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Hierarchy of Cosmological Baryon Generation

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The hierarchy of baryon generation by multiple species of both gauge and Higgs bosons whose interactions do not conserve baryon number are considered. A procedure for computing the final baryon asymmetry in terms of the masses and coupling strengths of all the various species is presented. If the lightest gauge boson has a mass below 10^{15} GeV, then the final asymmetry depends only upon this boson and any lighter Higgs bosons.

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Recent theoretical work on grand unified theories (GUT's) of particle interactions has provided a new solution to an old problem in cosmology: a fundamental explanantion of the observed amount of matter in the universe, quantified as the baryon number–to–specific-entropy ratio kn_B/s, and the lack of any significant amount of antimatter in the observed portion of the universe up to the scale of clusters of galaxies.[1] The observed asymmetry has the value $kn_B/s = 10^{-10.8 \pm 1}$, the uncertainty primarily due to our poor knowledge of the baryon density.[2] It has been shown that, with (i) baryon-number–nonconserving processes mediated by superheavy bosons predicted by GUT's, (ii) C and CP nonconservations in the interactions of superheavy gauge or Higgs bosons, and (iii) a departure from thermal equilibrium (provided by the rapid expansion of the early universe), a baryon asymmetry can arise dynamically in an initially symmetrical universe.[3,4] Although two groups have considered in detail the effects of a single superheavy species,[5,6] an arbitrary GUT has more than one such species present. The purpose of this Letter is to outline a simple procedure which for many GUT's allows the straightforward calculation of the final asymmetry which evolves due to a whole hierarchy of superheavy species whose interactions do not conserve baryon number.

Towards that end, we first summarize the relevant results for the effects of a single species. In a general GUT which spontaneously breaks down to $[SU(3)]_c \otimes [SU(2)]_L \otimes U(1)$, there are five generic classes of superheavy bosons which couple to ordinary fermions and whose interactions do not conserve baryon number[7]: two isodoublet, color triplets of vector particles (XY with charges $\pm\frac{4}{3}$, $\pm\frac{1}{3}$; and $X'Y'$ with charges $\pm\frac{1}{3}$, $\mp\frac{2}{3}$); and three color triplets of scalar particles (H, an isosinglet with charge $\pm\frac{1}{3}$; S, an isosinglet with charge $\pm\frac{4}{3}$; and S', an isotriplet with charges $\pm\frac{4}{3}$, $\pm\frac{1}{3}$, $\mp\frac{2}{3}$). In a given GUT there may be several representatives of each class, each with its own mass and coupling strength. The XY and

$X'Y'$ bosons behave similarly and we refer to them generically as "gauge" bosons; the H, S, and S' Higgs bosons all behave similarly and we refer to them generically as "Higgs" bosons.

The effect of one given species on kn_B/s depends in general on its mass M, its coupling strength α, and on the size of the CP nonconservation in its decays. The dependence upon mass and coupling strength appears in the combination

$$K=(16\pi^3 g_*/45)^{-1/2}\alpha m_p/M$$
$$=(2.9\times10^{17}\ \text{GeV})\alpha M^{-1}(160/g_*)^{1/2}, \quad (1)$$

where g_* = total number of relativistic degrees of freedom $= g_{\text{bosons}}+\frac{7}{8}g_{\text{fermions}}$ [$g_*\simeq160$ in the minimal SU(5)], $m_p=G^{-1/2}=1.22\times10^{19}$ GeV, and typically, $\alpha_{\text{gauge}}=\frac{1}{45}$ and $\alpha_{\text{Higgs}}\simeq10^{-4}-10^{-6}$. K is approximately the ratio of the decay rate of the superheavy boson to the expansion rate when $T=M$. The important reactions for baryon generation are the decays (D) and inverse decays (ID) of superheavy bosons, and K determines whether or not these reactions are occuring on the expansion timescale. For $K\ll1$, D and ID do not occur, and a departure from equilibrium results; for $K\gg1$, D and ID do occur, and thermal equilibrium is almost maintained. We parametrize the CP nonconservation by ϵ, the mean net baryon number generated when a superheavy boson-antiboson pair decays.

Each species can generate a baryon asymmetry and/or damp a preexisting asymmetry, depending on these parameters. Damping occurs for $T\approx M$, primarily by ID, followed by D (e.g., $\bar{q}+\bar{q}\to X$, $X\to q+l$). Generation of an asymmetry is proportional to ϵ and occurs for $T\lesssim M$. Quantitatively, for gauge and Higgs bosons, respectively,

$$(kn_B/s)_i=(kn_B/s)_{i-1}\exp(-5.5K_i)+1.5\times10^{-2}\epsilon_i/[1+(16K_i)^{1.3}], \quad (2)$$

$$(kn_B/s)_i=(kn_B/s)_{i-1}\exp[-(0.26\text{ to }1.6)K_i]+0.5\times10^{-2}\epsilon_i/[1+(3K_i)^{1.2}]. \quad (3)$$

Here $(kn_B/s)_{i-1}$ is the preexisting asymmetry and $(kn_B/s)_i$ is the resultant asymmetry which develops because of species i, whose parameters are K_i, ϵ_i, and M_i. These are results of numerical integrations of the Boltzmann equations.[8]

