isospin.

Although the quantum number H and the weak isospin are defined unambiguously only for zero-mass fermions where helicity and chirality are equivalent, this restriction is irrelevant in all the unified models under consideration,[1-4] where the quark and lepton masses are negligible on the scale of the grand unification mass. A physical process, such as proton decay or neutron-antineutron oscillations, is described by combining the B-nonconserving n-point functions at the quark-lepton level with spectator quarks and possibly gluons to describe hadrons. However, all these processes of building hadrons out of quarks conserve B and L, and all the B and L nonconservation occurs in an n-point function at the grand unification mass scale where the zero-mass approximation is good. Thus even though weak isospin, H, and the BL parity may not be defined for the hadron states, the selection rules for B and $B-L$ conservation still apply.

As an example consider an initial nucleon state described by the Massachusetts Institute of Technology bag model. The nucleon at rest does not have a well-defined weak isospin, H, or π_{BL}. However, the bag model wave function is a state of three zero-mass quarks, which can be expanded in eigenfunctions of these quantum numbers, all with $B=1$ and $L=0$. The final state is obtained by looking at all possible transitions allowed by the particular weak decay model from all terms in this expansion. If these are all four-point functions which conserve weak isospin and conserve H because they are vector exchange, then they all conserve $B-L$. Thus all terms in the expansion of the final state have $B-L=1$ and the proton decay process conserves $B-L$.

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[a]On leave from Department of Physics, Weizmann Institute of Science, Rehovot, Israel.

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Asymptotic-Freedom Scales

Michael Creutz
Physics Department, Brookhaven National Laboratory, Upton, New York 11974
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Using Monte Carlo methods with Wilson's lattice cutoff, the asymptotic-freedom scales of SU(2) and SU(3) gauge theories without quarks are calculated.

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The standard SU(3) Yang-Mills theory of the strong interaction is asymptotically free.[1] The effective coupling constant decreases logarithmicly at short distances. This ultraviolet behavior permits perturbative analysis of high-momentum-transfer processes. In this paper, I present a calculation of the parameters setting the scale of this phenomenon in pure SU(2) and SU(3) gauge theories.

In a non-Abelian gauge theory with an ultraviolet cutoff, the bare charge g_0 goes to zero with the logarithm of the cutoff parameter[2]

$$g_0^2 = \frac{1}{\gamma_0 \ln(1/\Lambda_0^2 a^2) + (\gamma_1/\gamma_0)\ln[\ln(1/\Lambda_0^2 a^2)] + O(g_0^2)}. \tag{1}$$

Here γ_0 and γ_1 are the first two coefficients in the perturbative expansion of the Gell-Mann–Low function[3]

$$\gamma(g_0) = a\,dg_0/da = \gamma_0 g_0^3 + \gamma_1 g_0^5 + O(g_0^7). \tag{2}$$

The cutoff length a is the lattice spacing in a lattice formulation. The parameter Λ_0 is the asymptotic-freedom scale associated with the renormalization scheme being used. For SU(N) gauge groups the coefficients in Eq. (2) are[2,4]

$$\gamma_0 = \tfrac{11}{3}(N/16\pi^2), \tag{3}$$

$$\gamma_1 = \tfrac{34}{3}(N/16\pi^2)^2. \tag{4}$$

These first two coefficients are independent of renormalization prescription.

Equation (1) defines the scale Λ_0 and can be rewritten

$$\Lambda_0 = \lim_{a\to 0} \frac{1}{a}[(\gamma_0 g_0^2(a))]^{(-\gamma_1/2\gamma_0^2)} \exp\left(\frac{-1}{2\gamma_0 g_0^2(a)}\right). \tag{5}$$

My Monte Carlo results for these scales in the pure gauge theories (no quarks) are

$$\Lambda_0 = (1.3\pm 0.2)\times 10^{-2}\sqrt{K}, \quad \mathrm{SU}(2); \tag{6}$$

$$\Lambda_0 = (5.0\pm 1.5)\times 10^{-3}\sqrt{K}, \quad \mathrm{SU}(3). \tag{7}$$

Here I have used Wilson's lattice regulator[5] and K is the string tension, the coefficient of the linear potential between widely separated sources in the fundamental representation of the gauge group. I will return later to the method of calculating these numbers.

