small but nonzero coupling. The curves suggest that as $N \to \infty$ the β function develops a kink at $\lambda^2 = 2.7$.[13] Transitions of this sort have been obtained in studies of one-plaquette models of lattice theories.[14]

We conclude this article with an observation concerning the reliability of finite lattice calculations of the crossover regions. Note from Fig. 1 that the value of a^2T where the strong-coupling calculation matches the weak-coupling scaling law is 0.3–0.4, and is roughly independent of N. But $(a^2T)^{-1/2}$ is a measure of each system's linear correlation length, and it is about 1.6–2.0 when measured in units of the lattice spacing. This is quite small so that ordinary methods of analysis such as strong coupling expansions, finite size scaling, computer simulations, and analytic solutions of small systems should be adequate here.

The calculations presented in this article are just exploratroy. As explained in the last paragraph, they appear to be at least reasonable and self-consistent. It would be good to calculate higher orders and to run computer simulations for various SU(N) gauge groups.

This work was supported in part by the National Science Foundation and in part by the U. S. Department of Energy under Contract No. EY76-S-02-2220.

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[1]H. Fritzsch, M. Gell-Mann, and H. Leutwyler, Phys. Lett. 47B, 365 (1973).
[2]See, for example, H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974).
[3]A. M. Polyakov, unpublished.
[4]M. Creutz, to be published.
[5]J. B. Kogut, R. B. Pearson, and J. Shigemitsu, Phys. Rev. Lett. 43, 484 (1979), and to be published.
[6]J. B. Kogut and L. Susskind, Phys. Rev. D 11, 395 (1975).
[7]W. Caswell, Phys. Rev. Lett. 33, 344 (1974).
[8]W. A. Bardeen, A. J. Buras, D. W. Duke, and T. Muta, Phys. Rev. D 18, 3998 (1978).
[9]In the content of Euclidean expansions the same observation was made by B. DeWit and G. 't Hooft, Phys. Lett. 69B, 61 (1977).
[10]J. Kogut, Institute for Theoretical Physics, Santa Barbara, Report No. NSF-ITP-80-16, 1980 (unpublished).
[11]A. Hasenfratz and P. Hasenfratz, to be published; R. Dashen, private communication.
[12]D. Gross, private communication.
[13]With use of momentum-space regularization $\lambda^2 = 2.7$ maps into $g_{\rm ms}{}^2 N/4\pi \approx 0.73$.
[14]D. Gross and E. Witten, Phys. Rev. D 21, 446 (1980); S. Wadia, to be published.

Flavor Goniometry by Proton Decay

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Unification of strong and electroweak forces implies that protons (and bound neutrons) decay. Modes like $\bar{\nu}\pi$, $e^+\pi$, and μ^+K^0 are expected, while modes like $\mu^+\pi$ and e^+K^0 are "Cabibbo" suppressed. Branching ratios can reveal much about the nature of the unifying group and the origin of fermion masses. Plausible models of unification and flavor mixing give surprisingly different predictions for two-body branching ratios.

PACS numbers: 12.20.Hx, 13.30.Ce

Once, weak decays of hadrons involved only one flavor-mixing parameter—the Cabibbo angle.[1] With N-fermion families, $(N-1)^2$ parameters describe the flavor mixing of electroweak phenomena.[2] Most electroweak-chromodynamic unifications predict a new interaction which violates the conservation of baryon number. Protons and bound neutrons decay into states with $S=0$ or 1 containing one antilepton (e^+, μ^+, or $\bar{\nu}$). Branching ratios of these $\Delta B = \Delta L = -1$ decays are *not* necessarily determined by the $(N-1)^2$ flavor-mixing parameters, a point stressed by many workers.[3-6] "Grand" unification reduces three gauge coupling constants to one, but introduces many new unknowns: mass ratios of superheavy bosons, and additional flavor-mixing parameters which play no role in electroweak phenomenology. Branching ratios of $\Delta B=-1$ decays determine some of these otherwise inaccessible parameters. Their measurement can shed new light on the origin of fermion masses and flavor-mixing angles. Only a few hundred observed $\Delta B=-1$ events suffice to open this new window.

