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Determination of the Asymptotic Ratio of the Deuteron *D*- and *S*-State Amplitudes

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The value 0.02649 ± 0.00043 is deduced for the deuteron asymptotic D- to S-state ratio from measurements of the $^{208}\mathrm{Pb}(d,p)^{209}\mathrm{Pb}$ tensor analyzing powers at sub-Coulomb bombarding energies $E_d = 7$, 8, and 9 MeV. The quoted uncertainty includes statistical errors, beam-polarization measurement errors, and systematic errors which result from a choice of optical-model parameters used to describe the reaction. The result is compared to several theoretical deuteron wave functions.

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In spite of years of experimental effort, many properties of the deuteron are still poorly determined. A continued study of the deuteron is required if stringent tests of n-p interaction theories are to be possible. Although the measurability of the deuteron D-state probability has recently been called into question,[1] the asymptotic ratio of the D- and S-state amplitudes (η) is accepted as a well-defined measurable quantity. Two methods have been proposed in the literature to measure η.

It has been suggested[2] that the tensor analyzing powers in p-d elastic scattering are sensitive to η. The sensitivity is a result of a pole in the scattering amplitude at an unphysical angle θ_p, where $\cos\theta_p < -1$. The extraction of η requires an extrapolation of tensor analyzing power and cross-section angular distributions to θ_p. Such extrapolations have been performed.[3,4] Of course, in any experiment designed to measure a fundamental quantity to high accuracy, an accurate error analysis is most important. The difficulties in performing a rigorous error analysis of the extrapolation method have been discussed by Colby and Haeberli.[5]

The other method to measure η was proposed in a recent Letter by Knutson and Haeberli.[6] This method is based on the observation that distorted-wave Born-approximation (DWBA) calculations predict that the tensor analyzing powers of (d,p) stripping reactions have a strong (essentially linear) dependence on η. Moreover, at bombarding energies below the Coulomb barrier, the fundamental assumptions in DWBA are particularly well satisfied and the dependence on parameters in the theory other than η is drastically decreased. Specifically, the dependence on the nuclear optical-model parameters which describe the incoming deuteron and outgoing proton scattering waves decreases because the scattered particles do not penetrate into the nuclear interior. As a result, the scattering waves are perturbed Coulomb wave functions. By adjusting the value of η to fit measured tensor analyzing power angular distributions of $^{208}\mathrm{Pb}(d,p)^{209}\mathrm{Pb}$ at $E_d = 9$ MeV, Knutson and Haeberli determined that[7] $\eta = 0.0234 \pm 0.0017$. This value of η is smaller than that of many published deuteron wave functions. For example, the wave functions of Refs. 8–10 predict $\eta = 0.0262$, 0.0260, and 0.0260, respectively. Unfortunately, because of the large error bar, the measurement is unable to conclusively rule out these wave functions and a measurement with a smaller error is desired.

The sources of error in Ref. 6 were primarily from uncertainties in the beam polarization measurement and uncertainties in the nuclear optical-model parameters. The former error can be reduced by careful polarimeter calibration. The latter uncertainty can be reduced by taking measurements at lower energies, at the expense of a more difficult experiment because of the reduced (d,p) cross section. As will be discussed in this Letter, very little sensitivity to the optical-model uncertainties remains, provided that the bombarding energy is no larger than approximately 8 MeV.

In this Letter, we report new measurements of the ^{208}Pb(d,p) ^{209}Pb tensor analyzing powers at E_d = 7, 8, and 9 MeV. The measurements were made with a highly accurate polarimeter with the result that uncertainties in the beam polarization measurement contribute less than ± 0.0002 to the uncertainty in η. Comparison of our E_d = 9 MeV data with those of Ref. 6 indicates that the T_{20} measurements of Ref. 6 were in error by 8% because of polarimeter calibration inaccuracies. When corrected for this error, the value of η from Ref. 6 becomes 0.0253. Transitions to the $\frac{9}{2}^+$, $\frac{5}{2}^+$, $\frac{1}{2}^+$ states in ^{209}Pb (E_x = 0.0, 1.57, and 2.03 MeV) were used in the determination of η because of their relatively large tensor analyzing powers. Data taken at E_d = 8 MeV are shown in Fig. 1. The displayed errors are the result of counting statistics only. Data taken at E_d = 7 and 9 MeV are similar to that shown in Fig. 1.