At first sight these small numbers are rather surprising, coming as they do from a theory with no small dimensionless parameters. However the value of Λ_0 is not independent of renormalization scheme.[6] Since it is defined in the weak-coupling limit, perturbative calculations to one-loop order can relate different definitions. Hasenfratz and Hasenfratz have recently done a lengthy analysis relating this Λ_0 to the more conventional scale Λ^{MOM} defined by the three-point vertex in Feynman gauge and at a given scale in momentum space. Their results are

$$\Lambda^{\mathrm{MOM}} = 57.5\Lambda_0, \quad \mathrm{SU}(2); \tag{8}$$

$$\Lambda^{\mathrm{MOM}} = 83.5\Lambda_0, \quad \mathrm{SU}(3). \tag{9}$$

These large factors partially cancel the small numbers in Eqs. (6) and (7); combining them gives

$$\Lambda^{\mathrm{MOM}} = (0.75\pm 0.12)\sqrt{K}, \quad \mathrm{SU}(2); \tag{10}$$

$$\Lambda^{\mathrm{MOM}} = (0.42\pm 0.13)\sqrt{K}, \quad \mathrm{SU}(3). \tag{11}$$

If we accept the string model[8] connection between K and the Regge slope α'

$$K = 1/2\pi\alpha' \tag{12}$$

and use $\alpha' = 1.0\ (\mathrm{GeV})^{-2}$, then we conclude for SU(3)

$$\Lambda^{\mathrm{MOM}} = 170\pm 50\ \mathrm{MeV}. \tag{13}$$

Some caution may be necessary in the phenomenological interpretation of this number because I have not included effects of virtual quark loops.

I now turn to the method of calculation. For SU(2) gauge theory I worked with a lattice of 10^4 sites except at strong coupling where 8^4 sufficed. I used the heat bath Monte Carlo algorithm of Creutz.[9] For SU(3) I used a Metropolis[10] scheme similar in spirit to that employed by Wilson.[11] This is inherently less efficient than the SU(2) procedure; so most SU(3) running was on a 4^4 lattice. One value of coupling, $g_0^2 = 0.902$, was studied on a 6^4-site system. As the SU(3) lattices are rather small, the conclusions depend heavily on an assumed similarity with the SU(2) case.

After bringing the lattice into a typical equilibrium state at some value of the bare coupling constant, I measured the expectation values of rectangular Wilson loops $W(I,J)$ where I and J are the dimensions of the loop in fundamental lattice units. From these loops, I construct the quantities

$$\chi(I,J) = -\ln\left(\frac{W(I,J)W(I-1,J-1)}{W(I,J-1)W(I-1,J)}\right). \tag{14}$$

In this combination overall constant factors and perimeter behaviors of the loops will cancel out. For $I \gg J \gg 1$, $\chi(I,J)$ is proportional to the force between a quark and an antiquark separated by distance Ja. The motivation for introducing χ is that in a region where the loops are dominated by an area law

$$W(I,J) \sim e^{-KA}, \tag{15}$$

where $A = a^2 IJ$ is the loop area, it directly measures the string tension K

$$\chi \to a^2 K. \tag{16}$$

This happens both when I and J are large and when the bare coupling is large. However in the weak-coupling limit with I and J held fixed, χ should have a perturbative expansion

$$\chi(I,J) = a_1 g_0^2 + O(g_0^4). \tag{17}$$

For example, a simple calculation gives

$$\chi(1,1) \underset{g_0^2\to 0}{\sim} \begin{cases} \tfrac{3}{16}g_0^2, & \mathrm{SU}(2) \\ \tfrac{1}{3}g_0^2, & \mathrm{SU}(3). \end{cases} \tag{18}$$

