Effect of Pion Absorption on πd Elastic Scattering from Threshold to the Resonance Region

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The first relativistic calculation of elastic πd scattering from threshold to $T_\pi^{\rm lab} = 260$ MeV is presented, based upon a completely coupled set of integral equations. The effects of intermediate pion absorption as well as scattering through the nonpole part of the $\pi N\, P_{11}$ partial wave are consistently included and found to be important. A reasonable agreement with experiments is encouraging for further detailed studies.

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Motivated by various experiments on the recent elastic πd scattering,[1] $\pi^+ d \rightleftarrows pp$,[2] and the nucleon-nucleon scattering in the inelastic region,[3] there have been considerable efforts in the study of the coupled πNN-NN system from the point of view of three-body scattering theories.[4-11] It has then been observed[6,7] that in order to properly describe the coupling to the two-nucleon channel, a conventional three-body theory (e.g. that of Faddeev) is not enough. The final outcome of this observation is two sets of coupled integral equations[7,9]: one for the amplitudes $T(\pi NN(\pi d)\to \pi NN(\pi d))$ and $T(\pi NN(\pi d)\to NN)$ and the other for $T(NN\to NN)$ and $T(NN\to\pi NN(\pi d))$. In this note we shall report on the calculation of the πd scattering based on the first set of coupled equations.

In their most general (relativistic, off-mass-shell) form, these equations (neglecting πNN three-body forces as well as two-particle–irreducible $\pi NN \leftrightarrow NN$ vertices) read[11]

$$T_{\mu\nu} = t_\mu \delta_{\mu\nu} + \sum\nolimits_\eta t_\eta (1-\delta_{\mu\eta}) G_3 T_{\eta\nu} + h_\mu G_2 T_{N\nu}, \tag{1a}$$

$$T_{N\nu} = h_\nu + \sum\nolimits_{\eta\mu} h_\eta (1-\delta_{\eta\mu}) G_3 t_\mu [\delta_{\mu\nu} + G_3 \sum\nolimits_\epsilon (1-\delta_{\mu\epsilon}) T_{\epsilon\nu}] + \mathcal{V}_{NN}^{\rm OBE} G_2 T_{N\nu}. \tag{1b}$$

In the above expression μ, ν, etc. label pairs πN_1, πN_2, $N_1 N_2$, etc.; t_μ is the two-body t matrix; h_μ is the πNN vertex function; $\mathcal{V}_{NN}^{\rm OBE}$ is the one-boson–exchange (including π) NN potential; and G_2 (G_3) is the product of dressed single-particle propagators for NN (πNN). The amplitudes $T_{\mu\nu}$ and $T_{N\nu}$ satisfy two- (NN) and three- (πNN) body unitarity, provided that the vertex functions and propagators are properly renormalized.[12] Without the last term on the right-hand side, Eq. (1a) is just the relativistic Faddeev equation. Obviously, this last term admits the coupling to the NN channel; thus Eqs. (1a) and (1b) are capable of including the effect of π absorption in the πd scattering. As discussed, e.g., in Ref. 7, Eqs. (1a) and (1b) have various advantages over those (formally equivalent) semicoupled equations described in Refs. 5 and 12, and used by Rinat *et al.*[13]

Actual calculations were done with fully relativistic kinematics by adopting the isobar (separable) approximation to t_μ ($\mu = \pi N, NN$), together with the three-dimensional reduction of G_2 and G_3 in the manner of Blankenbecler and Sugar to preserve two- and three-body unitarity. The S and P πN interactions are taken from Schwarz, Zingl, and Mathelitsch,[14] except for P_{11} which will be discussed later. For NN interactions we have retained only the $^3S_1$-$^3D_1$ channel, as parametrized by Giraud, Fayard, and Lamot,[15] with deuteron D-state probabilities 4% and 6.7%, denoted as SF4 and SF6.7, respectively. As for the one-boson–exchange potential $\mathcal{V}_{NN}^{\rm OBE}$, we have taken here a nonstatic one-pion–exchange contribution alone whose analytic form involves h_μ and may be found, e.g., in Ref. 7. Its nonstatic feature is essential to maintain three-body unitarity.

