Energy Dependence of Pion Elastic Scattering from Nuclei across the (3,3) Resonance

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Improved agreement between calculation and $\pi^{\pm}$ elastic-scattering data across the (3,3) resonance is obtained by assuming the π-nucleon collision energy, ω, to be $\simeq 30$ MeV less than the incident pion energy. This effect has been observed for nuclei ranging from ^{9}Be to ^{208}Pb.

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A reliable model of elastic scattering is essential for a clear interpretation of nuclear structure from pion scattering experiments. The on-shell impulse approximation to a first-order optical model has been extensively applied to pion elastic scattering. Although this approximation accounts for the main features of the elastic differential cross sections, it fails to reproduce the observed energy dependence, in particular the character of the diffractive minima. To more closely reproduce the elastic data, several approaches have been used (see, for example, Refs. 1–6). However, these methods are either uneconomical in terms of computation time, or they fail to reveal any systematics in the fitted parameters.

We have attempted to develop a purely phenomenological method that preserved an energy-independent density distribution and furnished optical-model parameters with a predictable energy dependence. The unconstrained density distribution parameters that we obtained were consistent with electron scattering,[7] and the P-wave optical-model parameters had the general energy dependence of those predicted from the free π-N (pion-nucleon) phase shifts but were shifted in energy from that of the incident pion (see Fig. 1). Since the impulse approximation consists of taking the π-A (pion-nucleus) t matrix, $\tau(E = E_{\pi} + E_N)$, to be the π-N t matrix, $t(\omega = \omega_{\pi} + \omega_N)$, where ω represents the π-N collision energy as seen in the π-A c.m. (center-of-mass) system, the optimal choice for ω will reflect the fact that the nucleon is embedded in the nuclear medium.[8] Hence, it appeared possible to use conventional input parameters and search on the kinematic shift in energy, Δ, in ω_{π} from the incident pion energy, E_{π}. The results for ^{12}C with $\Delta = -28 \pm 4$ MeV are shown in Fig. 2 by the solid curve. The dashed curve represents calculations with $\Delta = 0$ MeV. Although the results at back angles are less than perfect, the overall improvement throughout the energy range is evident.

This one parameter fit was performed with a modified version of the computer code PIRK,[9] which solves a Klein-Gordon equation in coordinate space, linked to an optimizing package, MINUIT.[10] All calculations use a Kisslinger optical potential[11] and ground-state densities from electron scattering,[7] which are modified to remove the finite charge size of the proton. The π-A t matrix was constructed using the π-N phase shifts of Rowe, Salomon, and Landau.[12] The infinitely-heavy-nucleus approximations used by the code to calculate the π-A t matrix were replaced by the exact c.m. relations. The most significant modification was to calculate the on-shell π-N to π-A c.m. transformation, γ, exactly. The approximation method of Auerbach, Fleming, and Sternheim[13] was replaced with an on-shell equivalent of Goldberger and Watson,[14]

$$\gamma = \omega_{\pi}^{*}\omega_N^{*}/\omega_{\pi}\omega_N, \tag{1}$$

where ω_{π} (ω_N) is the collision energy of the pion (struck nucleon). Except for the superscripted

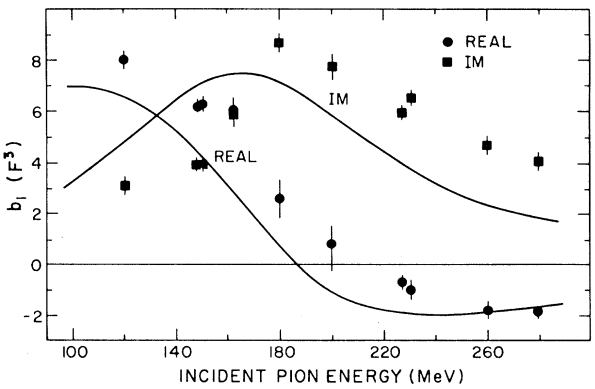

FIG. 1. Results of a search on the P-wave optical-potential coefficients, b_1, for $\pi^{\pm}$ + ^{12}C. The curves are the predictions for the optical-potential coefficients from the free π-N phase shifts.

