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Quark Magnetic Moments and $E1$ Radiative Transitions in Charmonium

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In the long-wavelength limit, $\Gamma(\psi' \to \gamma + \chi_J) = \text{const} \times (2J+1) p_\gamma^3$ and $\Gamma(\chi_J \to \gamma + \psi) = \text{const} \times p_\gamma^3$. The corrections to these expressions of order p_γ/m_c (m_c is the mass of the charmed quark) are calculated. These corrections are found to be proportional to $\langle \vec{L} \cdot \vec{S} \rangle_{\chi_J} \times \kappa$, where $\langle \vec{L} \cdot \vec{S} \rangle_{\chi_J} = 1, -1, -2$, for $J = 2, 1, 0$, and κ is the anomalous magnetic moment of the quark. Angular distributions of photons in the decays $\psi' \to \gamma\chi$ and $\chi \to \gamma\psi$ also are predicted; small but probably measurable deviations from the pure $E1$ limit are found.

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The decays $\psi' \to \gamma\chi$ and $\chi \to \gamma\psi$ provide valuable information about the electromagnetic properties of heavy quarks, about nonrelativistic bound state models,[1] and about the competing processes

$$\chi_J \to \text{hadrons}, \qquad (1)$$

for which there exist predictions based on quantum chromodynamics (QCD).[2]

In the long-wavelength limit of the nonrelativistic quark model,

$$\Gamma(\psi' \to \gamma + \chi_J) = C'(2J+1) p_\gamma^3, \qquad (2)$$

$$\Gamma(\chi_J \to \gamma + \psi) = C p_\gamma^3, \qquad (3)$$

where p_γ is the photon momentum and C', C are known constants.[3] The ratios of rates implied by (2) may be confronted with experiment directly; published data[4] on $\psi' \to \gamma + \text{anything}$ are consistent with them but with large quoted errors. Little is known about the validity of the ratios implied by (3). If (2) and (3) are assumed, one can extract from measured values of $B(\psi' \to \gamma\chi \to \gamma\gamma\psi)$ the ratios of χ total widths, which are expected to be in the range of several megaelectronvolts.[2,5] The ratio $\rho \equiv \Gamma(\chi_0 \to \text{all})/\Gamma(\chi_2 \to \text{all})$ appears experimentally[6,7] to be $\rho \gtrsim 10$, whereas QCD predicts[2] the ratio of two-gluon (gg) emission rates to be

$$[\Gamma(\chi_0 \to gg)/\Gamma(\chi_2 \to gg)](\approx \rho) = \tfrac{15}{4}. \qquad (4)$$

This potential discrepancy has led us to reexamine the basis for Eqs. (2) and (3). Others[8] are investigating the interesting possibility that strong radiative corrections dramatically alter prediction (4).[9]

The predictions (2) and (3), as well as photon angular distributions expected in the same limit, have been discussed previously.[10-17] Possible corrections also have been suggested,[14-17] but (with the exception of Ref. 17, with which we differ in details) these have not yet been calculated in an explicit expansion in p_γ/m_c, where m_c is the charmed-quark mass. It is the purpose of this note to present the results of such a calculation.

The leading terms of the interaction Hamiltonian describing photon emission by a quark may be written in the form[17-19]

$$H_I = -\frac{|e|e_c}{2m_c}(\vec{A}^*\cdot\vec{p}+\vec{p}\cdot\vec{A}^*) - \mu\vec{\sigma}\cdot\vec{H}^* - \frac{1}{2m_c}\left(\mu - \frac{|e|e_c}{4m_c}\right)(\vec{\sigma}\cdot[\vec{E}^*\times\vec{p}] - \vec{\sigma}\cdot[\vec{p}\times\vec{E}^*]). \tag{5}$$

(We have omitted some terms[18] which cancel for $c\bar{c}$ systems.)
Here $e_c = \frac{2}{3}$ is the quark charge, and μ is its magnetic moment:

$$\mu = (|e|e_c/2m_c)(1+\kappa) \tag{6}$$

The quark's anomalous magnetic moment is κ.

We will be interested in the matrix elements of H_I to order p_γ^2. The dominant term, of order p_γ, is the first one, which is just the electric dipole interaction $-|e|e_c\vec{r}\cdot\vec{E}^*$. It contributes to $E1$ transitions in $\psi'\to\gamma\chi$ and $\chi\to\gamma\psi$. The second term contributes in order p_γ^2 to both $E1$ and $M2$ transitions. The third, the spin-orbit term (which arises from the Foldy-Wouthuysen reduction[20] of the Dirac Hamitonian), contributes in order p_γ^2 to $E1$ transitions.

