which relates the ratio of strange (m_s) to nonstrange (m_n) quark masses to the ratio of electromagnetic radii for neutral and charged kaons. Combining the recently measured[5] neutral-kaon radius $\langle r_{K^0}^2\rangle = -0.054 \pm 0.026$ fm^2 with the present charged-kaon radius measurement, we find $m_s/m_n \geq 1.39 \pm 0.28$.

In conclusion, our best determination of the negative-kaon radius is $\langle r_{K^-}^2\rangle^{1/2} = 0.53 \pm 0.05$ fm as obtained from the fit to the directly measured form factor constrained to unity at zero momentum transfer.

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Do Quarks Interact Pairwise and Satisfy the Color Hypothesis?

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A simple three-quark Hamiltonian based on a quark-antiquark interaction which accurately describes all $q\bar{q}$ mesons has been solved numerically. Good agreement with the ground-state baryon mass spectrum is obtained if the qq and $q\bar{q}$ interactions differ by an overall factor of 2 as expected from an octet color-exchange mechanism. The interactions in a three-quark system appear to be very well described by only a sum of pair interactions.

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One of the major differences between quantum electrodynamics (QED) and quantum chromodynamics (QCD) is the nature of the interaction between the field quanta. In QCD the field quanta (colored gluons) interact whereas in QED the photons do not interact. This difference is a consequence of the non-Abelian gauge transformations postulated for QCD which are specifically phrased in terms of the SU(3) color group. The major consequences of the color hypothesis for strong interactions between quarks are the following:

(1) At small distances one-gluon exchange[1] between a pair of quarks (i, j) yields a Coulomb-like interaction with an overall strength which depends on color via the product $\vec{F}_i \cdot \vec{F}_j$, where the eight components of $\vec{F}$ are the generators of SU(3) in color space. As the distance between quarks is decreased the effective coupling constant approaches zero corresponding to the concept of asymptotic freedom.

(2) At large distances the interaction between quarks is expected to lead to confinement, although as yet there is no rigorous derivation of this hypothesis (infrared slavery). Most recent work assumes that the interaction between quarks at large separations can be approximated by a string,[2] a boundary condition (bag models[3]), or a linear potential with an overall strength again depending on $\vec{F}_i \cdot \vec{F}_j$.

(3) For more than two quarks the possibility of genuine multibody interactions occurs because of the allowed multigluon vertices. At small distances a three-gluon vertex with each gluon being emitted or absorbed from a different quark is envisaged as a source of a three-body interaction. At larger distances one might also expect multi-

gluon vertices to give rise to a three-quark confinement which is more complex than simply summing over the confining interactions between pairs of quarks.

As a test of some of these consequences, we have studied the two- and three-body quark systems using a pseudorelativistic potential approach. In the two-body system (quark-antiquark) we have recently[4] improved and extended the simple potential model of other workers.[5] By use of relativistic kinematics, effective sizes for quarks, and diagonalization procedures, the two-body Hamiltonian (which neglects the small electromagnetic terms)

$$H_{ij} = (m_i^{\,2} + p^2)^{1/2} + (m_j^{\,2} + p^2)^{1/2} + \langle V_{\mathrm{SR},\,ij}\rangle + V_{\mathrm{LR},\,ij} + \langle V_{\mathrm{A}}\rangle \tag{1}$$

has been evaluated nonperturbatively to an accuracy of 200 keV. In Eq. (1), m_i (m_j) is the quark (antiquark) mass and $\vec{p}$ the momentum in their relative coordinate. The three potential terms correspond to a short-range (SR) interaction with electric and magnetic components suggested by a one-gluon exchange mechanism, a long-range (LR) linear interaction which provides confinement, and an annihilation (A) term to account for the fact that isoscalar 0^- and 1^- mesons can annihilate into at least two and three gluons, respectively. Short-range terms are averaged over the finite size of the quarks as indicated by angular brackets. Full details of these interactions are too lengthy to list here. A complete description is given in Ref. 4.

By a suitable choice of potential parameters, quark sizes, and masses we have obtained[4] an accurate description of the properties of all known $q\bar{q}$ mesons ranging from the pion to upsilonium. The annihilation terms in Eq. (1) are vital[1,4] for a detailed understanding of pseudoscalar and vector mesons but have no counterpart in the quark-quark potential.