Special attention must be paid to the $\pi N\, P_{11}$ interaction as the pion absorption (emission) proceeds through this channel. The following points should be noted: (i) The total P_{11} t matrix may be written[7,9] as

$$t^{\rm tot} = t_{\rm P} + t_{\rm NP}, \tag{2a}$$

$$t_{\rm P} = h G_N h, \tag{2b}$$

where $t_{\rm P}$ is the direct nucleon-pole contribution with h the πNN vertex function and G_N the dressed nucleon propagator (we call t_P the *pole part*), while t_{NP} is the remaining back ground [hereafter referred to as *nonpole part*: NP-P_{11}]; (ii) t_P and t_{NP} enter Eqs. (1a) and (1b) separately,[7,9,16] the former being decomposed into h and G_N which appear as h_μ and part of G_2, respectively, whereas the latter joins as one of the t_μ's; and (iii) a particularly important feature is that t_{NP} itself is *unitary*,[7,17] thus imposing a strong constraint on the forms of h and G_N.

With those features mentioned above t^{tot} has been determined if we assume that t_{NP} is rank-1 separable: $t_{NP}(p';p;s)=g(p')\tau_{NP}(s)g(p)$ (s is the πN c.m. energy squared). Details will be found elsewhere,[18] and we give here a brief summary. The vertex h (which is formally expressed as $h = r + rG_{\pi N}t_{NP}$, where r is the two-particle-irreducible vertex, and $G_{\pi N}$ the πN propagator) is *complex*, and depends upon s and upon two range parameters β and γ characterizing, respectively, $g(p)$ and $r(p)$. The parameters involved have been determined by χ^2 fit to the scattering volume $a_{\pi N}$ ($=-0.079 m_\pi^{-3}$) and the phase shift.[19,20] We have constrained the fit by the nucleon pole position and its residue which is proportional to the πNN coupling constant, $f_{\pi NN}^2$ ($=0.082$). We have adopted two models. (1) GG: γ is set equal to β, with $\beta = 2.7\ \mathrm{fm}^{-1}$. (2) GR: $\gamma \neq \beta$, $\beta = 2.87\ \mathrm{fm}^{-1}$, and $\gamma = 4.77\ \mathrm{fm}^{-1}$. Both of them reproduce $\delta(P_{11})$ quite well (Fig. 1). As for the P_{11} interactions in previous πd calculations with π absorption consideration, only a pure nucleon-pole term[13] and a two-term separable potential[9] have been considered. We stress in this respect that the present P_{11} model is the first one to be compatible with unitarity while reproducing quite well the phase shift as well as the πNN coupling constant.

Our calculations have been performed from threshold to $T_\pi^{\mathrm{lab}} \simeq 260$ MeV. The threshold is characterized by the *complex* scattering length $a_{\pi d}$. Experimentally, both Re$a_{\pi d}$ and Im$a_{\pi d}$ are known with large uncertainties. For the real part we have[21] Re$a_{\pi d} = -(0.052^{+0.022}_{-0.017}) m_\pi^{-1}$. The imaginary part is proportional to α, the low-energy S-wave production coefficient for $pp \to \pi^+ d$. Richard-Serre *et al.*[2] found $\alpha = 180 \pm 20\ \mu$b while Spuller and Measday suggested[2] $200 < \alpha < 300\ \mu$b.

Disregarding the small πN P_{13} and P_{31} contributions for the moment (which we found to be less than 5%) we obtained, with GG and SF4 interactions, Re$a_{\pi d} = -0.030 m_\pi^{-1}$. This result was found to be independent of the choice of P_{11} models and the deuteron D-state probability. The effect of the P_{11} wave was tested as follows: (i) Without P_{11}, we found Re$a_{\pi d}$(No P_{11}) $= -0.027 m_\pi^{-1}$; (ii) with t_{NP} alone, we obtained an attractive (positive) contribution of ~10% to Re$a_{\pi d}$(No P_{11}); (iii) t_P alone was included to give a repulsive (negative) contribution of ~20%. Thus both t_P and t_{NP} in combination contribute repulsively resulting in 10% increase in magnitude. The theoretical value $-0.030 m_\pi^{-1}$ does not look very consistent with the yet unrefined experimental result. However, one should recall that it is quite sensitive to the isosymmetric combination of the πN(S-wave) scattering lengths, $a_1 + 2a_3$, which is not known definitely. The πN t matrices used here[14] give $a_1 + 2a_3 = -0.012 m_\pi^{-1}$, while the most recent analysis[20] leads to $-0.029 m_\pi^{-1}$. With this in mind, we have performed the calculation where all the (S $+P$) πN input is fitted to this analysis[20] and found Re$a_{\pi d} = -0.047 m_\pi^{-1}$.