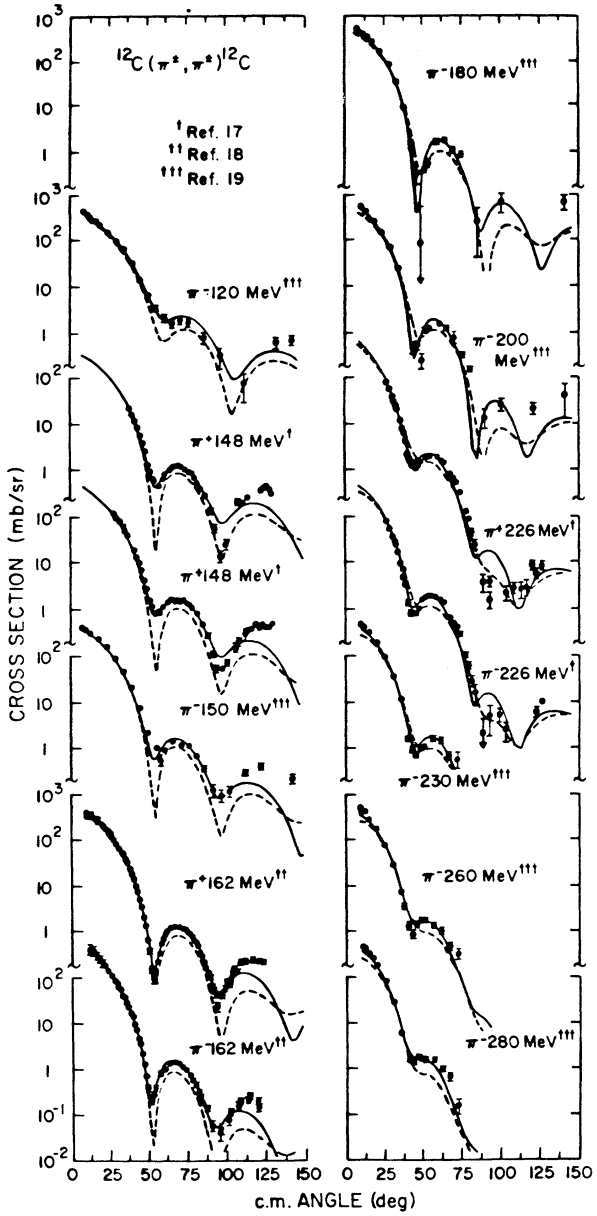

FIG. 2. PIRK calculations for $\pi^{\pm} + {}^{12}C$. The dashed curves are $\Delta = 0$ calculations and the solid curves are calculations with $\Delta = -28$ MeV. The displayed calculations have not been renormalized.

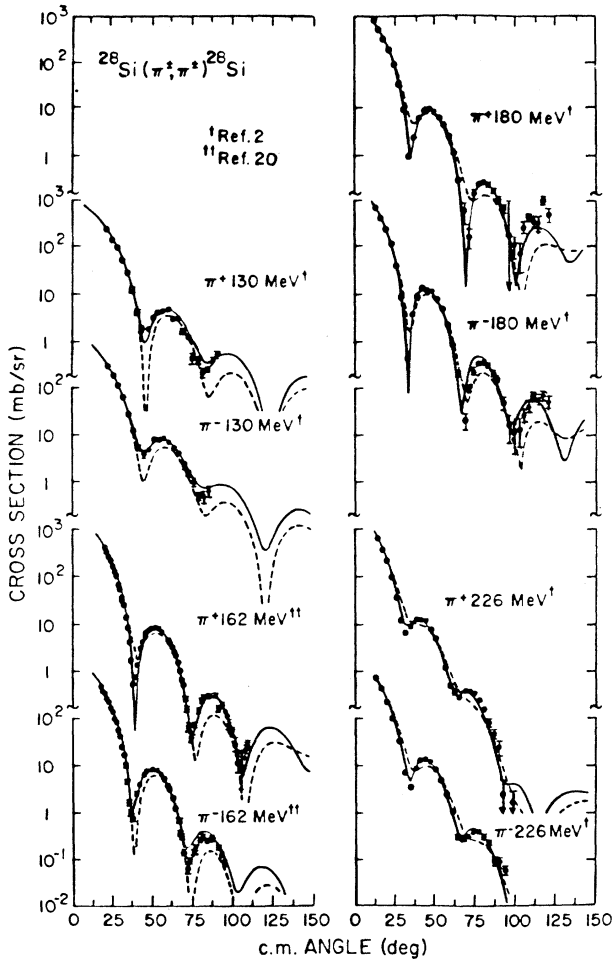

FIG. 3. Same as Fig. 2 but for ^{28}Si with $\Delta = -27$ MeV.

