$\mu \to e\gamma$ in Theories with Dirac and Majorana Neutrino-Mass Terms

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The $\mu \to e\gamma$ decay as induced by massive neutrino mixings is studied. In models with either pure Dirac or Majorana mass terms it is suppressed by small neutrino masses. When both Dirac and Majorana terms are present, one can avoid this mass suppression and the Glashow-Iliopoulos-Maiani cancellation is generally not complete. Then it is suppressed by small mixing angles. However, one instance is found where both suppression mechanisms can be avoided, yielding a "large" $\mu \to e\gamma$ rate.

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In the standard SU(2)⊗U(1) model of weak and electromagnetic interactions there are a number of exact global conservation laws. In particular the lepton flavors—electron number, muon number, etc.—are separately conserved. This is related to the fact that in this theory there is no direct coupling between leptons and quarks, and neutrinos are massless. Explorations of grand unification of strong, weak, and electromagnetic interactions have led us to expect that global symmetries are likely to be broken in a more complete theory. In grand unified theories lepton flavors will no longer be conserved exactly: Leptons and quarks are directly coupled, and except for the simplest case, neutrinos are massive. In this paper we shall study the decay $\mu \to e\gamma$ as induced by intermixing massive neutrinos. We shall pay particular attention to the class of theories where the neutrino masses contain both Dirac and Majorana types of terms.

Although recent interests in the question of massive neutrinos are stimulated by grand-unified-theory considerations, our discussion will be carried out mostly by use of the language of SU(2)⊗U(1) models.[1] Specific grand-unified-theory realizations of the models discussed in this paper will be mentioned as illustrative examples.

If neutrinos have nonzero masses, their mass matrices are not expected to be diagonal when defined with respect to neutrino fields having definite transformation properties under the gauge group. The left-handed neutrino fields ν_{eL}, $\nu_{\mu L}$, and $\nu_{\tau L}$, which form SU(2) doublets with e_L, μ_L, and τ_L, will be orthogonal combinations of mass eigenstates ν_i (corresponding to mass eigenvalues m_i). Such mixings will give rise to lepton-flavor nonconservation. The most accessible effects will perhaps be neutrino-flavor oscillations $\nu_{\mu L} \leftrightarrow \nu_{eL}$, etc. Currently this line of research is being actively pursued. Here we turn to another lepton-flavor-changing process: $\mu \to e\gamma$. Stringent limit already exists for this decay[2] and a sensitive search in the next generation of meson-factory experiments will be possible.

(I) *Theories with either Dirac or Majorana neutrino-mass term.*—Namely the neutrino mass terms are either

$$\mathcal{L}_{\rm D} = \bar{\nu}_{aL} D_{ab} \nu_{bR} + \text{H.c.} \tag{1}$$

or

$$\mathcal{L}_{\rm M} = \bar{\nu}_{aL}{}^c A_{ab} \nu_{bL} + \text{H.c.}, \tag{2}$$
$$a, b = e, \mu, \tau.$$

The Dirac mass terms in (1) are present when the standard SU(2)⊗U(1) model is augmented with right-handed neutrino fields in singlet representations. The Majorana masses in (2) can come, for example, from the vacuum expectation value of a Higgs scalar in triplet representation (with weak hypercharge $Q - T_3 = 1$).[1] The mass matrix D can be diagonalized:

$$U_{ai} D_{ab} V_{bj} = \hat{D}_{ij} = D_i \delta_{ij}, \tag{3}$$

with

$$\nu_{aL} = U_{ai} \nu_i, \tag{4}$$

and

$$\nu_{aR} = V_{ai} \nu_i; \tag{5}$$

$i, j = 1, 2, 3$. For simplicity we shall take U and V to be real. They are orthogonal matrices. Similarly the matrix A, which is symmetric,

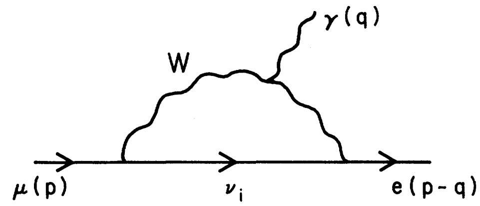

FIG. 1. The one-loop diagrams for the $\mu \to e\gamma$ decay as mediated by massive neutrinos ν_i. W's are the gauge bosons, with unitary gauge propagators. We do not explicitly display diagrams with photon emission from external charged lines. They contribute only to the $\overline{\psi}_e \gamma_\lambda \epsilon^\lambda \psi_\mu$ amplitude which vanishes because of current conservation.

can be diagonalized:

$$U'_{ai} A_{ab} U'_{bj} = \hat{A}_{ij} = A_i \delta_{ij}. \qquad (6)$$

For the two cases at hand we have neutrino masses $m_i = D_i$ or $m_i = A_i$. Thus the eigenvalues of D and A must necessarily be small.

