320 sec.

A slot, 0.03 cm wide, parallel to L_y and situated at $L_x/2$ was cut in each of the copper plates. An imposed vertical stream flowed through the slots (see Fig. 1). The flow was varied periodically and followed the relation $q(t) = q_0 + q_1 \cos\omega t$ ($\omega = 2\pi/T$, $q_1 < q_0$, $q_0 \simeq 2\times10^{-3}$ cm^3/sec).

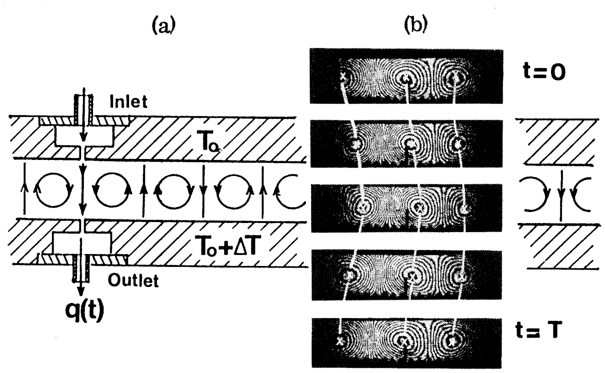

FIG. 1. (a) Experimental apparatus with a sketch of the generator of the disturbance. (b) Interferometric pictures visualizing the shifting of the roll structure during one period of perturbation. The position of the centers of the rolls vs time is marked by white lines. (For the purpose of visualization the disturbance has been increased considerably.)

The vertical velocity component v_z of the convective motion was measured by LDA in the middle plane of the container, and continuously recorded.

A visualization of the temperature gradients in the x-z plane was also accomplished by a differential interferometry technique[8] (see Fig. 1). In these pictures the fringes represent the lines of temperature horizontal isogradient.

The vertical velocity component $v_z(x,t)$ of the convective field in the middle plane of the container may be described by

$$v_z(x,t) = \tilde{V}(x,t)\cos[k_c x + \varphi(x,t)], \qquad (1)$$

with k_c, the critical wave number of the spatial pattern without disturbance; $\tilde{V}(x,t)$, the amplitude of the convective pattern; $\varphi(x,t)$, the phase contribution to the convective pattern.

The oscillating flow, imposed through the slots, induces time-dependent variations of the convective structure. The first feature that we observe is that the velocity variations are periodic in time and have the same frequency as that of the imposed flow; no noticeable harmonics were generated. Moreover, the magnitude of the velocity variations is spatially modulated. Two regions may be distinguished in the container. The first one, which we label the "amplitude modulation region," is located in the vicinity of the slots. In this region, velocity variations are mainly proportional to the mean velocity of the convective structure, that is to say the *amplitude* of the velocity field is time modulated:

$$\tilde{V} = \tilde{V}_0 + \tilde{V}_1(x)\cos(\omega t + \alpha_0). \qquad (2)$$

The second region, which we label the "phase modulation region," extends to the rolls further away from the slots. In this second region, velocity variations are maximum where the mean velocity vanishes and vanish where the mean velocity is maximum. These features suggest a periodic shifting accompanied by a deformation of the rolls structure. Since the roll position may be defined by the phase factor $\varphi(x,t)$ in Eq. (1), periodic shifting of the rolls is equivalent to a phase modulation

$$\varphi(x,t) = \varphi_0(x)\cos[\omega t + \theta(x)]. \qquad (3)$$

The second feature that we observe is that the phase modulation experiences a spatial damping and this damping increases with frequency; that is, $\varphi_0(x)$ is a decreasing function with the distance from the slots and with the frequency ω.

The third feature observed is that, in the phase modulation region, a time phase shift occurs between the velocity variations and the imposed flow. This phase shift $\theta(x)$ increases with the distance from the slots and with the perturbation frequency.

In our experiment, where the periodic perturbation extends across the entire y dimension of the container and where the rolls have a two-dimensional structure parallel to this flow, no phase perturbations are expected to propagate along the y axis (so that $\partial^n \varphi/\partial y^n = 0$, $\forall n$). So we study the propagation of phase perturbations in x direction. The phase dynamics has been predicted[6] to be governed by a diffusive process:

$$\partial\varphi/\partial t = D_{\parallel}\partial^2\varphi/\partial x^2. \qquad (4)$$

The diffusion coefficient may be written $D_{\parallel} = f(\epsilon, k_c)\xi_0^2/\tau_0$, where $\xi_0 = 0.385d$ is the characteristic length scale for amplitude perturbations, $\tau_0 = (d^2/\kappa)(1.95P+1)/38.44P$ is the characteristic time scale for amplitude perturbations, and $\epsilon = (R-R_c)/R_c$ is the relative distance to the onset of convection,[7] and

$$f(\epsilon, k_c) = \frac{\epsilon - 4.31\delta^2/k_c^2}{\epsilon - 1.44\delta^2/k_c^2}, \quad \delta = k - k_c.$$

