tic and $\frac{9}{2}^+$ cases, the form factor is dominated by the $M1$ moment and the agreement is reasonably good. However, it should be noted that the H-F prediction for the static dipole moment is approximately one-half the experimental value. As pointed out in Ref. 8, the choice of $g_s^{\text{eff}} = 0.48$ for the proton can reproduce the experimental value of the static dipole moment. At the same time this choice of g has only a small effect ($\lesssim 10\%$) on the $M1$ form factor. This may be understood from the fact that at the peak of the $M1$ form factor the convection current contribution is 4 times larger than the spin magnetization part.

The results of Rad *et al.*,[3] Bertozzi,[9] Creswell *et al.*,[9] and Sasanuma and co-workers[10] demonstrate the ability of the H-F technique to predict the lower multipole Coulomb densities of axially symmetric nuclei, and show that on a microscopic level ^{181}Ta is a very good rotational nucleus. The lack of detailed agreement of the calculations with the higher Coulomb multipole densities and with the present data is not understood. These discrepancies clearly indicate the need to investigate the validity of the specific assumptions in the H-F calculations as well as the sensitivity of the results to the details of the nucleon-nucleon force if detailed agreement with the experimental measurements is to be achieved.

This work was supported by the U.S. Department of Energy, Contract No. DE-AC02-76ERO-3069. One of us (M.V.H.) is a recipient of a Weizman Foundation Fellowship.

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Detailed Test of the Interacting Boson Approximation in a Well-Deformed Nucleus: The Positive-Parity States of ^{168}Er

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(Received 25 September 1980)

The interacting boson approximation has been applied to the deformed nucleus ^{168}Er. The parametrization used corresponds to a description close to the SU(3) limit of the model. The calculation, which is the most detailed test of the interacting boson approximation to date, correctly reproduces the complete sequence of positive-parity collective bands below the pairing gap and provides an excellent overall description of their decay properties.

PACS numbers: 21.60.Ev, 21.60.Fw, 27.70.+q

The interacting boson approximation (IBA)[1] has now been applied to a large number of nuclei with widely varying structure, and has met with a great deal of success in predicting many different properties[2] but no detailed test has yet been made in a deformed nucleus. The underlying SU(6) group structure of the model basis leads[1] to three limiting symmetries, SU(5), SU(3), and O(6),

corresponding in the geometrical description to vibrational, rotational, and γ-unstable nuclei, respectively. While simple analytic expressions can be extracted to describe nuclear properties in these limits, the detailed description of a particular nucleus generally requires a complete diagonalization of the model Hamiltonian, and such a technique then leads naturally to a unified description both of nuclei close to the limits, and of those in between. A thorough test of the collective structure predicted by the model for a given nucleus naturally requires a complete experimental knowledge of the collective states in that nucleus, and of their decay properties. In this respect, level schemes deduced via the reaction (n, γ) provide an excellent basis for comparison, since the nonselective nature of the reaction can ensure the population of a wide range of states, regardless of their structure. In fact, the combination of the techniques of curved crystal spectrometer and average resonance capture (ARC) in such (n, γ) studies has recently led to the establishment of a level scheme[3] for ^{168}Er which is undoubtedly the most detailed and complete currently available for an even-mass deformed nucleus. This scheme is *known to be complete*[4] for all levels with $J<6$ and $E_x<2$ MeV and thus offers a unique opportunity to compare the predictions of the IBA with the most well understood and developed of the geometrical descriptions, that of the familiar symmetric rotor, and thereby to assess its usefulness and applicability for a large class of nuclei. Indeed, recent controversy[5] concerning the overall validity of the underlying basis of the IBA is centered on the region of deformed nuclei, since it has been suggested that the presence of a deformed field is incompatible with the adoption of a basis consisting only of particles paired to $L=0$ and 2. Thus it is particularly timely to test the IBA in such nuclei.

The natural starting point in an IBA description of a symmetric rotor is the SU(3) limit, and the philosophy was to attempt such a description for all the positive-parity levels below the pairing gap, in as simple a framework as possible. The calculations have been performed in the IBA-1 scheme, with the code PHINT,[6] which makes no distinction between neutron and proton bosons. A truncated "multipole expansion" of the IBA Hamiltonian was used in the calculations, namely

$$H_{s,d} = -\kappa Q \cdot Q - \kappa' L \cdot L + \kappa'' P \cdot P. \quad (1)$$

The SU(3) limit of the IBA describes a symmetric rotor with degenerate β and γ bands. This limit arises naturally from the Hamiltonian of Eq. (1) when the last term is zero. In such a case,