We shall calculate all decays for a photon of right-handed circular polarization moving along the $+z$ direction:

$$\vec{A}(\vec{r}) = -\frac{1}{\sqrt{2}}(1, i, 0)e^{i(\vec{k}\cdot\vec{r}-\omega t)}. \tag{7}$$

The $\chi\to\gamma\psi$ or $\psi'\to\chi\gamma$ decays may be described by helicity amplitudes A_λ or A_λ', in which λ labels the projection of the spin of the χ state along the $+z$ or $-z$ axis, respectively. The radiative widths are given in terms of these amplitudes by

$$\Gamma(\psi'\to\gamma\chi) = \frac{p_\gamma^3}{3}\sum_{\lambda\ge 0}|A_\lambda'|^2, \tag{8}$$

$$\Gamma(\chi\to\gamma\psi) = \frac{p_\gamma^3}{2J+1}\sum_{\lambda\ge 0}|A_\lambda|^2. \tag{9}$$

We define

$$\epsilon \equiv \xi p_\gamma/4m_c, \tag{10}$$

where $\xi=-1$ for $\psi'\to\gamma\chi$, $\xi=+1$ for $\chi\to\gamma\psi$. The results are presented in Table I. We have decomposed the contributions into the dominant $E1$ piece (called $E1'$ in Ref. 15) and the smaller $E1$ and $M2$ pieces from the second and third terms in (5).

The smaller $E1$ pieces in Table I are proportional to the anomalous magnetic moments.[21] These pieces are the only ones that can contribute to deviations from the ratios (2) and (3) in order ϵ. The $M2$ contributions are incoherent with the $E1$ contributions in the sums (8) and (9) so that they can only affect the rates to order ϵ^2.

The second interesting feature of the $O(\epsilon)$ $E1$ contributions in Table I is their proportionality to $\langle\vec{L}\cdot\vec{S}\rangle$ (=1, −1, −2 for J = 2, 1, 0). This may be seen independently by explicit reference to multipole decompositions.[22]

The modifications of the rates, to $O(\epsilon)$, are then

$$\Gamma(\epsilon)/\Gamma(0) = 1 + 2\epsilon\kappa\langle\vec{L}\cdot\vec{S}\rangle_\chi. \tag{11}$$

Deviations of this form from the rates (2) and (3) imply an anomalous magnetic moment of the charmed quark.[23] The observed rate[7] for $\psi\to\gamma\eta_c$ is no larger than theoretical expectations[1] and may be somewhat smaller, suggesting $\kappa\le 0$. If $\kappa<0$, the rates for $\psi'\to\gamma\chi_2$ are enhanced, while those for $\psi'\to\gamma\chi_1$ and $\psi'\to\gamma\chi_0$ are depressed, with respect to Eq. (2). We see a (statistically unconvincing) trend in this direction in present data.[4,5]

The correction term in (11) is of opposite sign for the decays $\psi'\to\gamma\chi$ and $\chi\to\gamma\psi$. Such corrections, therefore, are unable to account for the

TABLE I. Helicity amplitudes A_λ' and A_λ for $\psi'\to\gamma\chi(\xi=-1)$ and $\chi\to\gamma\psi$ ($\xi=+1$). Here $\epsilon\equiv\xi p_\gamma/4m_c$. Overall factors of $(3C'/2)^{1/2}$ and $(3C/2)^{1/2}$ have been omitted (see Ref. 3).

	Dominant $E1$ piece		$E1$ piece		$M2$ piece
		$\chi(J=2)$			
$A_2=$	$\sqrt{6}\{1$	$+$	$\epsilon\kappa$	$+$	$\epsilon(1+\kappa)\}$
$A_1=$	$\sqrt{3}\{1$	$+$	$\epsilon\kappa$	$-$	$\epsilon(1+\kappa)\}$
$A_0=$	1	$+$	$\epsilon\kappa$	$-$	$3\epsilon(1+\kappa)$
		$\chi(J=1)$			
$A_1=$	$\sqrt{3}\{1$	$-$	$\epsilon\kappa$	$+$	$\epsilon(1+\kappa)\}$
$A_0=$	$\sqrt{3}\{1$	$-$	$\epsilon\kappa$	$-$	$\epsilon(1+\kappa)\}$
		$\chi(J=0)$			
$A_0=$	$\sqrt{2}\{1$	$-$	$2\epsilon\kappa\}$		

apparent suppression of the χ_0 contribution in $\psi' \to \gamma\chi \to \gamma\gamma\psi$ with respect to the χ_2 contribution.[24] The contradiction with the prediction (4) remains a problem to be resolved, if at all, by strong-interaction dynamics.[8]

