ite-element algorithm to connecting the astronomical and nuclear limits will be communicated later.[14]

The representations of equilibrium shapes and shape perturbations in the basis described above are ideally suited to the study of multiplicity of shapes of rotating liquid drops. The key to their success lies in the large number of degrees of freedom that are used in the basis and in the ability of the basis functions to represent shapes with virtually any symmetry. This latter advantage is responsible for the discovery of the asymmetric shape family reported here.

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Measurement of the Spectrum of Electric-Field Fluctuations in a Plasma by Laser-Fluorescence Spectroscopy

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Laser-fluorescence spectroscopy has been applied to measure the spectrum of electric wave fields with high temporal resolution in a pulsed hollow-cathode discharge. A low-frequency and a high-frequency component can be identified.

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Collective modes excited above thermal level influence drastically most transport coefficients of a plasma, and their analysis still poses a challenging task to the experimentalist. Two main methods have evolved and have been successfully applied: Thomson scattering[1] by microwave or laser irradiation yielding the spectrum of the electron density fluctuations, and optical studies, which allow the derivation of the spectrum of the electric field fluctuations from optical transitions stimulated in the vicinity of forbidden lines by these fields.[2]

In a model-type experiment using microwaves, Burrell and Kunze[3] demonstrated the feasibility of applying laser fluorescence spectroscopy to the second method, and of thus obtaining local measurements of fluctuating fields with high resolution even for lower turbulence levels. The principle of this technique is as follows. The beam from a tunable dye laser is focused into the plasma containing helium atoms, and the wavelength of the radiation is tuned in the vicinity of forbidden and allowed transitions. Absorption of laser radiation occurs by single-quantum as well as by two-quantum transitions involving absorption of a photon from the laser beam and absorption or emission of a quantum from the electric field fluctuations present in the plasma. Both processes may be observed as enhancement of line radiation from the upper level or from other levels strongly coupled to the first one by collisions.

The unexpected observation of low-frequency electric fields in the pulsed hollow-cathode discharge of Ref. 3 prompted us to investigate such fields with high spectral and temporal resolution thus also testing the possibilities of this technique.

The hollow-cathode discharge is of the type investigated in Ref. 4. The cathode consists of two solid copper cylinders of 20 mm in diameter facing each other at a distance of 5 mm and connected by an external loop. A hollow copper cylinder encloses this assembly and serves as anode. Holes allow the laser beam to enter and to leave the hollow-cathode region as well as permitting the observation of the radiation emitted by the plasma. A stationary glow discharge of 10 mA is maintained by a voltage of 200 V at a filling pressure of 3.5 mbar helium. According to the measurements of Ref. 4, the electron density is of the order of 10^{11} cm^{-3}. Superimposed on this dc plasma is a 4-kV pulse of 80 nsec duration from a 60-Ω coaxial cable. The time constant of the circuit as determined by the inductance of 400 nH and the resistors is 5 nsec, which is fast enough for the internal dynamics of this special high-current glow discharge.

Figure 1 shows the currents I_1 and I_2 to the upper and lower cathode plate: They are modulated nearly 100%, their period is 15 nsec, and they establish a phase shift of 180° already within 3 nsec after current rise. The role of both currents is of random nature because their phase can be exchanged by a slight increase of the inductivity of one of the cathode leads. The total current ranges between 30 and 40 A, and the applied voltage drops to 1.5 to 1.0 kV during the discharge.

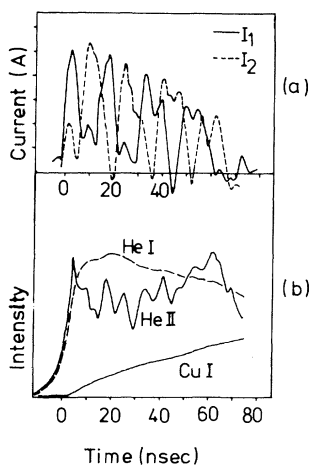

FIG. 1. Time histories of (a) current I_1 and I_2 to upper and lower cathode plate (5 A/div), and of (b) He I line at 447 nm, He II line at 469 nm, and Cu I line at 325 nm.

Figure 1(b) shows the time histories of three emission lines, the signals having been corrected for the internal time delay of the photomultiplier 1P28. The He II line at 469 nm is closely correlated with the total current and only five times weaker than the He I line of 447 nm, which rises within 10 nsec to a slowly decreasing plateau. The Cu I line at 325 nm increases nearly linearly until the end of the current pulse.