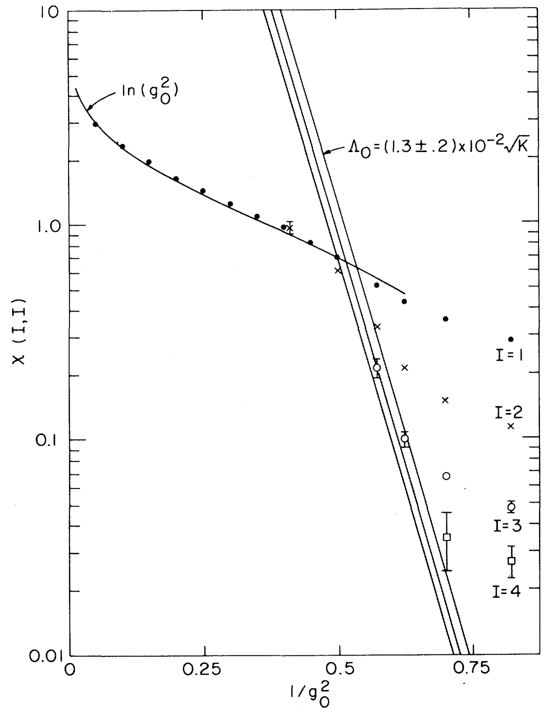

FIG. 1. The quantities $\chi(I, I)$ for SU(2) gauge theory as a function of $1/g_0^2$.

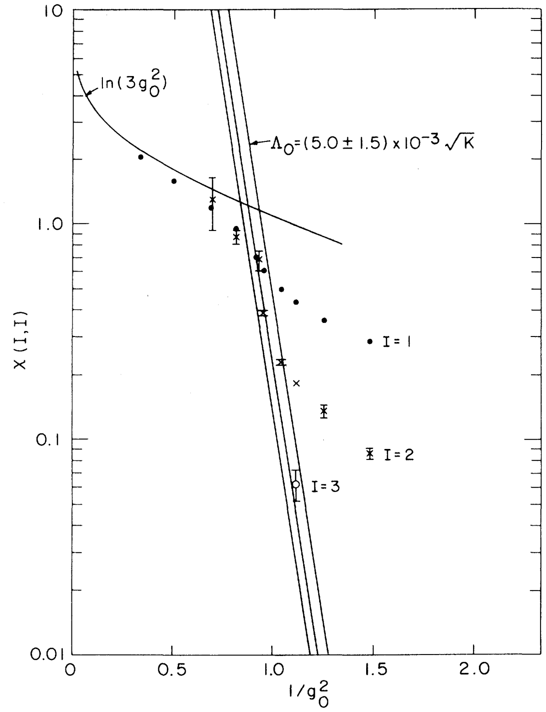

FIG. 2. The quantities $\chi(I, I)$ for SU(3) gauge theory.

This power behavior is radically different than the essential singularity expected for the right-hand side of Eq. (16)

$$a^2K \underset{g_0^2\to 0}{\sim} \frac{K}{\Lambda_0^2}(\gamma_0 g_0^2)^{(-\gamma_1/\gamma_0^2)} \exp\left(-\frac{1}{\gamma_0 g_0^2}\right). \quad (19)$$

This is a consequence of Eq. (5) coupled with the renormalization prescription of holding the physical string tension K fixed as the cutoff is removed.[9,12] In summary, for strong coupling, i.e., $g_0^2>1$, we expect all $\chi(I,J)$ to be equal to the coefficient of the area law, as in Eq. (16), but as g_0^2 is reduced, small I and J should give a χ deviating from a^2K. Thus the envelope of curves of $\chi(I,J)$ for all I and J plotted versus the coupling should be the true value of a^2K as a function of g_0^2. Use of this envelope avoids ambiguities in the fitting procedure used in Ref. 9.

In Fig. 1, I plot the value of $\chi(I,I)$ for $I=1-4$ versus $1/g_0^2$ for the gauge group SU(2). The error bars on the points represent the standard deviation of the mean taken from an ensemble of five configurations. At stronger couplings, the large loops have large relative errors but are consistent with χ approaching the values from smaller loops. On this graph, I show the strong-coupling limit for all $\chi(I,J)$,

$$\chi(I,J)=\ln(g_0^2)+O(1/g_0^4). \quad (20)$$

The weak-coupling limit for a^2K as given in Eq. (19) appears in the graph as a band corresponding to the value of Λ_0 in Eq. (6). The error in Λ_0 is a purely subjective estimate.