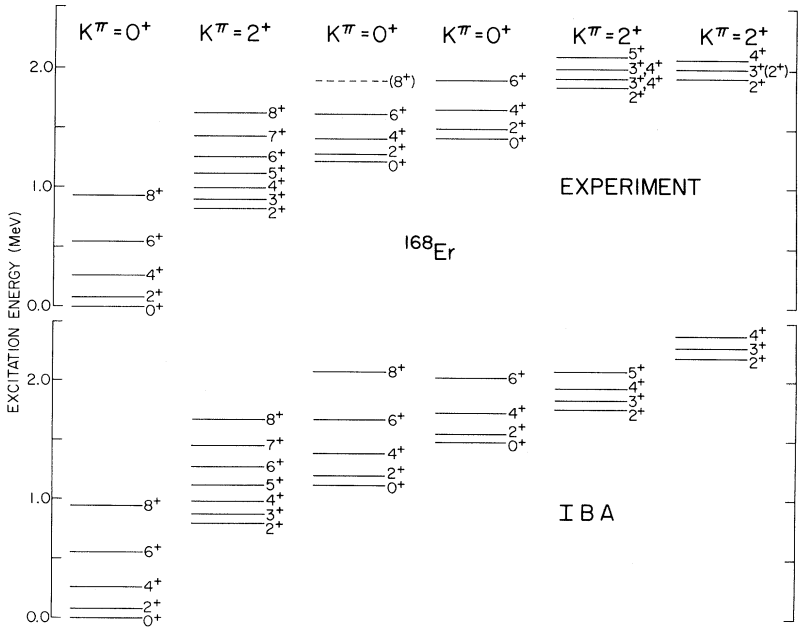

FIG. 1. Levels (Ref. 3) in ^{168}Er compared with the results of an IBA calculation for 16 bosons. Only experimental bands below the pairing gap, estimated as ~2 MeV, have been included. The constants κ and κ' of Eq. (1), calculated from the energies of the first two 2^+ states, are 0.004 and -0.010 MeV. The parameter κ'' was the only one varied to optimize the agreement with the experimental energies and the value chosen was 0.015 MeV.

the energies of states can be described by the expression

$$E = (0.75\kappa - \kappa')J(J+1) - \kappa C(\lambda, \mu), \quad (2)$$

where $C(\lambda, \mu)$ is the Casimir operator.[1] The first term describes the spacings within rotational bands while the second, dependent only on the $Q \cdot Q$ interaction, determines the full set of intrinsic energies of the collective bands. Thus the constants κ and κ' in the SU(3) limit can be extracted from the energy of the first and second 2^+ states. Any perturbation to the rigorous SU(3) limit can be expected to be in the direction of the O(6) limit which occurs at the end of the shell. The following procedure was therefore adopted. The values of κ and κ' were extracted, as described above, from the energies of the 2_1^+ and 2_2^+ states. This results in a level scheme in which the ground and γ bands are well reproduced, but in which the β band is degenerate with the γ band, and in which the higher lying bands lie in degenerate groups also, corresponding to the multiphonon excitations ($\beta\gamma$, $\gamma\gamma$, etc.) of the geometrical model. The pairing term of Eq. (1), which is important in the O(6) limit, was then introduced and the parameter κ'' varied to obtain the final calculated sequence of levels, which are compared with experiment in Fig. 1. It should be emphasized, therefore, that the complete sequence of bands shown in the lower part of Fig. 1 results effectively from the choice of *a single parameter*, given that κ and κ' were fixed from two experimental energies. No attempt to further improve the agreement between theory and experiment was made.

It can be seen from Fig. 1 that the entire experimental sequence of states belonging to $K=0$ and 2 bands has been well reproduced. Note that, although for convenience the experimental bands have been labeled by K quantum numbers, there is no prior assumption of K quantum numbers or band structure inherent in the IBA description. Nevertheless, not only do level sequences resembling β and γ bands appear, but also the calculated scheme encompasses the higher lying sequences which would correspond to multiphonon excitations. Also, given the *known completeness* of the experimental level scheme[3] mentioned earlier, it can be stated that in fact all observed positive-parity bands below 2 MeV with $K<5$ are predicted. The one exception to this is a $K^\pi = 3^+$ band at 1653 keV, which lies outside the bases of all the usual collective descriptions, and can only be accounted for in the IBA-1 by incorporating a g boson.

TABLE I. Comparison of experimental and theoretical $B(E2)$ branching ratios from states of the γ band in ^{168}Er.

