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CP Nonconservation without Elementary Scalar Fields

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Dynamically broken gauge theories of electroweak interactions provide a natural mechanism for generating *CP* nonconservation. Even if all vacuum angles are unobservable, strong *CP* nonconservation is not automatically avoided. In the absence of strong *CP* nonconservation, the neutron electric dipole moment is expected to be of order 10^{-24} $e\cdot$cm.

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In this Letter, we show that there is a natural mechanism for generating *CP* nonconservation in dynamically broken electroweak gauge theories. Our proposal requires no new gauge interactions beyond those discussed previously.[1-4] Spontaneous *CP* nonconservation can appear when we carry out Dashen's[5] procedure to identify the correct chiral vacuum. Whether *CP* nonconservation occurs is determined in principal by *only* the gauge group and the fermion representation content of the theory.

These theories have no elementary scalar fields and no fermion bare masses. Even though there are no observable vacuum angles,[6] strong *CP* nonconservation[7] is not automatically avoided.[8] We state a criterion for the absence of strong *CP* nonconservation; if it is satisfied, *CP*-nonconserving phases in the quark mass matrix are naturally suppressed by a factor of order 10^{-9}. Additional *CP* nonconservation appears in the electroweak interaction and in the gauge interaction responsible for chiral symmetry breaking. We predict that the electric dipole moment of the neutron is of order 10^{-24} $e\cdot$cm.

In a theory of weak interactions without elementary scalar fields, a new gauge interaction[1,2] ("hypercolor," with gauge group G_H) is required, in addition to the familiar color [$G_C = \mathrm{SU}(3)$] and electroweak [$G_W = \mathrm{SU}(2)\otimes \mathrm{U}(1)$] interactions. Hypercolor becomes a strong interaction at mass scale $m_H \simeq 1$ TeV, and drives the breaking of G_W down to $\mathrm{U}(1)_{\mathrm{EM}}$.

A theory with gauge group $G_H\otimes G_C\otimes G_W$ alone cannot be realistic. A "sideways" interaction[3,4] (with group G_S) is needed to break explicitly all chiral symmetries not gauged by G_W. G_S is dynamically broken to a subgroup containing $G_H\otimes G_C$ at a mass scale $m_S \simeq 100$ TeV. (We need not speculate here on the origin of the G_S breakdown.)[4,9,10]

All fermions are in at most four irreducible representations of G_S.[3] In the effective gauge theory which describes physics below 100 TeV, each of these representations transforms reducibly under $G_H\otimes G_C$. If we neglect the broken sideways interactions and the weak interactions, the $G_H\otimes G_C$-invariant effective Hamiltonian $\mathcal{H}_0$ respects a global (chiral) flavor-symmetry group G_f. G_W is a subgroup of G_f.

When hypercolor and color become strong, G_f is dynamically broken to a subgroup S_f. Many Goldstone bosons result. Three of these are absorbed by the weak $W^\pm$ and Z^0 bosons.[1,2] The remaining Goldstone bosons acquire mass from the chiral-symmetry-breaking perturbation $\mathcal{H}'$ generated by the weak and sideways interactions.

The ground state of $\mathcal{H}_0$ is highly degenerate; the vacua are parametrized by the coset space

G_f/S_f. The perturbation $\mathcal{H}'$ lifts the degeneracy and picks out the true chiral-perturbative vacuum, the limit of the ground state of $\mathcal{H}_0+\epsilon\mathcal{H}'$ as $\epsilon\to 0$. As Dashen[5] explained, we can identify the correct vacuum by minimizing an effective potential which, in lowest-order perturbation theory, is

$$V(g)=\langle\Omega|U^{-1}(g)\mathcal{H}'U(g)|\Omega\rangle. \qquad (1)$$

Here $g\in G_f$, $U(g)$ represents G_f in the Hilbert space of states, and $|\Omega\rangle$ is the S_f-invariant vacuum. In the calculations described below, it is convenient to regard the symmetry group of the vacuum as fixed. Then, from among all G_f-equivalent perturbation $\mathcal{H}'(g)=U^{-1}(g)\mathcal{H}'U(g)$, we choose that one which minimizes the energy of the S_f-invariant vacuum $|\Omega\rangle$.