asterisk denoting the π-N c.m. system, all variables are in the π-A c.m. system. The π-A t matrix in the impulse approximation is then given by

$$\tau(E) \simeq t_{KG}(\omega) = 2\omega\gamma\langle \vec{k}_{\pi}^{*}(\omega^{*})|t_{RS}(\omega^{*})|\vec{k}_{\pi}^{*\prime}(\omega^{*})\rangle,$$
$$|\vec{k}_{\pi}^{*}| = |\vec{k}_{\pi}^{*\prime}|, \quad (2)$$

where the factor 2ω makes the connection between the Klein-Gordon (KG) and the relativistic Schrödinger (RS) equations,[13] and the kinematic variables (k_{π}, k_{π}^{*}, ω_N, ω_N^{*}, ω_{π}^{*}, and ω_{π}) are trivially related by the on-shell relations. The form factor does not depend upon ω, and the corresponding coordinate space calculations use the unshifted π-A c.m. momentum and energy, E_{π}. The kinematic energy shift, Δ, is incorporated by the relation $\omega_{\pi} = E_{\pi} + \Delta$ affecting only the values of the S- and P-wave optical-model parameters used in the coordinate space calculations. Previous attempts to introduce a single energy shift were not successful on more than one nucleus or at pion energies greater than 150 MeV, presumably because of their choice of kinematics.[15,16]

This method has been extended beyond ^{12}C, and the systematics still apply.[24] Figures 3 and 4 display similar results for pion elastic scattering

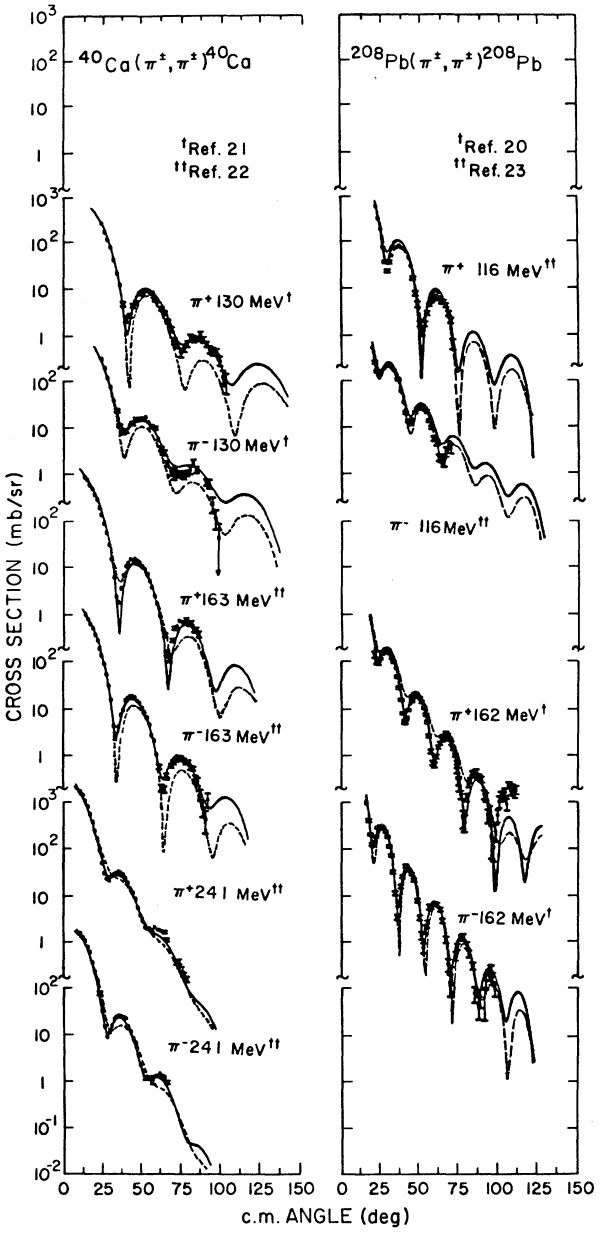

FIG. 4. Same as Fig. 2 but for ^{40}Ca with $\Delta = -30$ MeV and ^{208}Pb with $\Delta = -26$ MeV. To avoid obscuring the effect of Δ on a first-order optical potential, corrections for ^{208}Pb (which is not self-conjugate and has a large Coulomb barrier) have not been included.