We analyze $\Delta B=-1$ decays in an O(10) model. As we note below, this is not as restrictive as it seems since our results are valid in a larger class of unified models. Moreover, if the known fermion families (possibly supplemented with right-handed neutrino fields) are equivalent representations of the gauge group, then it can be only O(10) or SU(5). O(10) includes SU(5) as a special case and accommodates a simple predictive mechanism for fermion-mass generation.[7]

Taking a hint from ordinary weak interactions, we assume that the $\Delta B=-1$ interaction is dominated by gauge boson exchange, the Higgs bosons being more weakly coupled. Should this assumption be false, the first successful proton-decay experiment will see far more muons than electrons.

Gauge bosons responsible for $\Delta B=-1$ decay in O(10) comprise two approximately degenerate multiplets, irreducible under $SU(3)\otimes SU(2)\otimes U(1)$. Both are color triplets and weak doublets. They have different U(1) properties. One doublet, which we call type A, has electric charges $-\frac{1}{3}$ and $-\frac{4}{3}$. The type-B doublet has charges $\frac{2}{3}$ and $-\frac{1}{3}$. We denote the corresponding superheavy gauge boson masses by M_A and M_B. Only type-A bosons are present in ordinary SU(5), which corresponds to the $M_A/M_B \to 0$ limit of O(10). The opposite limit, $M_B/M_A \to 0$, corresponds to an $SU(5)' \otimes U(1)$ subgroup of O(10). SU(5)′ is not ordinary SU(5): It treats the left-handed positron field as a singlet and does not contain electric charge as a generator.

The induced four-fermion couplings responsible for $\Delta B=-1$ decays in an O(10) model are

$$G_A\epsilon^{ijk}[(\overline{D}_{ia}{}^c\gamma^\mu L_a{}^- - \overline{L}_a{}^+\gamma^\mu D_{ia})(\overline{U}_{jb}{}^c\gamma_\mu U_{kb}) + (\overline{L}^+\gamma^\mu U_{ia} - \overline{D}_{ia}{}^c\gamma^\mu \nu_a)(\overline{U}_{jb}{}^c\gamma_\mu D_{kb})]$$
$$+G_B\epsilon^{ijk}[(\overline{\nu}_a{}^c\gamma^\mu D_{ia} - \overline{U}_{ia}{}^c\gamma^\mu L_a{}^-)(\overline{D}_{jb}{}^c\gamma_\mu U_{kb}) + (\overline{U}_{ia}{}^c\gamma^\mu\nu_a - \overline{\nu}_a{}^c\gamma^\mu U_{ia})(\overline{D}_{jb}{}^c\gamma_\mu D_{kb})] \qquad (1)$$

with all fields left handed. Here, i, j, and k are color indices and a and b are flavor indices, c denotes charge conjugation, L, ν, U, and D denote charged leptons, neutrinos, $Q=\frac{2}{3}$ quarks, and $Q=-\frac{1}{3}$ quarks. The coupling constants are $G_A=g^2/M_A{}^2$ and $G_B=g^2/M_B{}^2$, where g is the O(10) gauge coupling.

While (1) was derived in an O(10) model, it is valid in any unified theory wherein no more than one doublet of gauge bosons of each type (A and B) contributes to $\Delta B=-1$ decay.

The branching ratios implied by (1) depend upon M_A/M_B and the angles and phases relating the fermion fields U, D, L, and ν in each family to mass eigenfields. So many parameters are involved in these relations that it is impossible to make any model-independent statement about $\Delta B=-1$ branching ratios. To be explicit, we consider two specific models describing the nature and origin of fermion masses and flavor mixing. We call these F masses and J masses, the former being simpler, the latter more phenomenologically successful.