Now we see how spontaneous CP nonconservation can arise naturally. We assume that the fermion representation content under G_S vacuum angles[6] can be simultaneously rotated to zero, and hence are unobservable. In particular, G_S might be simple.[11] We further assume that the breaking of G_S does not introduce CP nonconservation, so that the effective gauge theory below 100 TeV is CP invariant.[12,13] Thus the vacuum $|\Omega\rangle$ and the perturbation $\mathcal{H}'$ are CP invariant, and the effective potential is CP symmetric. However, the energy might be minimized by a CP-nonconserving $\mathcal{H}'(g)$; then $V(g)$ has a degenerate minimum, and CP is spontaneously broken.[14] The minimum of $V(g)$, and whether spontaneous CP nonconservation occurs, are determined once the pattern of G_S and G_f breaking is known.

The approximate flavor group G_f is determined by the $G_H\otimes G_C$ representation content of the fermions. We will assume that all nontrivial representations of $G_H\otimes G_C$ are complex, that the color and hypercolor interactions are vectorial, and that the maximal isospin is left unbroken when hypercolor and color get strong. [These assumptions ensure that G_W breaks down to $U(1)_{EM}$, and that the relation $M_W/M_Z=\cos\theta_W$ is satisfied.[2]] Then the fermions may be denoted $\psi_{Lri}{}^{(\rho)}$, $\psi_{Rri}{}^{(\rho)}$. The index ρ identifies the irreducible representation $\mathfrak{T}^{(\rho)}$ according to which $\psi^{(\rho)}$ transforms under $G_H\otimes G_C$. The gauge group $G_H\otimes G_C$ acts on the index i. The index r ($r=1,2,\ldots,n_\rho$) labels the various flavors of fermions which transform as the representation $\mathfrak{T}^{(\rho)}$.

The flavor group is

$$G_f=\prod_\rho[SU(n_\rho)\otimes SU(n_\rho)\otimes U_V{}^\rho(1)]\otimes U_A(1)\text{'s}, \qquad (2)$$

which breaks down to $S_f=\prod_\rho[SU(n_\rho)\otimes U_V{}^\rho(1)]$.

Under G_f, the fermions transform as

$$\begin{aligned}\psi_{Lr}{}^{(\rho)}&\to W_{r'r}{}^{L(\rho)}\psi_{Lr'}{}^{(\rho)},\\ \psi_{Rr}{}^{(\rho)}&\to W_{r'r}{}^{R(\rho)}\psi_{Rr'}{}^{(\rho)},\end{aligned} \qquad (3)$$

where $W^{L(\rho)}$ and $W^{R(\rho)}$ are unitary $n_\rho\times n_\rho$ matrices. If S_f is the diagonal subgroup with $W^L=W^R$, then the elements of G_f/S_f can be labeled by a set of unitary matrices $\{W^{(\rho)}=W^{L(\rho)\dagger}W^{R(\rho)}\}$.

Each representation $\mathfrak{T}^{(\rho)}$ of $G_H\otimes G_C$ has a hypercolor and color anomaly given by

$$\partial^\mu J_{5\mu}{}^{(\rho)}=T_\rho{}^H(g_H{}^2/8\pi^2)\,\mathrm{tr}F_H\tilde{F}_H + T_\rho{}^C(g_C{}^2/8\pi^2)\,\mathrm{tr}F_C\tilde{F}_C, \qquad (4)$$

where $T_\rho{}^H$ ($T_\rho{}^C$) is the trace of the square of the hypercolor (color) generators in the representation $\mathfrak{T}^{(\rho)}$. (We have assumed that G_H is simple.) Only those linear combinations of the $J_{5\mu}{}^{(\rho)}$'s with vanishing hypercolor and color anomalies generate $U_A(1)$ symmetries which are included in G_f. Thus the $W^{(\rho)}$'s satisfy the constraint

$$\prod_\rho[\det W^{(\rho)}]^{T_\rho{}^H}=\prod_\rho[\det W^{(\rho)}]^{T_\rho{}^C}=1. \qquad (5)$$