from ^{28}Si, ^{40}Ca, and ^{208}Pb. The single-parameter fits (solid lines), resulting in a value of $\Delta = -27 \pm 4$, -30 ± 4, and -26 ± 4 MeV for ^{28}Si, ^{40}Ca, and ^{208}Pb respectively, are a dramatic improvement compared to the unshifted calculations (dashed lines). We obtained similar fits for $\pi^{\pm}$ elastic scattering from all the nuclei that we investigated, including ^{9}Be, ^{13}C, ^{16}O, ^{18}O, ^{42}Ca, ^{44}Ca, ^{48}Ca, ^{54}Fe, and ^{58}Ni. All of these indicate a value for Δ of about -30 MeV. Typical values for the reduced χ^2 are about 2.0 to 20.0 for calculations renormalized within systematic experimental errors. Inclusion of the energy shift also leads to better agreement with the available pion total-cross-section data for ^{12}C and ^{16}O.[25] From our investigation of the possible sources for this effect, we conclude that the shift in energy probably is a result of Fermi momentum of the struck nucleon. The inclusion of an energy shift due to Coulomb effects or of a suitable off-shell model and an angle transformation in momentum-space calculations[26] does not change the value of Δ.

These results indicate that calculating the π-N t matrix at 30 MeV less than the incident pion energy improves agreement with the data for a wide range of target nuclei. The simplicity and apparent generality of this phenomenological kinematic parameter are suggestive of an underlying physical process that can be simply incorporated in the on-shell impulse approximation. It further suggests that many of the higher-order corrections either fortuitously cancel or may not be as important as is presently proposed for pion elastic scattering in the region of the (3,3) resonance.

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Mass Dependence of the Yrast-Yrare Interaction and Backbending in the Light Osmium Isotopes

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Yrast states with spins up to about $22\hbar$ have been identified in ^{176}Os, ^{178}Os, ^{180}Os, and ^{182}Os. In each case anomalies are observed in the yrast sequence. The yrare extensions of the ground-state and Stockholm bands in ^{178}Os and ^{180}Os are also observed and the magnitude of the yrast-yrare interaction matrix elements extracted and discussed.

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In most deformed nuclei in which the yrast states have been identified to high spin, backbending phenomena are observed. Particularly pronounced anomalies are known in $^{182}_{76}$Os, $^{184}_{76}$Os, and $^{186}_{76}$Os,[1,2] and it is now accepted that a band crossing between the rotation-aligned $(i_{13/2})^2$ neutron configuration (the Stockholm or s band) and the ground-state band (gsb) is the cause of the backbending.[3] Although the neutron Fermi level in these heavy osmium isotopes is high among the $i_{13/2}$ orbitals, implying reduced Coriolis effects and therefore reduced rotation alignment, a critical role is played by the large hexadecapole deformation which enhances the Coriolis mixing.[1] Since the hexadecapole deformation is expected to vanish as the neutron number is decreased, a study of the high-spin states in the lighter osmium isotopes is of considerable interest. Systematic studies over a wide isotopic range are of crucial importance given the recent cranking-model predictions of an oscillating interaction strength between the s band and the gsb, as a function of neutron number.[4-6]

We have populated high-spin states in the osmium isotopes from $^{176}_{76}$Os to $^{182}_{76}$Os using (^{16}O, $4n$) and (^{16}O, $5n$) reactions on metallic erbium targets and ^{16}O beams from the Australian National University 14 UD Pelletron accelerator. For each osmium isotope, high-resolution γ-ray measurements included singles excitation functions, γ-γ time-coincidence experiments, and γ-ray angular distributions with a Compton-suppressed Ge(Li) spectrometer.

The partial level schemes showing states associated with the high-spin yrast cascades are shown in Fig. 1. The yrast sequence in $^{182}_{76}$Os, the only case previously studied to high spin, has been extended to spin $24\hbar$. In the other cases