F masses result from the hypothesis that all charged fermion masses arise from Yukawa couplings to Higgs decimets with vacuum expectation values.[8] Down-quark masses are simply related to charged-lepton masses, as in naive SU(5),[9]

$$m_d/m_s = m_e/m_\mu, \quad m_s/m_b = m_\mu/m_\tau. \qquad (2)$$

These relations do not seem to be satisfied by nature. A third renormalization-group dependent relation, first derived by Chanowitz, Ellis, and Gaillard[3,10] correctly estimates $3m_\tau \simeq m_b$. With F masses, the suppression of such modes as $p \to K^0e^+$ or $\pi^0\mu^+$ can be expressed in terms of the K-M parameters,[2] and in practice, in terms of the Cabibbo angle. F masses are often implicitly assumed by those who conclude that the $\Delta B=-1$ Cabibbo-suppressed modes are tiny and lack interest. We consider F masses because they are simple. However, the failure of (2) suggests that this *Ansatz* is wrong.

In the alternative scheme of J masses, charged-fermion masses come from an interplay between a complex $\underline{10}$ and $\underline{126}$ of Higgs bosons with vacuum expectation values. This scheme is discussed elsewhere.[7,11] Instead of (2), we find the more reasonable relations

$$m_d/m_s = 9m_e/m_\mu, \quad 3m_s/m_b = m_\mu/m_\tau, \tag{3}$$

and the successful estimate of the τ mass is kept.

In our calculations of the nucleon decay $\Delta B = -1$ branching ratios, we neglect mixing to the third (or higher) fermion families since Cabibbo universality is approximately satisfied. We are reduced to a two-dimensional family space. The fermion mass matrices, in the case of F masses, may be written

$$D \propto L = \begin{bmatrix} m_e & 0 \\ 0 & m_\mu \end{bmatrix}, \quad 2U = \begin{bmatrix} [(m_c+m_u)+(m_c-m_u)\cos 2\theta_C] & (m_c-m_u)\sin 2\theta_C \\ (m_c-m_u)\sin 2\theta_C & [(m_c+m_u)-(m_c-m_u)\cos 2\theta_C] \end{bmatrix}, \tag{4a}$$

where θ_C is the Cabibbo angle. In the case of J masses, they are

$$L = \begin{bmatrix} 0 & (m_e m_\mu)^{1/2} \\ (m_e m_\mu)^{1/2} & (m_\mu - m_e) \end{bmatrix}, \quad D \propto \begin{bmatrix} 0 & -3(m_e m_\mu)^{1/2} \\ -3(m_e m_\mu)^{1/2} & (m_\mu - m_e) \end{bmatrix}, \quad U = \begin{bmatrix} 0 & (m_c m_u)^{1/2} \\ (m_c m_u)^{1/2} & \eta(m_c - m_u) \end{bmatrix}, \tag{4b}$$

where η is a complex phase and the Cabibbo angle is an implicit function of the other variables.

The substitutions that must be made in (1) to obtain explicit flavor couplings are those that diagonalize the mass matrices (4). In the case of F masses, L and D are simultaneously diagonal. The nontrivial substitutions are those that diagonalize U in (4a):

$$\begin{aligned} &U_1 \to c_\theta u - s_\theta c, \quad U_1{}^c \to c_\theta u^c - s_\theta c^c, \\ &U_2 \to s_\theta u + c_\theta c, \quad U_2{}^c \to s_\theta u^c + c_\theta c^c, \end{aligned} \tag{5}$$

where color indices are suppressed and s_θ (c_θ) $\equiv \sin\theta_C$ ($\cos\theta_C$).

In the case of J masses, the required substitutions are much more complicated because none of the matrices in (4b) is diagonal. We obtain

$$\begin{aligned} &L_1{}^- \to c_e e^- - s_e \mu^-, && L_2{}^- \to s_e e^- + c_e \mu^-, \\ &L_1{}^+ \to c_e e^+ + s_e \mu^+, && L_2{}^+ \to s_e e^+ - c_e \mu^+, \\ &D_1 \to c_d d + s_d s, && D_2 \to -s_d d + c_d s, \\ &D_1{}^c \to -c_d d^c + s_d s^c, && D_2{}^c \to s_d d^c + c_d s^c, \\ &U_1 \to c_u u + \eta^* s_u c, && U_2 \to -\eta s_u u + c_u c, \\ &U_1{}^c \to -\eta^* c_u u^c + s_u c^c, && U_2{}^c \to s_u u^c + \eta c_u c^c, \end{aligned} \tag{6}$$

where $s_e \equiv \sin\theta_e$, $c_e \equiv \cos\theta_e$, and similarly for s_d, c_d, s_u, and c_u, with $0 \le \theta_e, \theta_d, \theta_u \le \pi/2$. We determine these angles by diagonalizing the mass matrices in (4b):

$$s_e{}^2 = m_e/(m_e + m_\mu); \quad s_d{}^2 = m_d/(m_d + m_s); \quad s_u{}^2 = m_u/(m_u + m_c). \tag{7}$$