For a given set of optical-model parameters, η is the only free parameter in DWBA which affects the tensor analyzing powers and, as a result, each measured point in the angular distributions represents an independent determination of η. Weighted averages of the η values have been calculated according to bombarding energy and are listed in Table I for three different sets of optical potentials. The internal agreement of the η values obtained at a given energy is indicated by χ^2, which is also listed in Table I. The sensitivity of the η determination to the optical-potential parameters is indicated by the difference in η values obtained for the different potential sets.

Two (p,p) parameter sets and two (d,d) parameter sets are used, representing parameters obtained from fits to elastic scattering data both above and below the Coulomb barrier (approximately 12 MeV for deuterons or protons on Pb). As expected, the sensitivity to the optical-potential parameters is largest at E_d = 9 MeV, where the scattering waves penetrate farthest into the nuclear interior. The values of χ^2 at E_d = 9 MeV are also larger than at 7 or 8 MeV, indicating poorer internal agreement. As a result, the measurements at E_d = 9 MeV are not considered useful in determining η, but serve to establish an upper limit on the bombarding energies which are useful in obtaining η. In contrast, the determination of η is not sensitive to the optical-potential parameters at E_d = 7 or 8 MeV. The largest variation in η obtained by changing the potential parameters is less than 1%.

Because of ambiguities in optical-model fits, the potential parameter sets listed in Table I are not the only ones which reproduce the elastic scattering data. As is shown in the following, however, the parameter sets listed do give a reasonable measure of the uncertainties in η due to the choice of optical potentials. Even very crude

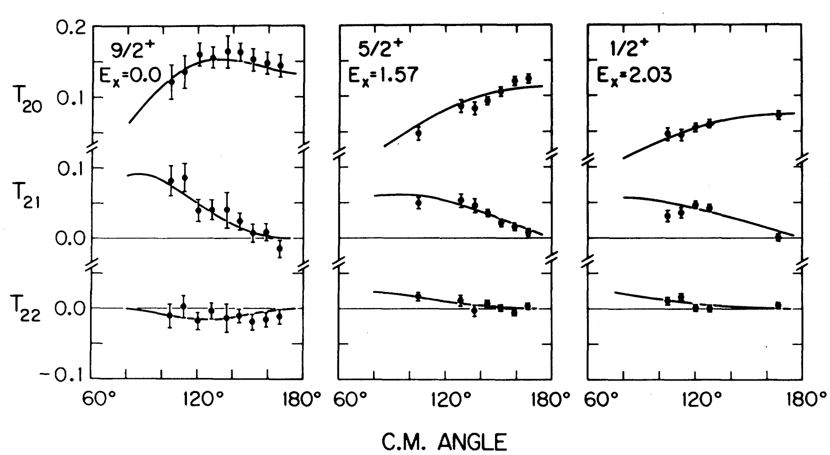

FIG. 1. Angular distributions of the tensor analyzing powers T_{20}, T_{21}, and T_{22} for the reaction ^{208}Pb$(d,p)^{209}$Pb at E_d = 8 MeV. The displayed errors are statistical only. The curves are the result of DWBA calculations using η = 0.026 49 and the optical-potential parameters of Refs. 11 and 12.

TABLE I. Values of η obtained by comparing tensor-analyzing-power data at $E_d = 7$, 8, and 9 MeV with DWBA calculations with use of the indicated optical-potential parameters.

Optical potentials (d,d)	(p,p)	$\eta \pm \delta\eta$ (χ^2) 7 MeV	8 MeV	9 MeV
a	b	0.026 44 ± 0.000 64 (1.3)	0.026 65 ± 0.0005 (0.9)	0.026 49 ± 0.000 32 (1.6)
a	c	0.026 38 ± 0.000 64 (1.3)	0.026 44 ± 0.0005 (0.9)	0.025 62 ± 0.000 32 (1.7)
d	b	0.026 22 ± 0.000 64 (1.3)	0.026 65 ± 0.0005 (0.9)	0.021 51 ± 0.000 27 (3.3)

[a] From Ref. 11, in which the parameters were obtained by fitting the ^{208}Pb$(d,d)^{208}$Pb cross section and vector analyzing power at $E_d = 12.3$ MeV.

[b] From Ref. 12, in which the parameters were obtained by fitting the ^{209}Bi$(p,p)^{209}$Bi cross section and analyzing power at $E_p = 10.76$ MeV.

[c] From Ref. 13, in which the parameters were obtained by fitting cross section and analyzing power (p,p) data taken above the Coulomb barrier on targets with $A > 40$.