Since the color-space product $\vec{F}_i \cdot \vec{F}_j$ has eigenvalues of $-\frac{4}{3}$ and $-\frac{2}{3}$ for color-singlet $q\bar{q}$ states and color-antitriplet qq states, respectively, and since a color-singlet qqq baryon only contains color-antitriplet pairs, then we can relate the $q\bar{q}$ potential (minus annihilation terms) to the qq potential by a simple division by 2. A test of this factor and point (3) above is obtained by studying the simplest three-quark Hamiltonian:

$$H_B = \sum_i (m_i^{\,2} + p_i^{\,2})^{1/2} + \sum_{i>j} \left[a\langle V_{\mathrm{SR},\,ij}\rangle + bV_{\mathrm{LR},\,ij}\right] \tag{2}$$

with the potentials being the same as in Eq. (1) and $a = b = \frac{1}{2}$ if the color relationship is adopted.

This three-quark Hamiltonian with all quarks confined ($b > 0$) and relativistic kinematics has been solved in the center-of-momentum frame to an accuracy of ~ 2 MeV for the ground-state multiplets presented here. Tensor and spin-orbit terms for the ground-state multiplets were not included explicitly since simple closure arguments yielded energy shifts of the same order as the numerical accuracy obtained. Details of the method used and the full set of results including higher orbital angular momenta for the three-quark problem are too lengthy to present here and will be reported on later.

The major elements of the calculation involve the use of a finite number of six-dimensional oscillator states and the use of fractional parentage techniques to accomplish symmetry requirements for the various subspaces. The total oscillator quantum number N is increased until a desired accuracy is achieved. For the results presented here $N\hbar\omega \leq 16\hbar\omega$ was adequate and limited the matrices to be diagonalized to less than 130×130. Because we are dealing with a fully confined system there is no need to use Faddeev techniques. The straightforward diagonalization procedure with a confined basis is faster and more convenient. Oscillators were chosen because of their simple symmetry properties and also because they are easily represented in both radial and momentum spaces.

If tensor and spin-orbit interactions are neglected then the total orbital angular momentum (L) and the total spin (S) of the three-quark system are good quantum numbers. The ground-state multiplets then have $L = 0$ and $S = \frac{1}{2}$ or $S = \frac{3}{2}$. The various members of these multiplets depend upon the flavor content of each baryon. Table I shows some of our results for $S = \frac{1}{2}$ and $S = \frac{3}{2}$ states and the corresponding experimental assignments. In column 1 are the experimental values and in the third column ($\epsilon = 0$) the results for the Hamiltonian of Eq. (2) when $a = b = 0.50$ (color hypothesis). The theoretical results are all slightly high rela-

TABLE I. Eigenvalues of the three-quark Hamiltonian assuming the color hypothesis.

System	Experiment Mass (GeV)[a]	Experiment Width (GeV)[a]	Theory Mass (GeV) $\epsilon = 0$	Theory Mass (GeV) $\epsilon = 0.182$
$S = 1/2$:				
N	0.939	–	1.005	0.939
Λ	1.115	–	1.165	1.108
Σ	1.193	–	1.248	1.198
Ξ	1.317	–	1.360	1.316
Λ_c	2.257 ± 0.010	–	2.283	2.249
Σ_c	2.426 ± 0.012	–	2.488	2.465
Λ_b	?	–	5.596	5.574
Σ_b	?	–	5.859	5.846
$S = 3/2$:				
Δ	1.232	0.115	1.321	1.283
Σ^*	1.385	0.035	1.448	1.416
Ξ^*	1.530	0.010	1.570	1.543
Ω	1.672	–	1.687	1.663
$\Sigma_c{}^*$	?	?	2.557	2.536
$\Sigma_b{}^*$	?	?	5.877	5.862

[a] For baryons with more than one charge state the mean value for the charge multiplet is given.

tive to experiment with deviations ranging from about 60 MeV for the nucleon to about 10 MeV for the Ω.