The d and s masses satisfy

$$(m_s - m_d)^2/m_s m_d = (m_\mu - m_e)^2/9m_\mu m_e. \tag{8}$$

From measured lepton masses, (7) and (8) we find $s_e = 0.069$ and $s_d = 0.197$. The Cabibbo angle depends upon the unknown phase η,

$$\sin\theta_C = |s_d c_u - \eta c_d s_u|, \tag{9}$$

and is constrained by the inequalities

$$\begin{aligned} |(m_d m_c)^{1/2} - (m_s m_u)^{1/2}| &\le [(m_d + m_s)(m_u + m_c)]^{1/2} \sin\theta_C \\ &\le |(m_d m_c)^{1/2} + (m_s m_u)^{1/2}| \end{aligned} \tag{10}$$

which are satisfied for reasonable quark masses.

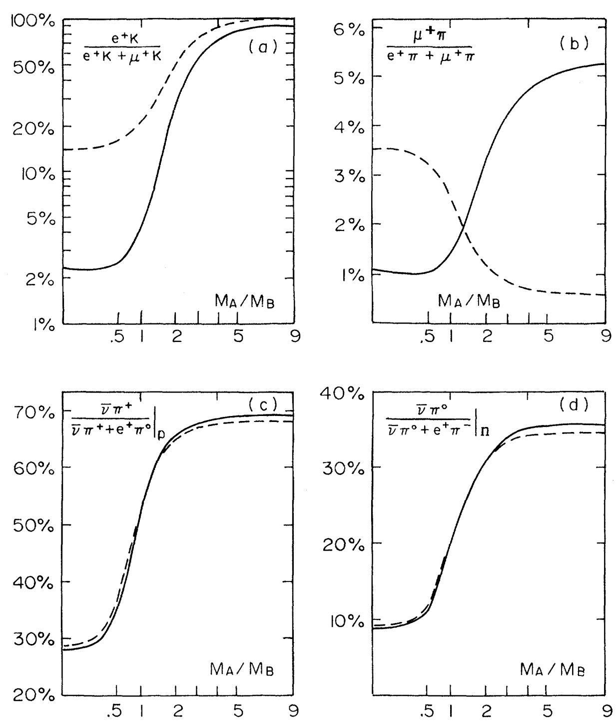

FIG. 1. Relative branching ratios of two-body modes with the same strangeness. The horizontal scales are linear in $(M_A/M_B)^{1/2}$. Continuous (dashed) lines correspond to F (J) masses. Figure 1(a) refers to proton decay; the e^+K^- and μ^+K^- decays of neutrons are forbidden. Figure 1(b) applies to either proton or bound-neutron decay. Figures 1(c) and 1(d) refer to protons and neutrons, respectively.

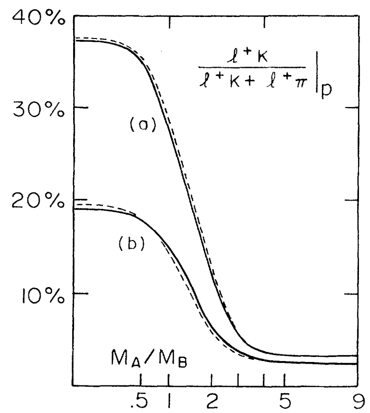

FIG. 2. Branching ratios of strange relative to nonstrange decay modes, with the same conventions as in Fig. 1. The curves labeled *a* correspond to the nonrelativistic model of Ref. 12, the curves labeled *b* are extracted from Ref. 13. Here $l^+K = e^+K + \mu^+K$ and $l^+\pi = e^+\pi + \mu^+\pi$.