Transition I_i	I_f,K	B(E2;J_i-J_f) (Relative) IBA	Exp[a]
2^+	0^+,0	66.	54.
	2^+,0	100	100
	4^+,0	6.0	6.8
3^+	2^+,0	2.7	2.6
	4^+,0	1.3	1.7
	2^+,2	100	100
4^+	2^+,0	2.5	1.6
	4^+,0	8.3	8.1
	6^+,0	1.0	1.1
	2^+,2	100	100
5^+	4^+,0	4.3	2.9
	6^+,0	3.1	3.6
	3^+,2	100	100
	4^+,2	98.5	122.
6^+	4^+,0	0.97	0.44
	6^+,0	4.3	3.8
	8^+,0	0.73	1.4
	4^+,2	100	100
	5^+,2	59.	69.
7^+	6^+,0	2.7	0.74
	5^+,2	100	100
	6^+,2	39.	59.
8^+	6^+,0	0.67	1.8
	8^+,0	3.5	5.1
	6^+,2	100	100
	7^+,2	29.	135.

[a]For all $\Delta I = 0$ or 1 transitions, up to and including the $7_\gamma^+ \rightarrow 6_g^+$, multipolarities have been determined (Refs. 3 and 7) and measured $M1$ components subtracted.

There are, however, emerging discrepancies near the pairing gap which may be symptomatic of admixtures of noncollective excitations which are outside the IBA basis. In particular the highest lying $K^\pi = 2^+$ band is predicted at somewhat too high an energy, and in addition, the calculation predicts a $K^\pi = 4^+$ band at 1620 keV, while the lowest 4^+ band found experimentally is at 2030 keV.

In order to compare experimental and theoretical branching ratios, $B(E2)$ values have been calculated. The $E2$ operator in the IBA is given by

$$T(E2) = \alpha(d^\dagger s + s^\dagger d)^{(2)} + (\beta/\sqrt{5})(d^\dagger d)^{(2)}. \quad (3)$$

Again, in the present calculation, no attempt was made to optimize the constants of Eq. (3). Instead, these were calculated from the measured values of $B(E2{:}0_1^+ \rightarrow 2_1^+)$ and $B(E2{:}0_1^+ \rightarrow 2_2^+)$ and thus it should be emphasized that there were *no free parameters* associated with the calculation of $B(E2)$ values in this study.

The results of these calculations for the γ band are given in Table I and it can be seen that the agreement between theory and experiment is excellent. The only significant apparent discrepancy is that for the $8^+ - 7^+$ transition, which was very weak experimentally and has an unknown $M1$ component.

It should be remarked that the branching ratios predicted for interband or intraband transitions alone are very close to those which would result from application of the Alaga rules.[8] Hence it can be inferred that, although the rigorous SU(3) symmetry of the IBA has been clearly perturbed, in that, for instance, the β and γ bands are now far from degenerate, the wave functions of the states are still very close to those of the limit so that in a geometrical description, the bands would be thought of as having good K quantum numbers. The most impressive success of the calculation is the correct prediction of the branching ratios *between different bands*, shown in Fig. 2 for those states in the γ and β bands for which branches to all possible bands have been observed. Since these ratios are dependent on the relative *intrinsic* structure of the bands, they cannot, of course, be obtained from the Alaga rules.

One crucial feature evident in Fig. 2 is the dominance of the branches from the β band to the γ band, over those to the ground band. This results from the basic symmetry properties of the IBA Hamiltonian since the β and γ bands belong to a different SU(3) representation than the ground state band, and the $E2$ operator cannot connect these different representations. The small $P \cdot P$ perturbation which breaks the rigorous SU(3) symmetry allows transitions between the two representations; however, the dominance of the β-γ transitions remains. In the pure geometrical description, β-γ transitions would be forbidden, since they would require the destruction of one type of vibration, and the creation of another. A similar decay pattern could probably be reproduced in the framework of a multiparameter, multiband mixing calculation in the Bohr-Mottelson description,[8] and it is interesting to note that

