Tunneling Theory of Charge-Density-Wave Depinning

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Introduction of a correlation length into the tunneling theory of depinning of charge-density waves by an electric field leads to a threshold field and to an expression for non-Ohmic conductivity in excellent agreement with experiments on the linear-chain compounds $NbSe_3$ and TaS_3. An extension of the theory leads to results for the frequency dependence of the ac conductivity, also in agreement with experiment.

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Early measurements[1,2] of the remarkable non-Ohmic conduction observed in the linear-chain compound $NbSe_3$ suggested an expression for the field-dependent conductivity of the form

$$I/E = \sigma_a + \sigma_b \exp(-E_0/E). \tag{1}$$

The second term on the right-hand side is attributed to moving charge-density waves (CDW's) that appear below Peierls transitions at 58 and 142 K. This form suggested Zener tunneling across a pinning gap, but the magnitude of the gap required, $\mathcal{E}_g$, is much smaller than the thermal energy of individual electrons. I suggested[3] a modified expression based on tunneling by coherent CDW's which have energies orders of magnitude larger than those of electrons. The theory gave the following expression for E_0:

$$E_0 = \pi \mathcal{E}_g{}^2/4\hbar e^* v_F = \mathcal{E}_g/2\xi_0 e^*, \tag{2}$$

where $\mathcal{E}_g$ is the pinning gap in the electron spectrum in the absence of the Peierls gap, $e^*/e = m/M_F$ is the ratio of band mass to the Fröhlich mass of the CDW (which includes lattice motion), and $\xi_0 = 2\hbar v_F/\pi\mathcal{E}_g$ is the coherence distance analogous to that in superconductors.

Later Fleming and Grimes[4] showed that there is no added conductivity below a threshold field E_T and this has been confirmed by many other experiments.[5,6] This suggested that an impurity pinning model based on sliding friction of the CDW proposed by Lee and Rice[7] is the appropriate one. On the other hand, Brill *et al.*[5] showed that data on $NbSe_3$ could be fitted approximately by a modified tunneling formula[4] in which E is replaced by $E - E_T$ (for $E > E_T$) in the denominator of the exponent in (1).

I show here that a threshold field appears in a different way if one introduces a correlation length L for the CDW, across which the field must be applied to be effective in accelerating the wave. If, as is the case for $NbSe_3$, $L \ll 2\xi_0$ two corrections must be made in (1). First, according to the Zener diagram (Fig. 1), there is no tunneling unless $e^*EL > \mathcal{E}_g$, and for $E > E_T = \mathcal{E}_g/e^*L$ tunneling can occur only over a fraction $[1-(E_T/E)]$ of the length L. Further, since fields outside the length L are not effective one must replace $2\xi_0$ in (2) by L. The result of these corrections is to replace the probability for tunneling across the gap, $\exp(-E_0/E)$, by

$$P(x) = (1 - x^{-1})e^{-(1/x)}, \tag{3}$$

where $x = E/E_T$, giving

$$\sigma = \sigma_a + \sigma_b P(x). \tag{4}$$

Only one parameter, E_T, is involved when $L \ll 2\xi_0$.

The universal function (3), plotted in Fig. 2, gives an excellent fit to the field dependence of

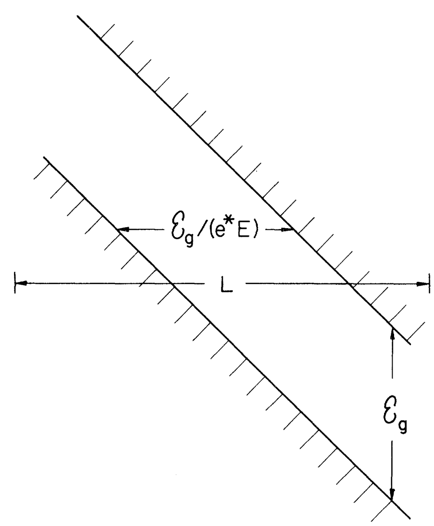

FIG. 1. Zener diagram showing one effect of a correlation length, L, on tunneling. The distance over which tunneling can occur is reduced by the gap length, $\mathcal{E}_g/e^*E$, giving an effective length $L - \mathcal{E}_g/e^*E$. There is a threshold field, $E_T = \mathcal{E}_g/e^*L$ below which tunneling cannot occur.

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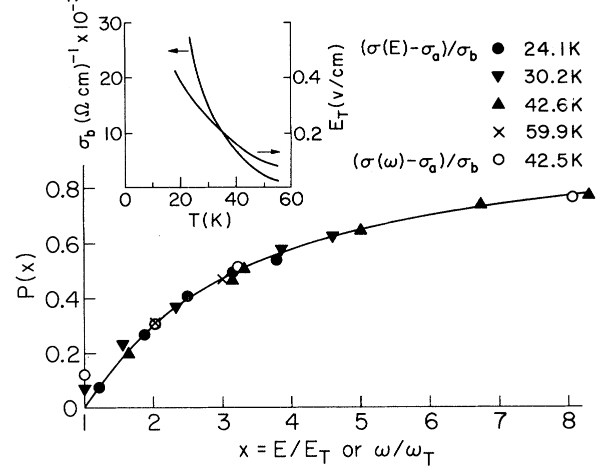

FIG. 2. Conductivity of $NbSe_3$ above threshold plotted in reduced units. The smooth curve is a plot of the tunneling probability $P(x)$ from Eq. (3). The dc data are not derived from experimental points but from smooth curves drawn through the points by Richard and Monceau (Ref. 6). The points for 59.9 K refer to the CDW for the 142 K transition, with $\sigma_b = 1500$ $(\Omega\ \mathrm{cm})^{-1}$ and $E_T = 2.75$ V/cm. The ac data are from experimental points in a plot of Grüner *et al.* (Ref. 8).

conductance in $NbSe_3$ taken at a variety of temperatures and for samples with varying impurity concentrations.[5,6] As an example, data of Richard and Monceau[6] on $NbSe_3$ at four different temperatures are plotted in reduced units in Fig. 2.