The exponential damping term in Eqs. (2) and (3) represents the damping of a prior asymmetry by inverse decays. (In general, there are several modes of damping; this is the slowest of these.) However, because there are conserved quantities, certain types of asymmetries cannot be damped at all. These are modes accompanied by nonzero values of charge Q, weak isospin I_3, net baryon minus lepton number, $B-L$ (in some theories), and (for gauge particles only), "5-ness," which measures the net number of particles in the $\overline{5}$* representation of SU(5). We expect Q and I_3 to be identically zero from $t=0$ as these quantities are both gauged. Asymmetries with net 5-ness are not generated or damped by XY, $X'Y'$ bosons, but they can be by Higgs bosons. XY and $X'Y'$ bosons may also be able to damp asymmetries with nonzero 5-ness if they are aided by gauge or light Higgs bosons whose interactions do not nonconserve B but do not conserve 5-ness (e.g., W_R, the boson which mediates right-handed interactions). For simplicity we will assume that the net 5-ness of all asymmetries is zero, although it is possible that an asymmetry with net 5-ness could evolve and never be damped.[9] Any asymmetry with a nonzero value of a conserved quantity [e.g., in SU(5), $B-L$] must be either truly "initial" or generated by non-GUT processes, and cannot be damped. Such asymmetries would survive the GUT epoch and contribute to the asymmetry observed today.

The second term in Eqs. (2) and (3) represents the baryon asymmetry produced by species i. In the limit $K\ll1$ (complete nonequilibrium), the saturation value which arises, $(kn_B/s)=1.5\times10^{-2}\epsilon$ (gauge) or $(kn_B/s)=0.5\times10^{-2}\epsilon$ (Higgs), is the familiar result of the out-of-equilibrium decay scenario.[4] In this limit, there is no significant damping and the saturation production simply adds to the initial asymmetry. In the limit $K\gg1$ (equilibrium almost maintained), the baryon excess produced is $(kn_B/s)\approx1.5\times10^{-2}\epsilon(16K)^{-1.3}$ (gauge) or $(kn_B/s)\approx0.5\times10^{-2}\epsilon(3K)^{-1.2}$ (Higgs). The power laws are an approximation to the actual dependence $(Kz_f)^{-1}$, where z_f is the solution of $Kz_f^{7/2}\exp(-z_f)=1$ (Ref. 5). In this limit any preexisting asymmetry which has no projection onto the eigenmodes with conserved quantities is completely erased, and the final asymmetry depends only on the parameters K_i and ϵ_i.

Although these results were obtained by integrating the Boltzmann equations for a single species from the Planck time to a final temperature $T\ll M_i$, we note that all effects occur $T\approx M_i$. Thus, we can apply (2) and (3) repeatedly to take into account the effects of all species ("impulse approximation"). We order all superheavy boson species according to decreasing mass, $M_i < M_{i-1}$. The universe is assumed to begin with an initial asymmetry $(kn_B/s)_0$, either as a true initial condition or produced by pre-GUT processes, which has zero projection on the conserved quantities Q, I_3, $B-L$, and 5-ness. As the universe expands and T decreases, interactions involving the most massive superheavy boson (species 1) cause the asymmetry to change at $T \approx M_1$ so that when its effect on kn_B/s is complete ($T < M_1$) the new baryon asymmetry is $(kn_B/s)_1$, determined by Eq. (2) or (3). This is then used as the input for species 2, etc., continuing until the effect of the lightest species has been taken into account. Thus, the general evolution of kn_B/s is reduced to a sequence of evolutions $(kn_B/s)_i$ with preexisting $(kn_B/s)_{i-1}$.[10]

Next, we mention two scenarios in which we can be more definite about the final outcome of the hierarchy. Note from (1) that as mass decreases (for fixed α), K increases and the damping of the previous asymmetry is more and more efficient. If the lightest gauge boson is lighter than 10^{15} GeV, then previously existing asymmetries are damped by a factor of at least 10^{12}, making it impossible for earlier asymmetries to contribute appreciably to the observed $kn_B/s \approx 10^{-10.8}$. The value of kn_B/s after this epoch, $T < M_g$, will be

$$(kn_B/s)_g = 1.5\times 10^{-2}\epsilon_g(16K_g)^{-1.3}. \qquad (4)$$

Suppose now, as expected in SU(5), that there is one Higgs boson lighter than M_g but heavier than 3×10^{13} GeV. For $\alpha_H \lesssim 10^{-4}$, this gives $K_H < 1$, so that it will not reduce the gauge-produced asymmetry significantly, but will simply add on its own production. In this case,

$$(kn_B/s)_F = 1.5\times 10^{-2}\epsilon_g(16K_g)^{-1.3} + 0.5\times 10^{-2}\epsilon_H. \qquad (5)$$

Figure 1 shows the results of actual numerical integration of the Boltzmann equations with two superheavy species for $M_g = 10^{15}$ GeV, $K_g = 10$, $\epsilon_g \approx 10^{-6}$, $M_H = 10^{14}$ GeV, $K_H = 0.1$, and $\epsilon_H \approx 10^{-8}$. These numerical results agree well with Eqs. (4) and (5), and illustrate the validity of the sequential treatment.