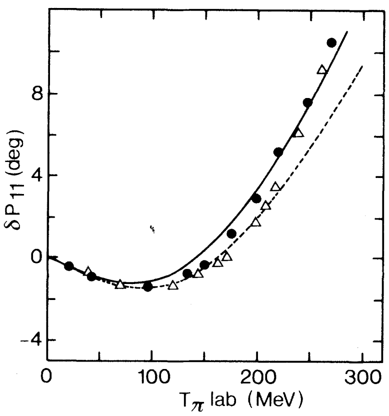

FIG. 1. P_{11} phase shift calculated with the GG (dotted curve) and GR (solid curve) models (see text). "Experiments" are from Refs. 19 (solid circle) and 20 (open triangle).

As for α we found 104 μb with SF4 and GG interactions. Contrary to Re$a_{\pi d}$, this quantity shows a considerable sensitivity to input models: (i) an increase in the deuteron D-state probability from 4% to 6.7% causes a decrease in α by 20%; (ii) the use of GR P_{11} model increases α by 30%, which indicates the sensitivity of α to the range parameter γ in the vertex function. To see this latter point more clearly, we took two GG-type interactions where, without fitting the phase shift, (1) β was varied while $f_{\pi NN}$ and $a_{\pi N}$ were kept fixed, or (2) $a_{\pi N}$ was varied with $f_{\pi NN}$ and β fixed. For case (1) we found an almost linear increase in α from 23 to 150 μb for β varying from 1.5 to 3.5 fm^{-1}, while Re$a_{\pi d}$ hardly changed. For case (2) the changes are $\Delta\alpha/\alpha < 5\%$, Δ(Re$a_{\pi d}$)

$\approx 0.02 m_\pi^{-1}$ as $a_{\pi N}$ goes from $-0.05 m_\pi^{-3}$ to $-0.09 m_\pi^{-3}$.

Adding the contribution from P_{31} and P_{13} waves, we have obtained $90 < \alpha < 150$ μb. Using GR and the πN input fitted to the data of Ref. 20, we found $\alpha = 165$ μb. The agreement with experiment is quite reasonable considering the present uncertainties of the latter. We remark in passing that α increases by 20% if we exclude NP-P_{11} contribution; so the inclusion of the full P_{11} channel is very important.

Encouraged by the above results we studied the elastic scattering at $T_\pi^{\text{lab}} = 47$, 142, 180, and 256 MeV with GG and SF6.7 interactions. We took into account all the small πN partial waves. At 47 MeV experimental data are as yet unreliable especially at large angles, and so a comparison between theory and experiment may not be meaningful. We just mention that our result is similar to that of Ref. 9 where the full P_{11} wave is also included (in a different manner, though) with relativistic kinematics kept only for the pion. At 142 and 180 MeV, our $d\sigma/d\Omega$ (Fig. 2) closely follows the non-P_{11} result. The only noticeable deviation occurs at backward angles, e.g., an increase by 10% at $\theta_{\text{c.m.}} = 180°$. At 142 MeV the agreement with the recent experiments[1] is very good, while at 180 MeV some deviation develops at large angles.

At 256 MeV the calculated $d\sigma/d\Omega$ with and without the P_{11} wave are almost identical up to $\theta_{\text{c.m.}} = 90°$ (Fig. 2). For $\theta_{\text{c.m.}} > 90°$ the result with full P_{11} shows a noticeable increase and at 180° it is not far from the point given by Frascaria *et al.*,[1] while the non-P_{11} result essentially stays flat. We point out that the pole and NP-P_{11} give, respectively, 0.82 and 0.28 mb at 180° (calculated without the small πN waves). When put together, they interfere destructively, which appears more effective at lower energies. Compared with the recent data,[1] our calculation reproduces the right shape, but not the magnitude for $\theta_{\text{c.m.}} > 90°$. With the GR P_{11} model, we noted an increase of about 12% in $d\sigma/d\Omega$ at backward angles.