The dye laser consists of an oscillator and an amplifier, both stages being pumped transversely by a long-pulse N_2 laser of 1 MW peak power and 18 nsec pulse duration. After passage through the amplifier cell the rest of the pumping pulse is absorbed in the dye cell of the oscillator, which is a somewhat modified type of that described in Ref. 5; the total end reflector is replaced by a combination of a prism and a grating with the dispersion perpendicular to the tuning grating assembly at the other end of the optical cavity. In this way the broadband background fluorescence of the dye could be sufficiently reduced. The laser beam was polarized almost completely, and after passing through the amplifier, it was focused in the midplane between the two cathode plates, the cross section of the pumped volume being 3×0.33 mm^2.

The dye used was stilbene 3 solved in water, thereby minimizing thermal problems. At 447 nm the laser beam had a peak power of 5 kW and a pulse duration of 5 nsec, and its spectrum showed about five longitudinal modes of 550 MHz separation.

The wavelength of the dye laser was tuned in the vicinity of the He I line at 447.1 nm, and the absorption processes were recorded as fluorescent emission at 90° to the incident beam, the wavelength position of a 0.25-m monochromator remaining fixed to observe the emission of the allowed line at 447.1 nm. The direction of polarization of the incident laser beam was in the line of sight. The detector was a 1P28 photomultiplier, and its signal was fed into a boxcar integrator (PAR model 162/163). The sample window of 1 nsec was set to the maximum of the fluores-

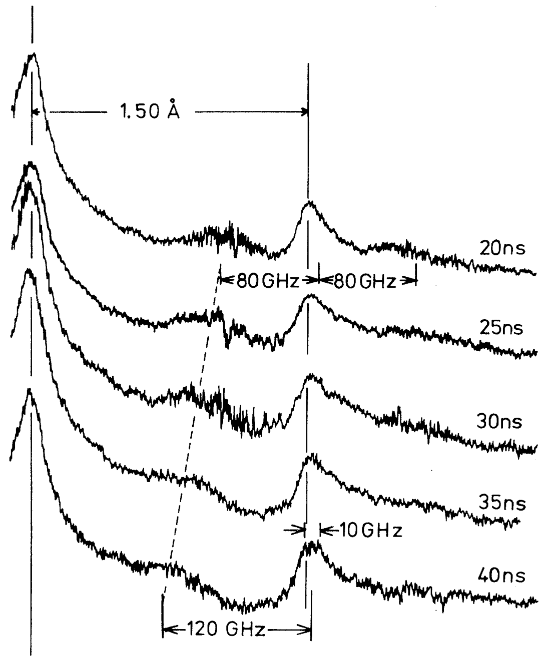

FIG. 2. "Spectrum" at different times after current rise.

cent signal with the laser tuned to the forbidden component. This peak occurred about 2 nsec later than the peak of the fluorescent signal if the allowed line itself was scanned. This delay is due to the finite time necessary for collisional transfer of the excitation from the 4^3F to the 4^3D level.

The "spectrum" thus obtained displays the peak fluorescent signal of the allowed transition as function of the wavelength of the incident laser radiation. At each wavelength position ten discharges were averaged. The spectra show the allowed and forbidden components and clearly two additional maxima on both sides of the forbidden component. They are considerably broader than the atomic line itself which is fictitiously broadened because of saturation. The two "satellite structures" also show strong fluctuations in contrast to the rest of the profile. In addition, they shift strongly with time as revealed by the next figure. Figure 2 shows five scans obtained successively at 5-nsec intervals. At the later times, two additional small maxima appear to develop (see, e.g., $t = 40$ nsec); they are separated from the peak of the forbidden component by about 5 GHz.

For the comparison of intensities, the fluorescent signal was measured as a function of incident laser power at the positions of the allowed and forbidden transitions. The saturation curve displayed a logarithmic law rather accurately over three decades, which is indicative of a high-quality Gaussian intensity distribution of the incident laser beam, and of the cross section observed in the plasma being larger than that of the laser. The ratio S_F of absorption at the wavelength position of the forbidden component to that of the allowed transition thus derived was about $S_F \simeq 0.006$ at $t = 20$ nsec. For the case of a thermal plasma,[6-8] this ratio would imply a relatively high electron density of about 10^{14} cm^{-3} and a