Figure 2 is essentially the same as Fig. 1 except the gauge group is now the physically interesting SU(3). As most of the points are from a 4^4 lattice, only $\chi(1,1)$ and $\chi(2,2)$ are plotted. At $g_0^2=0.902$ the run on a 6^4 lattice gave one point for $\chi(3,3)$. The strong-coupling behavior is now

$$\chi(I,J)=\ln(3g_0^2)+O(1/g_0^2). \quad (21)$$

Note that the corrections for SU(3) begin in a lower order of the strong-coupling expansion than for the SU(2) case in Eq. (20). These small lattices in and of themselves do not allow any precise conclusions; however, assuming a similarity with the SU(2) picture, I have plotted the band from Eq. (19) giving the Λ_0 in Eq. (7). Note that the strong-coupling deviation from the asymptotic-freedom behavior sets in rather abruptly at g_0^2

~1. This is in excellent agreement with the series results of Ref. 12.

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Search for Baryonium States in the Reaction $pp \to pp\bar{p}p$ at 11.75 GeV/c

S. Kooijman, M. W. Arenton, D. S. Ayres, R. Diebold, E. N. May, L. Nodulman, J. R. Sauer, and A. B. Wicklund
Argonne National Laboratory, Argonne, Illinois 60439

and

E. C. Swallow
Department of Physics,[a] *Elmhurst College, Elmhurst, Illinois 60126 and Enrico Fermi Institute, University of Chicago, Chicago, Illinois 60637*

(Received 28 April 1980)

A search was made for five-quark "pseudobaryon" and four-quark baryonium states with a 1000-event sample of the reaction $pp \to pp\bar{p}p$ at 11.75 GeV/c. For states with widths $\lesssim 10$ MeV/c^2 and masses $M_{\bar{p}p} < 2.2$ GeV/c^2 and $M_{pp\bar{p}} < 3.4$ GeV/c^2, upper limits on the product of the cross section and the branching ratio for forward-hemisphere production are measured to be ~ 15 nb.

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In this Letter we report on an experimental study of the reaction $pp \to pp\bar{p}p$ at 11.75 GeV/c carried out with the effective-mass spectrometer (EMS) at the Argonne zero-gradient synchrotron. Compared with topologically similar channels such as $p\pi^+\pi^-p$, the $pp\bar{p}p$ final state is quite rare and we know of no previous measurements of this reaction, although inclusive production of $\bar{p}p$ pairs has been investigated at higher energies.[1]

Several experiments have searched for narrow meson states, having a $qq\bar{q}\bar{q}$ quark configuration, that decay preferentially into baryon-antibaryon channels.[2,3] Although "baryonium" candidates that couple to $\bar{p}p$ have been reported at 1932, 2020, and 2200 MeV, their existence is not well established.[2] Indeed, recent measurements of the $\bar{p}p$ total cross section indicate that the relative couplings of these states to $\bar{p}p$ must be small,[3] opening to question their interpretation as multiquark states. Analogous five-quark ($qqqq\bar{q}$) states with baryon quantum numbers have been proposed,[4,5] which could be produced diffractively in pp collisions. A possible signature for these "pseudobaryon" states (P) would be their cascade into a baryonium (B) plus a proton,

$$\begin{array}{l} pp \to Pp \\ \quad\;\; \lfloor\!\to Bp \\ \qquad\;\; \lfloor\!\to \bar{p}p, \end{array} \tag{1}$$

which would result in correlated peaks in the $pp\bar{p}$ and $p\bar{p}$ mass spectra in our reaction. Even if the B states happen to have small branching ratios into $\bar{p}p$, the observation of a $P \to B$ cascade would provide evidence that P and B are not ordinary $q\bar{q}$ resonances. We note that evidence has been presented for a possible $S = -1$ five-quark state with a mass of 3.17 GeV/c^2 produced in K^-p col-