The factor $f(\epsilon, k_c)$ is very close to 1 when $k \to k_c$. For the rigid-rigid horizontal boundaries in our apparatus and with the physical properties of our silicone oil,[7] we get $D_{\parallel} \simeq \xi_0^2/\tau_0 = 3.3\times10^{-3}$ cm^2/sec. The solution of Eq. (4) with the initial condition $\varphi(x=0,t) = \varphi_0\cos\omega t$ is

$$\varphi(x,t) = \varphi_0\exp(-m_1|x|)\cos(m_2|x| - \omega t), \qquad (5)$$

with $m_1 = m_2 = (\omega/2D_{\parallel})^{1/2} \simeq (\pi\tau_0/T)^{1/2}/\xi_0$.

All our experiments were performed at $\epsilon = 0.16$. We have scanned the velocity field step by step along the x axis at $L_y/2$. At each step, spaced 0.07 cm from the previous one, the time dependence of the velocity was recorded during two periods of the perturbation. Six different periods were investigated: $T/\tau_0 = 18.75$, 37.5, 75, 187.5, 375, and 817.5 ($\tau_0 = 16$ sec).

As it has been pointed out previously, significant velocity variations exist at a distance up to $6d$ from the slots. This cannot be explained by an amplitude modulation whose characteristic penetration length is expected to be smaller or equal to the correlation length ξ^+. ($\xi^+ = \sqrt{2}\xi_0\epsilon^{-0.5} = 1.35d$ when $\epsilon = 0.16$.)[7] This demonstrates that we deal with another phenomenon of a different scale length. This phenomenon may be understood by considering the interferometric pictures, where a periodic shifting of the rolls structure clearly appears, demonstrating the phase modulation.

Our recordings of the velocity give (i = step number)

$$v_z(x_i,t) = \langle v_z(x_i)\rangle + \delta\tilde{v}_z(x_i)\cos[\omega t + \theta(x_i)]. \qquad (6)$$

The velocity variations may be related to the phase modulation given in Eq. (1) so that the phase

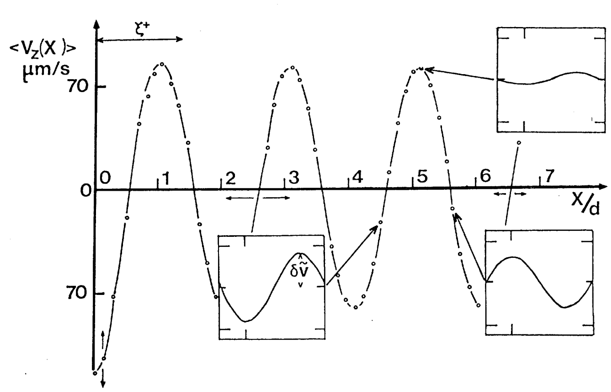

FIG. 2. Mean velocity $\langle v_z(x)\rangle$ vs the distance to the slots and detailed view of the velocity fluctuations $\delta\tilde{v}$ during one complete period of perturbation (an expanded scale has been used for $\delta\tilde{v}$: 1 div = 3.5 μm/sec).

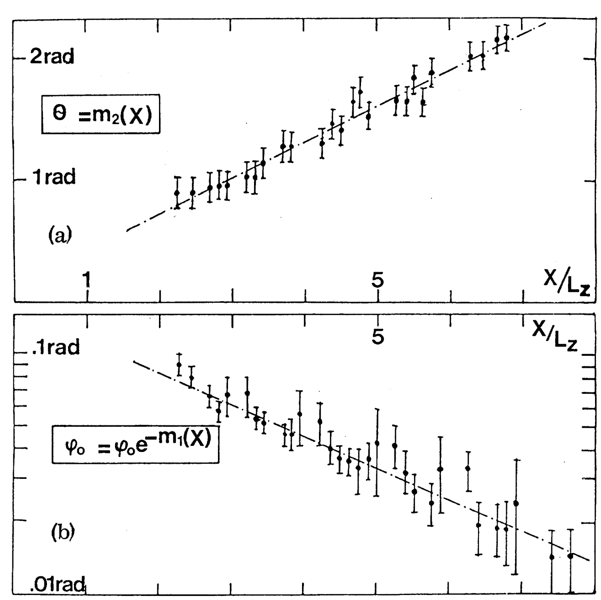

FIG. 3. (a) and (b), respectively, phase shift θ and phase modulation amplitude φ_0 vs the distance to the slots for $T = 187.5\tau_0$.