The muon-number–nonconserving decay $\mu \to e\gamma$ proceeds via the one-loop diagram shown in Fig. 1. To the amplitude

$$T(\mu \to e\gamma) = i\overline{\Psi}_e(p-q)\sigma_{\mu\nu}\epsilon^\mu q^\nu(a + b\gamma_5)\Psi_\mu(p) \qquad (7)$$

each ν_i diagram contributes[3]

$$a_i = -b_i = (g^2/8M_W^2)(em_\mu/32\pi^2)T_i, \qquad (8)$$

with

$$T_i = U_{\mu i} U_{ei}[\tfrac{10}{3} - m_i^2/M_W^2 + \cdots], \qquad (9)$$

where we have made an expansion in powers of the small parameter m_i/M_W. After summing over the index $i = 1, 2, 3$ the leading constant terms mutually cancel because of the orthogonality condition $U_{\mu i} U_{ei} = 0$, leaving an amplitude of the order $eG_\mu m_i^2/M_W^2$ and a branching ratio

$$B(\mu \to e\gamma) = \frac{\Gamma(\mu \to e\gamma)}{\Gamma(\mu \to e\nu\overline{\nu})} = \frac{3\alpha}{32\pi}\left| U_{\mu i} U_{ei} \frac{m_i^2}{M_W^2} \right|^2. \qquad (10)$$

This is the leptonic version of the Glashow-Iliopoulos-Maiani (GIM) suppression mechanism.[4] Even if one takes a neutrino mass that saturates the cosmological bound[5] 100 eV, we still have $B(\mu \to e\gamma) \leq 10^{-40}$.

(II) *Theories with both Dirac and Majorana neutrino-mass terms.*—Like the case in Eq. (1) we enlarge the standard SU(2) ⊗ U(1) model with right-handed neutrinos. ν_R is totally neutral with respect to the gauge group. The most general SU(2) ⊗ U(1)-invariant interactions lead to the following neutrino-mass term:

$$\mathcal{L}_{\rm DM} = \overline{\nu}_{aL} D_{ab} \nu_{bR} + (\overline{\nu}_c{}^c)_R B_{cd} \nu_{dR} + \text{H.c.} \qquad (11)$$

The Majorana mass term B is present unless we impose on the theory [as we did in Eq. (1)] an *ad hoc* global symmetry corresponding to lepton-number conservation. Equation (11) may be written in more compact form:

$$\mathcal{L}_{\rm DM} = \overline{n} H n, \qquad (12)$$

where n is a column of six self-conjugate fields,

$$n = \begin{pmatrix} \nu_{aL} + (\nu_a{}^c)_L \\ \nu_{bR} + (\nu_b{}^c)_R \end{pmatrix}, \quad a, b = e, \mu, \tau; \qquad (13)$$

H is a symmetric 6×6 Majorana mass matrix,

$$H = \begin{pmatrix} 0 & \frac{1}{2}D \\ \frac{1}{2}D^T & B \end{pmatrix}, \qquad (14)$$

which can be diagonalized

$$W^T H W = \begin{pmatrix} \hat{m} & 0 \\ 0 & \hat{M} \end{pmatrix}, \qquad (15)$$

with $\hat{m}_{ij} = m_i \delta_{ij}$ and $\hat{M}_{ij} = M_i \delta_{ij}$. Before proceeding to display the orthogonal matrix W, we shall make a simplifying assumption[6] that $B_{ab} = B\delta_{ab}$. This will allow us to present our results in more suggestive form. It does not affect in any essential way the physics conclusions which we shall draw. With this simplification, W takes on the form

$$W = \begin{pmatrix} U & 0 \\ 0 & V \end{pmatrix} \begin{pmatrix} \hat{C} & \hat{S} \\ -\hat{S} & \hat{C} \end{pmatrix}, \qquad (16)$$

where U and V are the orthogonal matrices in Eqs. (3)–(5); $\hat{C}$ and $\hat{S}$ are diagonal matrices:

$$C_{ij} = \cos\theta_i \delta_{ij}, \quad S_{ij} = \sin\theta_i \delta_{ij}, \quad \tan 2\theta_i = D_i/B. \qquad (17)$$