If on the other hand, this lighter Higgs boson is light enough or couples strongly enough so that $K_H \gtrsim 10$, then it can reduce the gauge-generated asymmetry to an extent such that the final asymmetry depends only on K_H and ϵ_H,

$$(kn_B/s)_f = 0.5\times 10^{-2}\epsilon_H(3K_H)^{-1.2}. \qquad (6)$$

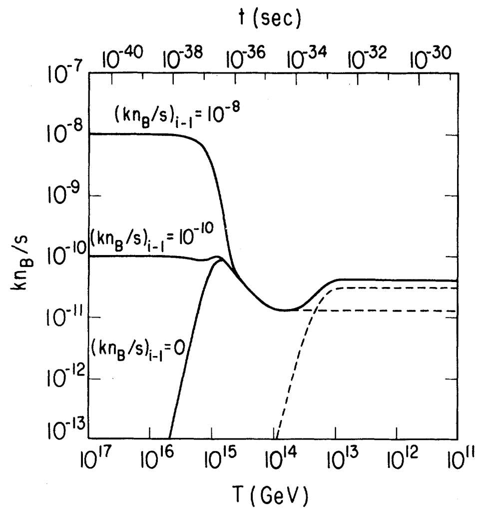

FIG. 1. The time development of kn_B/s under the effects of one gauge species ($M = 10^{15}$ GeV, $K = 10$, $\epsilon_g \cong 10^{-6}$) and one Higgs species ($M = 10^{14}$ GeV, $K = 0.1$, $\epsilon_H \cong 10^{-8}$) for three different prior asymmetries. The gauge particle interactions totally erase the prior asymmetry and produce kn_B/s equal to the value which continues across as a broken line [cf., Eq. (4)]. The contribution of the Higgs particle simply adds on later, as shown by the continuation of the solid curve. The asymmetry generated by the Higgs particle alone is also shown as a broken line. Note that the solid curve for $T \lesssim 10^{13}$ GeV is the sum of the two broken curves [cf., Eq. (5)]. The curves were produced by numerically integrating the Boltzmann equations with two superheavy species.

Finally, we mention some possible complications which might invalidate our sequential treatment. First, one superheavy species (X) might decay primarily into a lighter superheavy species (X') and due to CP nonconservation in its decays create an X'-$\bar{X}'$ asymmetry.[11] If for the lighter species $K_{X'} \lesssim 1$, then even if $\epsilon_{X'} = 0$, decays of the X''s can create a baryon asymmetry (if $K_{X'} \gg 1$, our results above apply). Also, some theories [e.g., SU(10)] have superheavy Majorana neutrinos in addition to the usual fermions, and these particles may play an important role in baryon generation.[12]

In the last few years, the cosmological production of the baryon asymmetry has been the topic of much discussion and speculation. In fact, with the strong damping of prior asymmetries by inverse decays, the existence of a superheavy gauge boson lighter than 10^{15} GeV (which could be inferred from observations of proton decay) would make cosmological baryon generation almost a necessity, unless an initial asymmetry existed with a conserved quantity (e.g., $B-L$ or 5-ness). Such an observation would also require matter-antimatter separation by mechanisms invoked in baryon symmetrical universes without baryon generation to occur after $T=10^{15}$ GeV, since inverse decays can damp local as well as global asymmetries. This must happen on scales much larger than $\sim ct$, the distance over which separation could have occurred since damping ceased ($T\sim 10^{15}$ GeV, $t\sim 10^{-36}$ s). Although the metric might be horizonless because of quantum gravity effects,[13] the distance over which effects could propagate since $T\sim 10^{15}$ GeV does not differ significantly from $\sim ct$, the usual horizon size.

Although we do not know which (if any) GUT is correct, an even larger question is that of the C and CP nonconservations in the superheavy system. In Eq. (5), with $M_g \simeq 3\times 10^{14}$ GeV and $M_{\rm H} \gtrsim 3\times 10^{13}$ GeV ($\alpha_{\rm H} \lesssim 10^{-4}$), we need $\epsilon_g \approx 10^{-5.7}$ or $\epsilon_{\rm H} \approx 10^{-8.5}$ to explain the observed asymmetry $kn_B/s \simeq 10^{-10.8}$. CPT and unitarity restrict $\epsilon \lesssim O(\alpha) \approx 10^{-2}$. In the minimal SU(5) (one $\underline{24}$ and one $\underline{5}$ of Higgs), it has been shown $\epsilon < 10^{-10}$ (Refs. 7, 11, and 14). However, the addition of just one more Higgs $\underline{5}$ again allows ϵ as large as $\sim 10^{-2}$ [the minimal SU(5) model also has difficulty explaining fermion mass ratios]. There is always the hope that ϵ can be related to the CP nonconservation in the K°-$\bar{K}^\circ$ (or analogous) systems, tying together the two observed nonconservations of matter-antimatter symmetry.

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