The weak interactions play a secondary role in determining the minimum of the effective potential[15]; we need consider only the broken sideways interactions. We integrate out the massive sideways gauge bosons to obtain a $G_H\otimes G_C\otimes G_W$-invariant effective Lagrangian. The leading G_f-breaking terms in the effective Lagrangian are four-fermion operators. Higher-dimension operators are suppressed by additional powers of $m_s{}^{-1}$. We simplify the discussion by assuming that $SU(2)_W$ commutes with G_S, so that there is an exact global U(1) symmetry which distinguishes weak doublets from weak singlets. After Fierz rearrangements, the most general four-fermion operator invariant under $G_H\otimes G_C\otimes G_W$ and the global U(1) which contributes to the effective potential has the form[16]

$$\mathcal{H}'=\Gamma^{\rho\sigma,a}_{rr'ss'}\bar{\psi}_{Lri}{}^{(\rho)}\psi_{Rr'j}{}^{(\rho)}\bar{\psi}_{Rsk}{}^{(\sigma)}\psi_{Ls'l}{}^{(\sigma)}t^{\rho\sigma,a}_{ijkl}. \qquad (6)$$

Here the $t^{\rho\sigma,a}_{ijkl}$ are $G_H\otimes G_C$-invariant tensors, indexed by a, and $\Gamma^{\rho\sigma,a}_{rr'ss'}=O(m_s{}^{-2})$ is a G_W-invariant tensor which can be calculated to arbitrary order in the sideways coupling $g_s{}^2$. To lowest order in $g_s{}^2$, we have $\mathcal{H}'=g_s{}^2(\mu_s{}^{-2})_{ab}J_L{}^{\mu a}J_{R\mu}{}^b$, where $\mu_s{}^2$ is the massive sideways gauge boson mass matrix, and the $J_{L(R)}{}^{\mu a}$ are the left- (right-) handed broken sideways currents. This interaction can be Fierz transformed into the form of Eq. (6).

To compute the effective potential, we note that the S_f-invariant vacuum $|\Omega\rangle$ has the property

$\langle\Omega|\bar\psi_{Lri}{}^{(\rho)}\psi_{Rr'j}{}^{(\sigma)}|\Omega\rangle=\delta^{\rho\sigma}\delta_{rr'}\delta_{ij}\Delta^\rho$. Therefore S_f invariance implies that

$$\langle\Omega|\bar\psi_{Lri}{}^{(\rho)}\psi_{Rr'j}{}^{(\rho)}\bar\psi_{Rsk}{}^{(\sigma)}\psi_{Ls'l}{}^{(\sigma)}t_{ijkl}^{\rho\sigma,a}|\Omega\rangle$$
$$=\Delta^{\rho\sigma,a}\delta_{rr'}\delta_{ss'}+\Delta'^{\rho,a}\delta^{\rho\sigma}\delta_{rs'}\delta_{r's}. \qquad (7)$$

The second term on the right-hand side of Eq. (7) is G_f invariant. Hence, the leading term in the effective potential is

$$V(W)=\Delta^{\rho\sigma,a}\Gamma_{rr'ss'}^{\rho\sigma,a}W_{rr'}{}^{(\rho)}W_{ss'}{}^{(\sigma)\dagger}+\text{const.} \qquad (8)$$

Hermiticity and CP invariance of the effective Hamiltonian require $\Gamma_{rr'ss'}^{\rho\sigma,a}=\Gamma_{s's r'r}^{\sigma\rho,a}=\Gamma_{rr'ss'}^{\rho\sigma,a*}$. CP invariance of the vacuum $|\Omega\rangle$ implies $\Delta^{\rho\sigma,a}=\Delta^{\rho\sigma,a*}$. Therefore, $V(W)=V(W^*)$. In general, we expect that for some range of sideways gauge-boson masses, the minimum of $V(W)$ occurs for W complex, $W\neq W^*$. Then the minimum is degenerate, and CP is spontaneously broken.