Thus, ν_{aL} and ν_{aR} all are superpositions of the six mass eigenstates: ν_i and N_i with eigenvalues m_i and M_i being $\frac{1}{2}[B \mp (B^2 + D_i^2)^{1/2}]$, respectively. We assume at least three (m_i) masses are small.

In this category of models there will be six $\mu \to e\gamma$ diagrams like Fig. 1, with intermediate lines being ν_i and N_i. We have computed the amplitudes without making any assumption on the size of the intermediate fermion mass. With inessential electron and muon masses neglected, the result for the amplitude as defined in Eq. (8) is

$$T_i = U_{\mu i} U_{ei} \cos^2\theta_i F(m_i^2/M_W^2) + U_{\mu i} U_{ei} \sin^2\theta_i F(M_i^2/M_W^2), \tag{18}$$

where

$$F(x) = 2(x+2)I^{(3)}(x) - 2(2x-1)I^{(2)}(x) + 2xI^{(1)}(x) + 1,$$

with

$$I^{(n)}(x) = \int_0^1 dz\, z^n/[z + (1-z)x]. \tag{19}$$

We note that the light-fermion-mass limit,

$$F(x) \to F(0) + xF'(0) = \tfrac{10}{3} - x, \tag{20}$$

is just the result shown in Eq. (9).

Within this category of models we can further differentiate two subclasses depending upon the range of M_i values.

(i) M_i are also small: $M_i \approx m_i$. Namely D_i and B are all small. Such models would have, besides the usual neutrino-flavor oscillations, also neutrino-antineutrino oscillations[7]: $\nu_{aL} \leftrightarrow (\nu_b{}^c)_L$.

For $\mu \to e\gamma$, after summing over all six amplitudes we still have complete GIM cancellation of the leading constant terms, just like the situation in section (I). This yields a rate of the same order of magnitude as that in Eq. (10).

(ii) M_i are large: $M_i \gg m_i$. In this case, as first suggested by Gell-Mann, Ramond, and Slansky,[8] D_i and B can be large, so long as $D_i/B \ll 1$. The $\mu \to e\gamma$ amplitude in Eq. (18) can be simplified because θ_i are small:

$$T_i = U_{\mu i} U_{ei} \theta_i^2 [F(M_i^2/M_W^2) - F(0)],$$

with

$$F(x) - F(0) = 6x[I^{(3)}(x) - I^{(2)}x]. \tag{21}$$

This agrees with the result first obtained by Altarelli *et al.*[9] For superheavy N_i's it is appropriate to take the limit of $M_i^2/M_W^2 \to \infty$; we obtain

$$F(\infty) = \tfrac{4}{3} \tag{22}$$

and

$$B(\mu \to e\gamma) = (3\alpha/8\pi)|U_{\mu i} U_{ei} \theta_i^2|^2. \tag{23}$$

Thus we see that in this category of models with $M_i \gg m_i$ the GIM cancellation is generally not effective. However, Eq. (17) shows that the mixings (θ_i) of N_i in the ν_{aL} states,

$$\theta_i \cong (m_i/M_i)^{1/2}, \tag{24}$$

must be extremely tiny, again leading to a strong suppression of the decay process.

An interesting example in this class of models is the minimal O(10) grand unified theory[10] with Higgs scalars in (besides the $\underline{45}$) $\underline{10}$ and $\underline{16}$ representations only. ν_L and ν_R are members of the 16-dimensional spinor representation. Witten[11] has pointed out that the Dirac neutrino-mass terms are related to charge-$\frac{2}{3}$ quark masses; $D_i = m_{q_i}$ and the Majorana masses B are induced by two-loop radiative correction $B \cong \epsilon(\alpha/\pi)^2(M_q/M_W)M_{10}$ with ϵ being some mixing angle and M_{10} being the O(10)/SU(5) gauge boson masses. He estimates that neutrino masses will be $m_i \cong 10^{0\pm 2}$ eV and $M_i \cong 10^{9\pm 2}$ GeV. Thus in this model the mixing angle of Eq. (24) will be $10^{-9\pm 2}$ again leading to an infinitesimal $B(\mu \to e\gamma) < 10^{-40}$.