As for the polarization parameters, the largest change was observed in t_{20}. For example, at 142 MeV, we found $t_{20}(180°) = -0.08$ with the GG interaction and 0.15 with GR, more consistent with the experiment by Holt *et al.*[1] than the value without absorption (-0.73).

To comment on the work by Rinat *et al.*,[13] their differential cross sections at 142 and 180 MeV decrease towards backward angles relative to the result without pion absorption, contrary to our findings. However, the backward cross section at 256 MeV increases appreciably when the pion absorption is included, similar to our results. A direct comparison of their work with ours is rather difficult as they not only employed semicoupled equations but they took only the pole term for πN P_{11}.

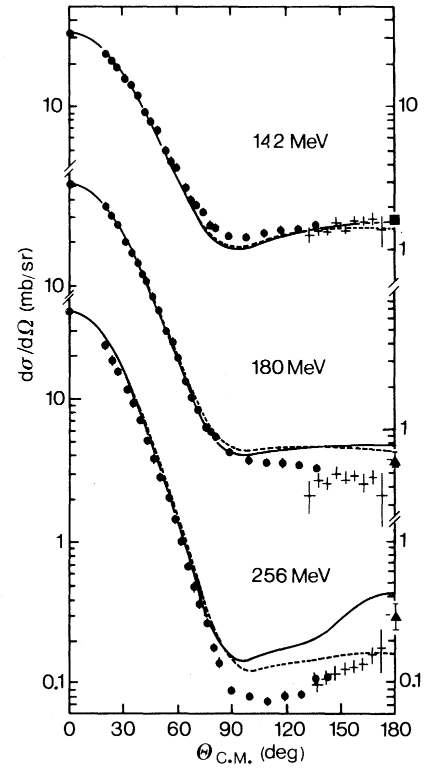

FIG. 2. πd differential cross sections. The GG P_{11} is included (solid curve), or neglected (dotted curve). The experimental data are from Ref. 1, Gabathuler *et al.* (solid circles), Stanovnik *et al.* (crosses), Holt *et al.* (solid square), and Frascaria *et al.* (solid triangles).

In conclusion, we have presented the first calculation of the πd scattering from $T_\pi^{\text{lab}} = 0$ to 260 MeV consistently incorporating the full P_{11} wave. The overall agreement of theory with the current experimental data is satisfactory. Above the $\Delta(3,3)$ resonance, there still remains a noticeable disagreement which presumably might show that some important physical ingredient is missing in the theory, for example dibaryon resonances.[22] On the theoretical side, we are currently studying the elastic πd and the associated $\pi^+ d \rightleftharpoons pp$ process together with a careful examination of the β-meson effect and the possible importance of inelasticities in the πN channels. On the experimental side, we welcome more accurate measurements of $d\sigma/d\Omega$ at backward angles for $T_\pi > 180$ MeV, as well as of $a_{\pi d}$ and polarizations.

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Microscopic Description of Nucleon-Nucleus Total Reaction Cross Sections

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Microscopic calculations of the total reaction cross sections for protons on ^{12}C, ^{27}Al, ^{40}Ca, and ^{208}Pb, and neutrons on ^{27}Al and ^{208}Pb have been made, which provide for the first time an excellent description of the data for projectile energies from 15 MeV through 1 GeV. The calculations are based on the experimental nucleon-nucleon total cross sections and explicitly include the effects of the real nuclear potential, the Coulomb potential, Pauli blocking, and Fermi motion.

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The total reaction cross section (σ_R) for the interaction of a nuclear projectile with a nucleus is perhaps the most basic nuclear reaction observable. Attempts to calculate this quantity[1-5] in a microscopic manner have generally begun with the experimental nucleon-nucleon scattering amplitudes or total cross sections and nuclear charge densities. Only above approximately 300 MeV can such calculations generally reproduce the data (e.g., Ref. 1). No calculations have been reported, however, which include the effects of the real nuclear potential, Pauli blocking, and Fermi motion. It is generally felt that all of the above effects are important for a realistic description of σ_R at lower energies.

In this Letter, we consider the energy and mass dependence of the nucleon-nucleus σ_R for which good quality data exist over a wide range of energy and mass. Extension can then hopefully be made to the more complicated nucleus-nucleus system. It will be interesting to see whether such calculations can reproduce the apparent[3,4] nongeometric behavior of nucleus-nucleus σ_R.

Viewing the nucleon-nucleus interaction semi-