Values of the parameters σ_b and E_T can be derived either by fitting $\sigma = \sigma_a + \sigma_b P(E/E_T)$ to low field data or by plotting σ as a function of $1/E$ and extrapolating to $E = \infty$, with E_T derived from the limiting slope. Values derived by the two methods are in excellent agreement. Plots of σ_b and E_T as functions of temperature are given in the inset of Fig. 2. Values of L are estimated to be as large as 10^{-3} cm, so that tunneling occurs over very large distances near threshold.

In the case of impurity pinning, $\mathcal{E}_g$ is proportional and L inversely proportional to impurity concentration c_i, so that E_T varies as c_i^2 in agreement with experiment.[5] One expects in analogy with the expression for acceleration of the current in superconductivity theory[9] that $L = 2l$, where l is the mean free path of the electrons.

Commensurability pinning occurs in TaS_3, where a CDW forms along the chain direction with a wavelength four times the lattice spacing. Non-Ohmic conduction has been studied by Takoshima *et.*,[10] who find a threshold field, $E_T \sim 100$ V/cm, above which there is a rapid rise in conductances of several orders of magnitude.

As they point out, the pinning field can be derived from an expression for the commensurability energy derived by Lee, Rice, and Anderson.[11] For $N=4$, their expression for the gap reduces to

$$\mathcal{E}_g = \Delta_P{}^4/E_F W^2, \tag{5}$$

where $2\Delta_P$ is the Peierls gap of the CDW, E_F the Fermi energy, and $W \sim 4E_F$ the bandwidth. With $\Delta_P \sim 600$ K and $E_F \sim 2400$ K, $\mathcal{E}_g \sim 0.6$ K $\sim 10^{-16}$ erg. In this case, the coherence distance, ξ_0, is of the order 2×10^{-4} cm and e/e^*, estimated from $E_0 \sim 600$ V/cm, is of the order of 3×10^3; a reasonable value. Estimating the correlation distance L from the threshold field $(e^*LE_T = \mathcal{E}_g)$, I find $L \sim 2\times10^{-3}$ cm, about five times larger than $2\xi_0$. Thus the exponential does not need to be modified, but the factor $(1-x^{-1})$ should be included in the tunneling probability. Since $E_0 \sim 10E_T$ for TaS_3,

$$P(x) \sim (1-x^{-1})e^{-(10/x)}. \tag{6}$$

The tunneling theory gives reasonable orders of magnitude for commensurate as well as impurity pinning.

It is possible to relate non-Ohmic behavior with the frequency-dependent ac conductivity. In a theoretical study of tunnel junctions as detectors and mixers, Tucker[12] derived an expression for the current resulting from a voltage $V = V_0 + V_1 \cos\omega t$ applied across a non-Ohmic junction. He found for small V_1 the following simple result for the ac component of the current:

$$I_1 = (eV_1/2\hbar\omega)[I_0(V_0 + \hbar\omega/e) - I_0(V_0 - \hbar\omega/e)], \tag{7}$$

where $I_0(V_0)$ is the dc current for voltage V_0. To apply this expression to transport by CDW's, I take $V_0 = EL$, the voltage across the correlation length L, and replace e by e^*. Resistive terms are assumed to dominate. When $V_0 = 0$, the ac response is

$$I_1 = (e^*V_1/\hbar\omega)I_0(\hbar\omega/e^*), \tag{8}$$

or

$$\sigma(\omega) = \sigma_{dc}(\hbar\omega/e^*). \tag{9}$$

Thus the ac conductivity should follow exactly the same relation as a function of frequency that the dc does of voltage. The threshold frequency is $\omega_T = \mathcal{E}_g/\hbar$. Plotted in Fig. 2 are data of Grüner *et al.*[8] on the frequency-dependent conductivity of $NbSe_3$ at 42 K. Here x is ω/ω_T, with $\omega_T/2\pi$

$=1.5\times10^7$ Hz. This implies $\mathcal{E}_g=\hbar\omega_T=9.5\times10^{-20}$ erg. Such a small gap makes sense only because of the large number of electrons involved in motion of the CDW.

The success of the tunneling theory when applied to $NbSe_3$ and TaS_3 suggests the possibility of applying the theory to non-Ohmic conduction observed in other linear-chain compounds.

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Massive Neutrinos and the Large-Scale Structure of the Universe

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If neutrinos dominate the mass density of the Universe, they play a critical role in the gravitational instability theory of galaxy formation. For neutrinos of mass $30m_{30}$ eV, a maximum Jeans mass $M_{\nu m}\approx4\times10^{15}m_{30}^{-2}M_\odot$ is derived. On smaller scales, neutrino fluctuations are damped, and the growth of baryon fluctuations is greatly inhibited. Structures on scales $>M_{\nu m}$ may collapse, forming galaxies as in Zel'dovich's "pancake" theory.

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The idea that massive neutrinos may solve the missing-mass problem in galaxies and in clusters of galaxies, as well as dominating the mass content of the Universe, has been explored by many authors.[1,2] The recent experimental reports of electron-neutrino mass detection[3] and neutrino oscillations,[4] coupled with the natural appearance of a neutrino-flavor mass spectrum with splittings of order of those between the three leptoquark families in extensions of grand unified theories to SO(10),[5] suggest that we take seriously the astrophysical implications of a neutrino

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