occurrence of P wave and higher composite states at $P \lesssim 1/r$ appears to be a natural low-energy angular-momentum suppression.

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Where is the Top Quark?

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Arguments are presented suggesting that the top-quark analog of the J/ψ should lie at 38 ± 2 GeV. Should there exist a fourth $Q=\frac{2}{3}$ quark h, the first $\bar{h}h$ state must be heavier than 300 GeV.

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Is there a top quark to complete the third family of quarks and leptons, and if so, what is its mass? Experiments at PETRA[1] show that the $\bar{t}t$ analog state of J/ψ does not exist at a mass less than ~36 GeV. This contradicts arguments[2] that its mass should be ~27 GeV. Experiments at Cornell Electron Positron Storage Ring[3] suggest that the b quark has conventional weak couplings. This would be in contradiction with certain theories[4] in which there is no top quark at all. In this note, I develop another line of thought which predicts that the missing $\bar{t}t$ state should lie in the vicinity of 38 GeV, a prediction which can soon be tested.

Assume that there are N conventional families of quarks and leptons, and that $N \geq 3$. $M(Q)$ denotes the $N \times N$ mass matrices for $Q=\frac{2}{3}$ quarks, $Q=-\frac{1}{3}$ quarks, and $Q=-1$ leptons, and it may be written

$$M(Q) = \sum_a F^a(Q) G^a, \tag{1}$$

where F^a are Q-dependent real numbers and G^a are Q-independent complex $N \times N$ matrices. Should there be just *one* term in the sum, all three mass matrices are proportional: All flavor-mixing angles vanish, and corresponding ratios of up-quark, down-quark, and charged-lepton masses coincide. This is contrary to observed facts. Should there be *three or more* terms in the sum, (1) implies no special relationship among the masses within the first three families. This leaves open the question of what happens if there are just *two* terms in the sum, the hypothesis upon which my subsequent analysis is based. [In an O(10) model, my hypothesis is fulfilled if all fermion masses arise from Yukawa couplings to a single complex 10 and a 126 of Higgs bosons with vacuum expectation values. Such a specific model gives more predictions than those we discuss here, particularly concerning neutrino masses and mixing. These are reserved for a subsequent work.]

If there are two terms in (1), it follows that $M(\frac{2}{3})$, $M(-\frac{1}{3})$, and $M(-1)$ are linearly dependent matrices,

$$M(-1)=\alpha M(\tfrac{2}{3})+\beta M(-\tfrac{1}{3}). \tag{2}$$

If all flavor-mixing angles vanished, then all three matrices may be chosen to be diagonal, and (2) becomes a vector relation implying

$$\left\|\begin{vmatrix} u & d & e \\ c & s & \mu \\ t & b & \tau \end{vmatrix}\right\| = 0, \tag{3}$$

where particle names are particle masses. The empirical relation[5]

$$ce = u\mu \tag{4}$$

simplifies (3) to give

$$t\mu = c\tau, \tag{5}$$

which is to say

$$M(-1) \propto M(\tfrac{2}{3}). \tag{6}$$

This solution to (2) survives intact when there is flavor mixing, whence $[M(\frac{2}{3}), M(-\frac{1}{3})] \neq 0$. I adopt (6) as an appealing *Ansatz*, in contrast to the empirically unacceptable relation

$$M(-1) \propto M(-\tfrac{1}{3}), \tag{7}$$

which follows from naive implementation of SU(5) symmetry.[6] Equation (6) encompasses the known "coincidence" (4), and the new prediction (5), though, unlike (7), it is not a natural consequence of any known theory.

In order to compare (5) with experiment, one must remember that fermion "masses" are momentum-dependent parameters. Quark masses, in particular, are subject to significant quantum-chromodynamic renormalization-group correction[7]

$$\frac{m(q)}{m(Q)} = \left(\frac{\ln(Q/\lambda)}{\ln(q/\lambda)}\right)^p, \tag{8}$$

where λ is the quantum-chromodynamic coupling parameter, q and $Q \gg \lambda$, and p is the anomalous dimension $p = 4(11 - 2f/3)^{-1}$, with f the number of operative quark flavors. The mass of a heavy $q\bar{q}$ state is taken to be $2m$, and we shall define $M_i \equiv m_i$ to be *the* mass of a heavy quark q_i, or equivalently, half the measured mass of the associated $q_i\bar{q}_i$ state ($2m_i$). Applied to the mass of the $\bar{t}t$ state, (8) becomes

$$2M_t = \left[\frac{\ln(2M_b/\lambda)}{\ln(2M_t/\lambda)}\right]^{12/23} \left[\frac{\ln(2M_c/\lambda)}{\ln(2M_b/\lambda)}\right]^{12/25} \times (2m_t)(2m_c), \tag{9}$$

where I have put $f = 4$ between $\bar{c}c$ and $\bar{b}b$ thresholds and $f = 5$ between $\bar{b}b$ and $\bar{t}t$ thresholds. Interpreting (5) to mean

$$m_t(q) = 16.88 m_c(q) \tag{10}$$

for any $q \gg \lambda$, I obtain from (9) an implicit determination of the mass of the $\bar{t}t$ state in terms of λ.

For λ = 100 MeV, one finds the new state at 39.6 GeV. For λ = 300 MeV, the result is 36.7 GeV. If the present *Ansatz* is correct, the missing $\bar{t}t$ state lies just above the explored region of energy in electron-positron collisions.