We cannot express θ_u in terms of lepton masses. To estimate it we use the empirically acceptable relation $m_u m_\mu = m_c m_e$ which implies $s_u = s_e$. From (9), our evaluation of θ_d, and the observed value of θ_C, we deduce $\text{Re}\eta \cong -0.4$. With these values of s_d, s_e, s_u, and η, we may perform the substitutions (6). Our results for branching ratios are insensitive to the poorly determined values of s_u and η.

Substitution of (5) (F masses) or (6) (J masses) into (1) gives the effective Lagrangian for nucleon decay into channels with quantum numbers of $(e^+, \mu^+, \bar{\nu})\pi$ or $(e^+, \mu^+, \bar{\nu})K$. Our results are displayed in the figures. Figure 1 shows ratios of decay modes into final states with the same strangeness: No hadronic symmetry other than isospin is required for these results. The substitution $\pi \to \rho$ would leave Figs. 1(b), 1(c), and 1(d) practically unchanged. Phase space corrections become important if K^*'s are substituted for K's in Fig. 1(a). Figure 2 compares $S=0$ and 1 final states. These results are model dependent, and we show two plots of the same quantity to demonstrate this. The upper curve is extracted from Gavela *et al.*[12]; the lower curve from Machacek.[13] Figure 2 is not sensitive to the choice between F and J masses. Despite evident uncertainties, there is a large (and plausible) domain in M_A/M_B for which the $\Delta S = 1$ decay modes are significant.

Figures 1(c) and 1(d) show that measurement of the relative importance of neutrino modes of nucleon decay determines the superheavy mass ratio M_A/M_B. This result is practically independent of the choice of flavor-mixing models. However, the branching ratios into Cabibbo-supressed modes [Figs. 1(a) and 1(b)] are seen to depend strongly upon the choice of F vs J masses. The ratio $eK/(eK+\mu K)$ is much larger for J masses than for F masses. The $\pi\mu^+$ decay mode, small but not too small, can also be used to discriminate between F and J masses.

In conclusion, we have demonstrated how sensitive nucleon-decay branching ratios are to the parameters entering a grand unified theory. Neutrino to charged-lepton branching ratios determine the mass ratio of intermediate vector bosons responsible for $\Delta B = -1$ decays. Electron to muon branching ratios are strongly dependent on the parameters describing the flavor-mixing patterns. Proton-decay experiments may reveal features of the underlying theory well beyond the mere breakdown of baryon- and lepton-number conservation.

This work is supported in part through funds provided by the U. S. Department of Energy under Contract No. DE-AC02-76ER03069 and by the National Science Foundation under Grant No. PHY77-22864.

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[a]On leave from CERN, CH-1211 Geneva 23, Switzerland.

[1]N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Gell-Mann and M. Levy, Nuovo Cimento 16, 705 (1960).

[2]For three-fermion families, the four parameters are the K-M angles and phase, M. Kobayashi and K. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

[3]A. Buras, J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Nucl. Phys. B135, 52 (1979).

[4]C. Jarlskog, Phys. Lett. 82B, 401 (1979).

[5]J. Ellis, M. K. Gaillard, and D. V. Nanopoulos, Phys. Lett. 88B, 320 (1979).

[6]R. N. Mohapatra and G. Senjanović, Phys. Rev. Lett. 44, 912 (1980); F. A. Wilczek and A. Zee, to be published.

[7]H. Georgi and D. V. Nanopoulos, Nucl. Phys. B159, 16 (1979).

[8]Neutrinos are special. But, because their masses are small and irrelevant to issues at hand, we shall not discuss them here.

[9]H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974).

[10]M. S. Chanowitz, J. Ellis, and M. K. Gaillard, Nucl. Phys. B128, 506 (1977).

[11]H. Georgi and C. Jarlskog, Phys. Lett. 86B, 297 (1979).

[12]M. B. Gavela *et al.*, Université de Paris-Sud, Orsay, Report No. LPTHE 80/6, 1980 (to be published).

[13]M. Machacek, Nucl. Phys. B159, 37 (1979).