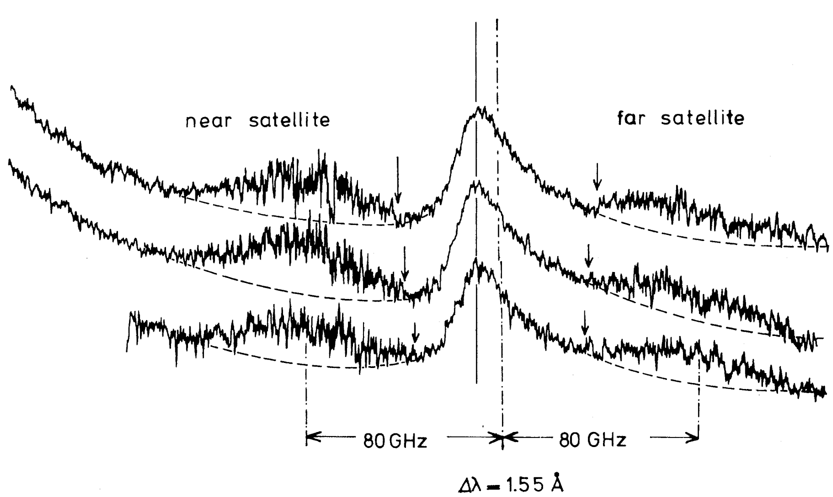

FIG. 3. Comparison of three spectra at $t = 20$ nsec. The dashed curves are a smooth continuation of the thermal profile. The center of symmetry at $\Delta\lambda = -1.55$ Å is derived from the onset of the field spectra marked by arrows.

wavelength separation between allowed and forbidden components of $\Delta\lambda = 0.150$ nm which agrees indeed with the observed value.

Several spectra were taken for each time of Fig. 2, and the results were surprisingly reproducible. Figure 3 shows three enlarged scans at $t = 20$ nsec, where the strongly noisy satellite structures have been made visible. The center of symmetry of both satellites is located at a wavelength separation of $\Delta\lambda \simeq 0.155$ nm from the allowed line; such a shift between allowed and forbidden components is produced by an average electric field of about 5 kV/cm in the plasma.[2] The initial centers of the satellites correspond to an angular frequency of about $\omega \simeq 0.5 \times 10^{12}$ sec^{-1}, and the total widths of these structures to $\Delta\omega \simeq 0.6 \times 10^{12}$ sec^{-1}. If we relate these observed wave fields to high-frequency electrostatic oscillations in the plasma, an electron density of $n_e \simeq 0.8 \times 10^{14}$ cm^{-3} increasing to $n_e \simeq 1.8 \times 10^{14}$ cm^{-3} at $t = 40$ nsec is required. Such rapid ionization is possible because of the high density of the neutral atoms, although the detailed mechanisms taking place in the discharge are not understood yet.

The spectral width $\Delta\omega$ of the wave field is larger than expected, but most probably it is not due to changes in the electron density from discharge to discharge since the form of the discharge current was extremely reproducible even in the finest details. We cannot exclude at present, however, a spatially inhomogeneous plasma. The strong fluctuations also point to a strongly turbulent plasma, where the amplitudes and phases of waves are rapidly changing. The far satellite is too strong in comparison with the near one although second-order perturbation theory still should be valid[9]; according to Griem[10] this again indicates the existence of rapidly growing and decaying modes coupled through nonlinear mechanisms. The strong asymmetry of the forbidden component, its increasing width, as well as the appearance of two small additional maxima at later times, point to low-frequency wave fields, too.

In support of these results obtained by laser fluorescence spectroscopy we have scanned the line profile also in emission at different times by using a monochromator and sampling sufficient discharges. Figure 4 shows one example for $t = 20$ nsec. The intensity scale now is linear in contrast to that of the previous figures, where it was approximately logarithmic because of saturation. The intensity ratio, the shift between allowed and forbidden component, and the "satellite structures" obtained by both methods are consistent. Finally, the satellites were seen at the correct position also near the singlet line at 492.2 nm.

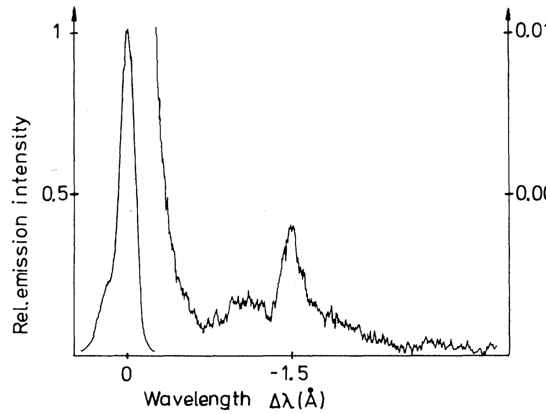

FIG. 4. Emission profile of the He I line at 447.1 nm for $t = 20$ nsec.

We have demonstrated thus that the spectrum of electric wave fields in a plasma indeed can be measured by laser-excited fluorescence, and detailed studies of the mechanisms in the discharge and the proper interpretation of the spectra are in preparation.

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