The existence of a fourth family of quarks and leptons is compatible with the *Ansatz* (6). Because there exists no fourth charged lepton σ with a mass less than 17 GeV,[1] one may put a lower limit on the mass of the putative fourth $Q = \frac{2}{3}$ quark h. From (6), I have the naive relation

$$h\tau = t\sigma \text{ or } h > 10t. \tag{11}$$

When renormalization-group corrected, this relation gives a lower limit of 300 GeV to the mass of the $h\bar{h}$ state.

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Model of Flavor Unity

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An SU(7) model is presented toward a flavor unification for known particles. The t quark is not a partner of the b quark. There are three types of neutrinos and several —so far unobserved—light detectable particles (masses < 300 GeV): a doubly charged lepton T^{--}, a $Q=-\frac{4}{3}$ quark x, and a $Q=\frac{5}{3}$ quark y. An intermediate mass scale is a necessity and there is no problem of magnetic monopoles.

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It is an attractive idea[1] to unify the electroweak,[2] strong, and flavor interactions. The minimal model[3] does not give a description of the flavor interaction. One of the most important predictions of the grand unified models is a huge difference in mass scales in the theory. At present we do not understand how the symmetry breaking that the theory requires is realized. We do know, however, that there exist light fermions. The existence of these light fermions requires a chiral invariance to protect them from acquiring masses of order grand-unification mass scale. This is achieved as a complex representation in gauge theories. Therefore, we require that *the known fermions* (ν_e, ν_μ, e, μ, u, c, d, *and* s) *should belong to a complex representation in the* $[\mathrm{SU}(3)]_c \otimes \mathrm{SU}(2) \otimes \mathrm{U}(1)$ *gauge group.*

We also know that there does not exist a massless quark or a massless charged lepton, which means that the fermion representation is not complex in the gauge group $[\mathrm{SU}(3)]_c \otimes [\mathrm{U}(1)]_{\mathrm{em}}$. The fermion representation is real under $[\mathrm{SU}(3)]_c \otimes [\mathrm{U}(1)]_{\mathrm{em}}$. The condition implies that if $(\underline{3}, a)$ and $(\underline{1}, b)$ (with $b \neq 0$) representations in $[\mathrm{SU}(3)]_c \otimes [\mathrm{U}(1)]_{\mathrm{em}}$ appear, they should be matched by $(\underline{3}^*, -a)$ and $(\underline{1}, -b)$. Therefore, all the $[\mathrm{SU}(3)]_c \otimes \mathrm{U}(1)_{\mathrm{em}}$ nonneutral fermions can acquire Dirac masses, among which the $[\mathrm{SU}(3)]_c \otimes \mathrm{SU}(2) \otimes \mathrm{U}(1)$ complex fermions are light. By requiring this, we exclude possible infrared problems of $[\mathrm{SU}(3)]_c \otimes [\mathrm{U}(1)]_{\mathrm{em}}$. Not all anomaly-free representations with arbitrary charge assignment satisfy this condition.[4]

The prototype SU(5) model[3] satisfies these conditions. However, the representation $\underline{10} + \underline{5}^*$ is too small to include flavor interactions. We find that the simplest group which satisfies the above requirements and can also include the second generation without repetition of representations is the SU(7) group with the left-handed fermions included in

$$\Psi_\alpha + \Psi^{\alpha\beta} + \Psi_{\alpha\beta\gamma} = [\underline{1}^*] + [\underline{2}] + [\underline{3}^*] \tag{1}$$

which is anomaly-free. The repeated indices mean antisymmetric combinations. The charge generator for the fundamental representation is given by $Q = \mathrm{diag}(-\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}, 1, 0, c, -c)$. The color and the electroweak indices are $\alpha = 1, 2, 3$, and $\alpha = 4, 5$, respectively. The charge assignment (2) for any c satisfies the reality in $[\mathrm{SU}(3)]_c \otimes [\mathrm{U}(1)]_{\mathrm{em}}$. Also, it satisfies the complexity condition in $[\mathrm{SU}(3)]_c \otimes \mathrm{SU}(2) \otimes \mathrm{U}(1)$ if $c \neq 0$. Both of these can be checked by classifying the 63 chiral fields under the respective subgroups. The weak hypercharge is given by

$$Y = Q - I_3 = \mathrm{diag}\,(-\tfrac{1}{3}, -\tfrac{1}{3}, -\tfrac{1}{3}, \tfrac{1}{2}, \tfrac{1}{2}, 1, -1)\,. \tag{2}$$

The other color-singlet $[\mathrm{U}(1)]_a$ and $[\mathrm{U}(1)]_b$ generators are

$$Y_a = \mathrm{diag}\,(\tfrac{1}{5}, \tfrac{1}{5}, \tfrac{1}{5}, \tfrac{1}{5}, \tfrac{1}{5}, -\tfrac{1}{2}, -\tfrac{1}{2})\,, \tag{3}$$

$$Y_b = \mathrm{diag}\,(\tfrac{2}{5}, \tfrac{2}{5}, \tfrac{2}{5}, -\tfrac{3}{5}, -\tfrac{3}{5}, \tfrac{1}{2}, -\tfrac{1}{2})\,. \tag{4}$$

The bare value of $\sin^2\theta_W{}^0$ at the grand-unification mass scale is[5,6] $\sin^2\theta_W{}^0 = 3/(8+12c^2)$, which is 0.15 for $c^2 = 1$. We fix $c = 1$ to include the τ-lepton