Now we minimize $V(W)$ to determine the correct chiral perturbation $\mathcal{H}'(W)$. $V(W)$ is stationary subject to the constraint in Eq. (5) if[17]

$$(M^{(\rho)}-M^{(\rho)\dagger})\Delta^\rho=i(\nu_H T_\rho{}^H+\nu_C T_\rho{}^C)\underline{1}, \qquad (9)$$

where

$$M_{rr'}{}^{(\rho)}\Delta^{(\rho)}=W_{rm}{}^{(\rho)}(\partial V/\partial W_{r'm}{}^{(\rho)})$$
$$=\Delta^{\rho\sigma,a}\Gamma_{r'mss'}^{\rho\sigma,a}W_{rm}{}^{(\rho)}W_{ss'}{}^{(\rho)\dagger}.$$

Here ν_H and ν_C are Lagrange multipliers. [Taking the trace of both sides of Eq. (9) and summing on ρ we obtain a relation between ν_H and ν_C. If G_S is simple, then $\nu_H=-\nu_C$.]

If we include contributions from higher-dimension operators to $V(W)$, the condition for an extremum of $V(W)$ is still of the form given in Eq. (9), but with a modified matrix $M^{(\rho)}$. For quarks, this matrix $M^{(q)}$ differs from the "current-algebra" quark mass matrix by approximately one part in 10^9. This relation holds because the mass $m_C\sim 300$ MeV at which color becomes strong is small compared to the mass $m_H\sim 1$ TeV at which hypercolor becomes strong.

Physics below $m_H\sim 1$ TeV can be described by an effective theory involving only quarks, leptons, gluons, electroweak bosons, and pseudo-Goldstone bosons. This effective Lagrangian is obtained by integrating out hypergluons and hyperfermions, as well as massive sideways bosons. We must also sum up all hard (order m_H) gluon exchanges. A series of G_C-invariant operators is generated. The operators which contribute to the effective potential are $A_{r'r}\bar q_{Lr}q_{Rr'}$, $B_{r'r}\bar q_{Lr}\sigma_{\mu\nu}\times\lambda^a q_{Rr'}G^{\mu\nu a}$, $C_{rr'ss'}\bar q_r q_{r'}\bar q_s q_{s'}$, and higher dimension operators. The coefficients A, B, and C depend on the hyperfermion $W^{(\rho)}$'s. A is the quark mass matrix; it contributes $2\Delta^q\,\mathrm{Re\,Tr}(W^{(q)}A)$ to $V(W)$. Relative to this term, the contribution from four-quark operators is suppressed by $(m_C/m_H)^3\sim 10^{-10}$. The operator $\bar q_L\sigma_{\mu\nu}\lambda^a q_R G^{\mu\nu a}$ is generated by graphs involving one hard-gluon exchange[15]; its contribution is suppressed by $[\alpha_C(m_H)/2\pi](m_C/m_H)^2\sim 10^{-9}$. Hence, the part of $V(W)$ involving $W^{(q)}$ is $2\Delta^q\,\mathrm{Re\,Tr}W^{(q)}A[1+O(10^{-9})]$, and therefore $M^{(q)}=W^{(q)}A[1+O(10^{-9})]$.

Equation (9) is solved by several sets of unitary matrices $W=\{W^{(\rho)}\}$. From all solutions, we choose the one (or ones) which minimizes $V(W)$. The W which minimizes the effective potential will satisfy one of three conditions:

(i) $W=W^*$.—In this case, CP invariance is not spontaneously broken, and no CP nonconservation occurs at all.