The mixing[25] of the ψ' with the nearby $\psi''(3.77)$,[26] a $^3D_1$ state, also can modify the rates (2). The pattern is different, and is found by a straightforward calculation to be proportional to the matrix element of the tensor operator in the χ states, which may be taken as $\frac{2}{5}$ for $J=2$, -2 for $J=1$, and 4 for $J=0$. Thus a fit to the decay rates of the form

$$\Gamma(\psi' \to \gamma\chi_2) = 5C' p_\gamma^3(1 + xp_\gamma - \tfrac{2}{5}y), \tag{12}$$

$$\Gamma(\psi' \to \gamma\chi_1) = 3C' p_\gamma^3(1 - xp_\gamma + 2y), \tag{13}$$

$$\Gamma(\psi' \to \gamma\chi_0) = C' p_\gamma^3(1 - 2xp_\gamma - 4y), \tag{14}$$

can permit the separation of the effects we have discussed previously ($\sim x$) from S-D mixing effects ($\sim y$).

A sensitive test for quark magnetic moments is the effect of $M2$ contributions in Table I on photon angular distributions.[14,15] If θ (θ') is the angle between the photon and either lepton in the ψ (ψ') rest frame (under the assumption that the ψ' is produced by e^+e^- and the ψ decays to e^+e^- or $\mu^+\mu^-$), the predicted angular distributions are of the form

$$W(\theta, \theta') \sim 1 + \beta_J \cos^2(\theta, \theta'), \tag{15}$$

where, to first order in ϵ, we find $\beta_0 = 1$ and

$$\beta_2 = \tfrac{1}{13} + \tfrac{240}{169}\epsilon(1+\kappa), \tag{16}$$

$$\beta_1 = -\tfrac{1}{3} - \tfrac{16}{9}\epsilon(1+\kappa). \tag{17}$$

The present data appear compatible with deviations from the $\epsilon = 0$ limits by $|\Delta\beta| \lesssim 0.2$, corresponding to $|\epsilon(1+\kappa)| \lesssim 0.1$. This is not a strong constraint in view of Eq. (10). Modest increases in systematic precision should lead to detection of the $O(\epsilon)$ effects in Eqs. (16) and (17), unless $|1+\kappa| \ll 1$. In that case, however, the effects (11) of an anomalous moment on the decay rates should be clearly visible.

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[3]Specifically, $C' = (4\alpha/27)e_Q^2|\langle 2p|r|2s\rangle|^2$ and $C = (4\alpha/9)e_Q^2|\langle 1s|r|2p\rangle|^2$. Their specific values need not concern us here, though they tend to be somewhat overestimated in quark models (see, e.g., Ref. 1).

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[7]Peck, Ref. 5.

[8]J. Ellis informs us that corrections to Eq. (4) are being calculated by R. Barbieri *et al.*

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[20]See, e.g., J. D. Bjorken and S. Drell, *Relativistic Quantum Mechanics* (McGraw-Hill, New York, 1964), pp. 48–51.

[21]We are indebted to Al Wattenberg for very enlightening discussions on this point. The part of H_I in Eq. (5) relevant to our calculation transforms as $x - iy + (p_\gamma/4m_c)[z(\sigma_x - i\sigma_y) + (1+2\kappa)(x - iy)\sigma_z]$. For $\kappa = 0$, the terms in square brackets are clearly of $J = 2$ ($M2$) form.

[22]See, e.g., J. D. Jackson, *Classical Electrodynamics*

(Wiley, New York, 1975), 2nd ed., p. 758, Eq. (16.94), and John Babcock and Jonathan L. Rosner, Ann. Phys. (N.Y.) 96, 191 (1976), Eq. (6.8). A simple integration by parts displays the $\langle \vec{L}\cdot\vec{S}\rangle$ contribution explicitly.

[23]Donald A. Geffen and Warren J. Wilson, Phys. Rev. Lett. 44, 370 (1980). These authors have suggested that such effects could be larger for heavy quarks than for light ones, as a result of the couplings $\gamma \to (q\bar{q}) \to (3g) \to c\bar{c}$, where $q\bar{q}$ denote u, d, s. For the suggestion that u, d, s might have observable anomalous magnetic moments, see also G. Grunberg and F. M. Renard, Nuovo Cimento 33A, 617 (1976); A. Bohm and R. B. Teese, Phys. Rev. D 18, 330 (1978); A. N. Kamal, Phys. Rev. D 18, 3512 (1978). The effects of light-quark anomalous moments are found to be small in analyses of resonance photoproduction: see, e.g., Babcock and Rosner, Ref. 22, and John Babcock and Jonathan L. Rosner, Phys. Rev. D 18, 4027 (1978).

[24]Analogies with light-quark systems led to the conclusion in Refs. 14 and 15 that the decay $\chi_0 \to \gamma\psi$ might be totally suppressed. These analogies also lead, however, to large $M2$ contributions in contradiction with data from Refs. 5 and 6.

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