In order to test the sensitivity of our calculations to the values of a and b we varied a and b by +10% individually and in combination. Table II shows our results for three typical particles (N, Δ, Ω). Increasing a by 10% alone, corresponding to 10% more attraction in the short-range potential, leads to large downward shifts of 100–160 MeV. Increasing b by 10% alone, corresponding to 10% more repulsion in the linear confinement, yields large upward shifts of 60–80 MeV. Increasing both a and b by 10% yields a large downward shift for the nucleon $(S=\frac{1}{2})$ of 100 MeV and a smaller downward shift (30–40 MeV) for the Δ and Ω ($S=\frac{3}{2}$ states). We conclude that the color hypothesis with $a=b=\frac{1}{2}$ is remarkably well satisfied in the present model. Small variations in a and b (~5%) would improve the agreement with experiment but an alternative argument is more appealing.

In obtaining a parametric $q\bar{q}$ interaction [Eq. (1)] we dropped explicit retardation terms. Momentum-dependent terms of the type

$$V(r_{ij})\vec{p}_i \cdot \vec{p}_j$$

do occur[6] in the reduction of a Bethe-Salpeter equation to a Pauli form. For a two-body system $\vec{p}_i = -\vec{p}_j = \vec{p}$ in its center-of-momentum frame and terms depending upon p^2 are presumably already included to a good approximation by small changes in the effective masses of the quarks. However, for the three-quark system the center-of-momentum frame has $\vec{p}_1 = -\vec{p}_2 - \vec{p}_3$ and terms of the type

$$\Delta V = -V(r_{12})p_3{}^2/4$$

appear which are not accounted for by the simple approach leading to Eq. (2).

In order to estimate ΔV we have approximated it by replacing V by ϵT (virial approximation) where T is the relativistic kinetic energy of particle 3 and ϵ is a parameter. We have also used finite-size quarks to average ΔV consistently with our $\langle V_{\mathrm{SR}}\rangle$ averaging procedure which avoids divergence problems at large momenta or short distances. Ths slightly modified Hamiltonian

$$H_B' = H_B - \epsilon \sum_i \langle [(m_i{}^2 + p_i{}^2)^{1/2} - m_i] P_i{}^2/4\rangle$$

was diagonalized for a few values of the parameter ϵ.

Choosing $\epsilon = 0.182$ and $a=b=\frac{1}{2}$ yields the nucleon mass at 0.939 GeV and all other results are then determined with no new parameters. The results are shown in the last column of Table I. The overall agreement with experiment for all known states is excellent and suggests that the retardation effect is a more appealing correction than small arbitrary changes in a and b.

In summary the present calculations yield strong support for the following statements:

TABLE II. Sensitivity of eigenvalues to deviations from the color hypothesis.

System	Mass, $\epsilon = 0$ (GeV) $a=0.50$, $b=0.50$	$a=0.55$, $b=0.50$	$a=0.50$, $b=0.55$	$a=0.55$, $b=0.55$
N	1.005	0.842	1.066	0.900
Δ	1.321	1.213	1.393	1.285
Ω	1.687	1.582	1.744	1.640

(1) The relationship of a factor of 2 between quark-antiquark and quark-quark interactions given by the color hypothesis is satisfied to the order of 5%. This relation holds for both the short-range and long-range confinement terms.

(2) The three-quark system is dominated by pairwise interactions. The effects of multigluon vertices as an additional three-body interaction are less than the order of 5% of the pair interactions. If retardation effects are included then the three-body interaction must have matrix elements which are less than the order of 20 MeV.

The first conclusion is important not only as a verification of the color hypothesis but also because it is needed if the potential model is to be used between quarks in different hadrons where the long-range linear terms must exactly cancel. Since it appears that Van der Waals interactions between hadrons do not exist[7] within the nonperturbative potential model used here there is now a real hope for a calculable model of quark dynamics for several hadron-hadron systems. This hope will be enhanced if we can assume that quarks only interact pairwise as suggested by our second conclusion. A more realistic picture will emerge, however, only when $q\bar{q}$ creation and annihilation processes are included in addition to the "primitive" states (q^3 and $q\bar{q}$ for baryons and mesons) calculated here.

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