(ii) $W\neq W^*$, $\nu_C\neq 0$.—In this case, strong CP nonconservation occurs. The effective Hamiltonian contains the CP-nonconserving term $\frac14(\nu_C/\Delta^q)\bar q i\gamma_5 q$; a current-algebra calculation[18] of the neutron electric dipole moment D_n yields $D_n\sim 2\times10^{-16}(\nu_C/m_u\Delta^q)$ $e\cdot$cm, where $m_u\sim 5$ MeV is the up quark mass. The natural scale of $\frac14\nu_C/\Delta^q$ is of order m_u and so we expect $D_n\simeq 10^{-15}$ $e\cdot$cm, which exceeds the experimental bound[19,20] by a factor of 10^9.

(iii) $W\neq W^*$, $\nu_C=0$.—In this case, CP invariance is spontaneously broken, but there is no large contribution to D_n. The coefficients B and C have phases of order 1, but since $M^{(q)}$ is Hermitian, $W^{(q)}A$ is Hermitian up to corrections of order 10^{-9}. The anti-Hermitian part of the quark mass matrix contributes of order 10^{-24} $e\cdot$cm to D_n. A similar contribution comes from the operator $\bar q_L\sigma_{\mu\nu}q_R F^{\mu\nu}$, where $F^{\mu\nu}$ is the photon field. Its coefficient is expected to have a phase of order 1 and a magnitude $[\alpha_C(m_H)/2\pi]\times(m_u/m_H{}^2)e\simeq 10^{-24}$ $e\cdot$cm.[15,21] The Lagrange multiplier ν_C vanishes if the minimum of the effective potential remains a stationary point when we remove the constraint that $U_A(1)$ rotations with a color anomaly are not allowed.

In any given model, the dynamics determines which possibility is realized. Only models of type (iii) can have CP-nonconserving interactions consistent with experiment.

In perturbation theory about the true chiral vacuum, the $W^{(\rho)}$'s appear explicitly in the weak and sideways currents. Minimizing the effective potential determines $W^{(\rho)}=W^{L(\rho)\dagger}W^{R(\rho)}$, but does not determine $W^{L(\rho)}$ and $W^{R(\rho)}$ separately. For quarks, we define $W^{L(q)}$ and $W^{R(q)}$ by demanding that the mass matrix $W^{(q)}A$ be diagonal. The quark matrices $W^{L,R(q)}$ commute with electric charge; there are unitary matrices $W_u^{L,R}$, $W_d^{L,R}$ for up and down quarks. The Kobayashi-Maskawa[22] mixing matrix is $W_u^{L\dagger}W_d^{L}$. If it contains phases which cannot be removed by redefining the phases of the quark fields, then the weak interactions violate CP invariance.

Mixing matrices appear in the broken sideways currents also. Typically, phases in these matrices cannot be absorbed by redefining fields, because both left-handed and right-handed fermions transform nontrivially under G_S. Sideways gauge bosons can couple to flavor-changing neutral currents. If the operator $C\bar{s}d\bar{s}d$ occurs in the effective interaction, ImC must be suppressed. If C is comparable to the coefficient of the term which generates the down quark mass,[10,23] and $\arg C \approx 1$, this operator induces a CP-nonconserving part of K^0-$\bar{K}^0$ mixing which is larger by 10^4 than what is observed.

The effective Lagrangian below 1 TeV contains pseudo-Goldstone bosons whose couplings to fermions can be CP nonconserving. Exchange of these bosons can generate milliweak CP nonconservation.[24]

Spontaneous CP nonconservation typically does not occur if the sideways interaction is vectorial.[15] But the sideways interaction must be nonvectorial to generate realistic quark masses.[3,4,23] Toy models in which spontaneous CP nonconservation occurs will be exhibited in Ref. 15.

We have proposed a natural mechanism for generating CP nonconservation in a gauge theory without elementary scalar fields or fermion bare masses. Although the strong CP problem is not automatically solved, strong CP nonconservation may be avoided in particular models. However, CP nonconservation in the broken sideways interactions is unavoidable, if weak CP nonconservation occurs at all. Hence, if the electric dipole moment of the neutron is not discovered soon,[20] the general approach to CP nonconservation which we advocate in this paper will become untenable.

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