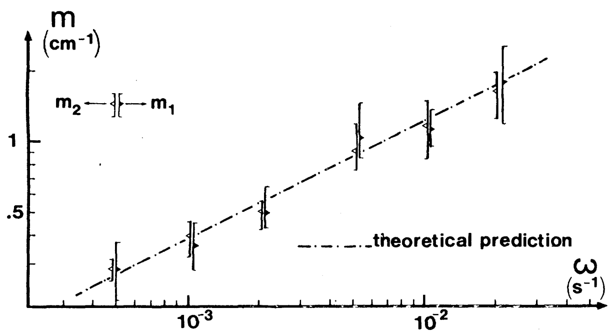

FIG. 4. Experimental value of m_1 and m_2 vs the frequency of the perturbation ($\omega = 2\pi/T$).

modulation may be worked out with use of a first-order expansion in terms of φ (since φ was < 0.1 rad); thus we may express the term φ_0 in Eq. (3) as

$$\varphi_0(x_i) = \delta\tilde{v}_z(x_i)/[\tilde{V}^2 - \langle v_z(x_i)\rangle^2]^{1/2}.$$

At each step we have measured $\langle v_z(x_i)\rangle$, $\delta\tilde{v}_z(x_i)$, and $\theta(x_i)$ (Fig. 2). From $\delta\tilde{v}_z(x_i)$ we deduced $\varphi_0(x_i)$. The plot of $\varphi_0(x_i)$ vs x_i [Fig. 3(b)] is well fitted by an exponential decrease $\varphi_0 \exp(-m_1 x)$, far from the slots. Thus we can introduce the characteristic phase penetration length $L_\varphi = (m_1)^{-1}$. The plot of the time phase shift $\theta(x_i)$ vs x_i [Fig. 3(a)] has, far from the slots, a linear asymptotic behavior $\theta(x) = m_2 x$.

We have found that the equality $m_1 = m_2$ holds for the different T measurements within our experimental errors. Notice the independent character of these two measurements. Moreover, the m values obtained are compatible with the relation $m^{-1} \propto \sqrt{T}$, typical of a diffusion process (Fig. 4).

The measured value of the diffusion coefficient $D_{\parallel \mathrm{expt}} = (3.5 \pm 1)\times 10^{-3}$ cm^2/sec is in good agreement with the theoretical prediction[6] and consistent with earlier determinations of ξ_0 and τ_0.[7,9]

For the smallest periods, $37.5\tau_0$ and $18.75\tau_0$, we have used $q(t) = q_1 \cos\omega t$ as the imposed flow. Since this imposed flow reverses periodically, a small roll is alternately created and destroyed at the slots which obviously generates a stronger initial phase modulation φ_0.

The different series of measurements demonstrate the diffusive behavior of the phase contribution to the velocity field.[10] The diffusion coefficient $D_\parallel$ agreed with the predicted value[6] within our experimental errors. This quantitative agreement is valid since we have performed our experiments with a high–Prandtl-number oil at a slightly supercritical Rayleigh number and with slow variation of phase and amplitude.[11] Actually a pure phase disturbance is rather difficult to generate and in our experiment the amplitude of the rolls structure was also perturbed. However, we found that the phase contribution can dominate the amplitude contribution. This happens when the amplitude modulation is damped faster than the phase modulation, that is when $L_\varphi > \xi^+$ which leads to $T > 2\pi\tau$ (where $\tau = \tau_0\epsilon^{-1}$ is the characteristic time of the slowing down[7]). We suggest that this kind of analysis may be applied to the similar interesting experiments of Berkovsky *et al.*[12] concerning thermoconvective waves.

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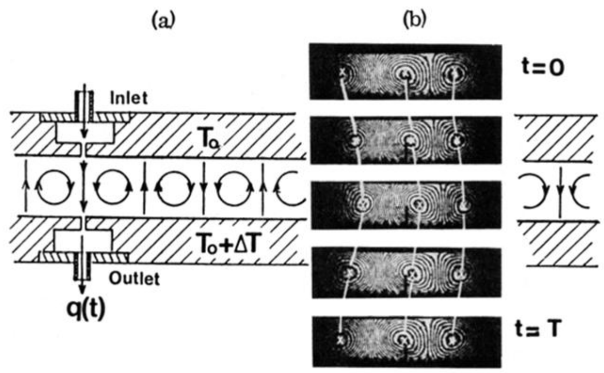

FIG. 1. (a) Experimental apparatus with a sketch of the generator of the disturbance. (b) Interferometric pictures visualizing the shifting of the roll structure during one period of perturbation. The position of the centers of the rolls vs time is marked by white lines. (For the purpose of visualization the disturbance has been increased considerably.)