[d] The $E_d = 9$ MeV parameters are from Ref. 14, where the parameters were obtained by fitting ^{208}Pb$(d,d)^{208}$Pb cross section and vector analyzing power at $E_d = 9$ MeV. The $E_d = 7$ and 8 MeV parameters were obtained by fitting ^{208}Pb$(d,d)^{208}$Pb cross section and vector-analyzing-power data taken at $E_d = 7$ and 8 MeV in conjunction with the present work. The parameters are 8 MeV, $V_0 = 122$, $r_0 = 1.06$, $a_0 = 0.72$, $W_I = 1.88$, $r_I = 1.65$, and $a_I = 0.88$; 7 MeV, $V_0 = 126.7$, $r_0 = 1.06$, $a_0 = 0.72$, $W_I = 1.58$, $r_I = 1.65$, and $a_I = 0.88$. The $E_d = 7$ and 8 MeV potentials had, in addition, a Coulomb distortion term $V(r) = -2400r^{-4}$ MeV fm^4 as proposed in Ref. 15.

changes in the potential parameters do not produce substantially larger variations in η than those shown in Table I. The value of η was calculated using optical parameter sets in which the real and imaginary radius and diffuseness were each changed by 5% and 10% and in which the potential strengths were each changed by 5%, 10%, 20%, and set to zero. The value of η was also calculated with all nuclear potential strengths set to zero so that the scattering waves were Coulomb wave functions. In no case were the changes in the $E_d = 7$ and 8 MeV values of η larger than 0.000 36. Since these optical potentials obviously no longer fit the elastic scattering data, the effect on η is unreasonably large and this number is to be regarded as an upper limit.

The values of η which are midway between the values given by the different optical potentials in Table I are, respectively, 0.026 33 and 0.026 55 at $E_d = 7$ and 8 MeV. The errors contributed by the uncertainties in the optical parameters are calculated from the differences in η in Table I and are, at $E_d = 7$ and 8 MeV, the same value ± 0.000 11. The average of the $E_d = 7$ and 8 MeV values of η, weighted by the quadruture sum of the statistical and optical-parameter errors, is

$$\eta = 0.02649 \pm 0.00043.$$

The quoted error includes a polarimeter calibration error of ± 0.000 16 added in quadruture. The DWBA calculations using this value of η are shown in Fig. 1. The χ^2 with use of this value of η and the $E_d = 7$ and 8 MeV measurements is 1.1.

The experimental value of η may have a dependence on three effects which have been neglected. In our DWBA calculations, we make use of the local-energy approximation[16] (LEA) because, at present, a suitable finite-range code is not available. The accuracy of the LEA has not been precisely determined at sub-Coulomb energies but it has been estimated[17] on the basis of semiclassical arguments that the accuracy is better than 5%. Tensor terms in the $d + {}^{208}$Pb optical potential have been neglected. The effect of these terms on the ^{208}Pb$(d,p)^{209}$Pb analyzing powers has not been determined although it is expected that, in view of the insensitivity of η to the central terms at $E_d = 7$ and 8 MeV, the tensor terms will also have a small effect on the ^{208}Pb$(d,p)^{209}$Pb tensor analyzing powers at $E_d \leq 8$ MeV. Finally, we have neglected the distortion of the deuteron in the Coulomb field of ^{208}Pb. Initial calculations[18] show that the effect is of the order of 2–3%. It is hoped that the present experiment will stimulate further interest in calculating these effects.

The experimental result is compared with several theoretical values of η in Table II. The first entry is an example of the separable po-

TABLE II. Values of P_D, Q, and η given by various deuteron wave functions.

Author	Ref.	P_D(%)	Q (fm^2)	η
Mongan IV	19	1.4	0.279	0.0406
Reid	8	6.5	0.280	0.0262
dTS	9	5.5	0.279	0.0255
dTSR	9	5.8	0.279	0.0260
Paris (1975)	20	6.8	0.290	0.0293
Paris (1980)	10	5.4	0.278	0.0260
Experiment				0.02649 ± 0.00043

tentials, which are used primarily in multiparticle calculations. As pointed out in Ref. 19, the multiparticle calculations made with a particular potential are strongly influenced by the deuteron properties of that potential. Thus, potentials such as the first entry in Table II which give the incorrect value of η will probably give incorrect results in multiparticle calculations. The other potential models all have a long-range one-pion-exchange potential tail and obtain the short-range potential phenomenologically. The Paris potential (1975) is obsolete but is included to illustrate the changes in η that have occurred with theoretical progress. The most recent of these potentials is the Paris potential (1980), in which the intermediate- and long-range potential are calculated assuming one-pion-exchange, two-pion-exchange, and ω exchange. The agreement between this potential and the present experimental result is indeed gratifying since the theoretical and experimental results were obtained independently.

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