The result in Eq. (22) represents a curious evasion of the Appelquist-Carazzone theorem,[12] which states that amplitudes corresponding to nonrenormalizable interactions should vanish in the limit when any of its internal particle masses approaches infinity. A detailed discussion of this theoretical point is presented in a separate communication.[13]

We should emphasize that the minimal O(10) example given above is still, numerically, an extreme case. For a wide range of M_i values that are more comparable to M_W the presence of heavy-neutrino mass eigenstates invariably enhances $\mu \to e\gamma$ decays. The branching ratio will be larger than the result in Eq. (10) by a factor of $(M_i/m_i)^2$ for $M_i < M_W$ or by a factor of $(2M_W^2/m_i M_i)^2$ for $M_i > M_W$. For example, with M_i equal to either 10 GeV or 1000 GeV, and m_i being 100 eV, $B(\mu \to e\gamma)$ will be on the order of 10^{-24}. Unfortunately, this is still far below the general level where one can hope for laboratory detection.

(III) *Theories with the most general neutrino-mass terms.*—Is the rate for $\mu \to e\gamma$ as induced by neutrino-mass mixings always small? Here we display a case where the suppressions by small neutrino masses and by small mixing angles are both avoided.

Consider theories with the most general neutrino-mass term of H in Eqs. (12) and (14):

$$H = \begin{pmatrix} A & \frac{1}{2}D \\ \frac{1}{2}D^T & B \end{pmatrix}. \tag{25}$$

Namely, it is a combination of Eqs. (11) and (22). This situation is realized, for example, in the

O(10) grand unified theory with the inclusion of a Higgs scalar in 126-dimensional representation.[14]

It is easy to convince oneself that the mass matrix in (25) with $A \neq 0$ can, unlike the situation in section (II), have very dissimilar eigenvalues: $M_i \gg m_i$ without the concomitant small mixing angles θ_i. With the approximation $B_i \gg D_i \gg A_i$, the solution now reads as

$$M_i \simeq B_i\,,\qquad m_i \simeq (D_i^2 - 4A_iB_i)/4B_i\,,\quad \theta_i \simeq D_i/2B_i. \tag{26}$$

Clearly this allows the results in Eqs. (21) and (23) to yield a "large" $\mu \to e\gamma$ rate if we make the appropriate fine tuning of parameters. The following numerical example illustrates our point: A_i, B_i, and D_i are the order of 10^{-1}, 10^3, and 10 GeV, respectively.[15] With the heavy-neutrino masses M_i of order 10^3 GeV, the light-neutrino masses $m_i \lesssim 1$ eV require fine tunings of better than one part in 10^8 in the cancellations between D_i^2 and $4A_iB_i$ in Eq. (26). Then a θ_i of order 10^{-2} and Eq. (23) lead to a branching ratio $B(\mu \to e\gamma)$ of order 10^{-13} if $U_{ei}U_{\mu i}$ are taken to be the order of Cabibbo angle.

We conclude that, in models with the most general types of neutrino mass terms, intermixing neutrinos themselves can in principle lead to a "large" rate for $\mu \to e\gamma$, although a fine tuning of parameters would be involved.

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[14]The 126 representation of O(10) contains both SU(2) triplet and singlet which can develop vacuum expectation value. This gives rise to A and B in Eq. (25). It is not "natural" (in the technical sense) to set either of them to be zero.

[15]A_i terms transform as members of weak isotriplets. They contribute to the violation of the $M_W = M_Z \cos\theta_W$ relation. Present experimental limit requires $A_i/f_{Yi} < 50$ GeV, where f_{Yi} are Yukawa couplings (see Cheng and Li, Ref. 1). Also the present experimental limit on universality violation is not very stringent ($\theta_i < 0.1$). Thus our numerical illustration with A_i of order 10^{-1} GeV and θ_i of order 10^{-2} is well within these limits.