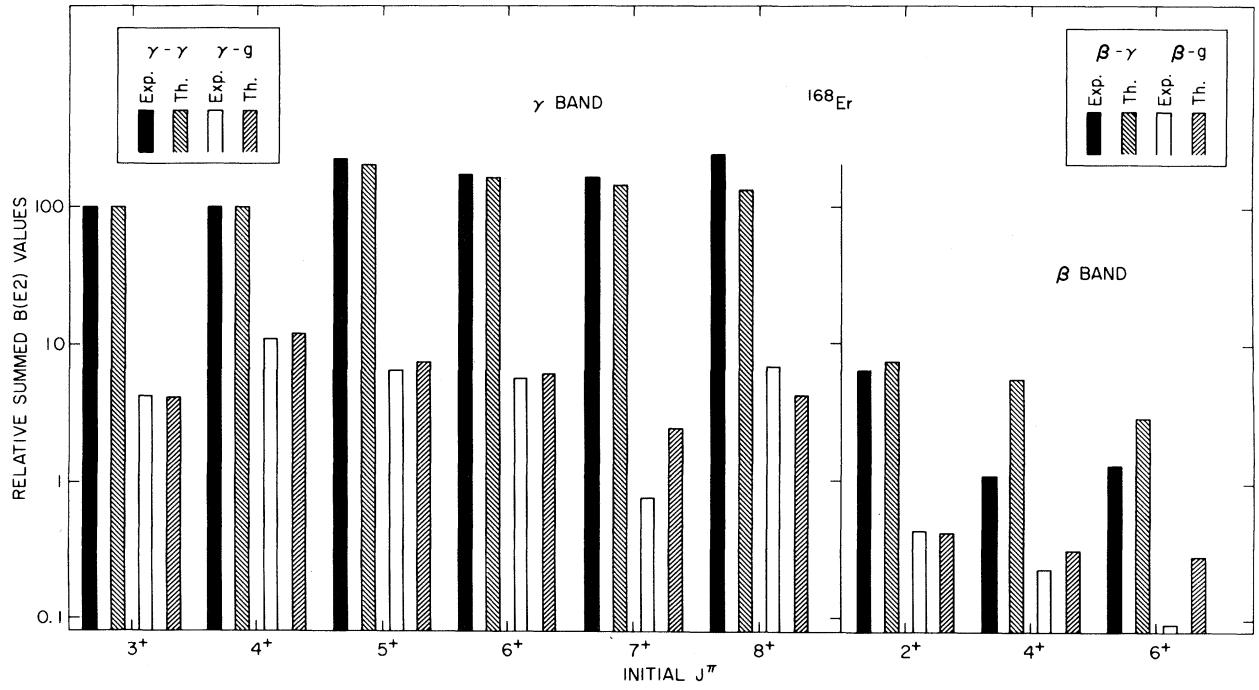

FIG. 2. Comparison of calculated and experimental values of the summed $B(E2)$ strengths from states in the γ and β bands to the γ and ground bands. For each initial state the bars represent the sum of all observed or calculated transitions to a given final band. The $B(E2)$ values have been normalized to 100 for an *intra*band transition in each case. To keep the figure uncluttered for the β band, only the *inter*band transitions are plotted.

the IBA wave functions can in fact be expressed in terms of a basis involving the K quantum number, and that such a transformation automatically results in finite admixtures between different (pure K) bands, even in the rigorous SU(3) limit.[1] Indeed, it is hardly surprising that the end results of the two descriptions can be made almost equivalent when applied to "good" rotational nuclei. The crucial point is the natural appearance in the IBA of the specific band admixtures which give rise to the strong β-γ branches. It would be interesting, therefore, to inquire whether the dominance of the β-γ branch occurs in other nuclei of this type. The consistent occurrence of this feature would further support the validity of the underlying basis of the IBA.

This calculation represents the most detailed test to date of the IBA. Although some discrepancies arise for states at higher energies, the overall agreement with the data is impressive, both in terms of the correct prediction of the complete set of positive-parity bands appearing below the pairing gap, and the reproduction of the experimental branching ratios, both interband and intraband. This agreement is particularly convincing in view of the extremely simple parametrization chosen for the model Hamiltonian.

Research has been supported by the U. S. Department of Energy under Contract No. DE-AC02-76CH00016.

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Exotic Heavy-Ion Reactions on ^{40}Ca: (^{14}C, ^{14}O) Double Charge Exchange and (^{14}C, ^{15}O) Rearrangement Transfer

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Double charge exchange is observed for the first time with heavy ions by using the reaction ^{40}Ca(^{14}C, ^{14}O)^{40}Ar at 51 MeV incident energy. Angular distributions for the rearrangement-transfer reaction ^{40}Ca(^{14}C, ^{15}O)^{39}Ar were also measured, and both exotic reactions are compared with the more usual one- and two-proton transfer reactions (^{14}C, ^{15}N) and (^{14}C, ^{16}O). Cross sections for the exotic processes are surprisingly large: ~10 μb/sr for (^{14}C, ^{14}O) and ~200 μb/sr for (^{14}C, ^{15}O).

PACS numbers: 25.70.Bc, 25.70.De, 25.70.Hi

It has long been realized that heavy-ion beams offer the possibility to study certain nuclear reactions that may be regarded as "exotic" because they involve an unusual reaction mechanism or a novel nuclear structure phenomenon. By using a ^{14}C beam, we have measured two reactions that belong in this category: (1) the double-charge-exchange (^{14}C, ^{14}O) reaction, and (2) the rearrangement-transfer (^{14}C, ^{15}O) reaction.

The interest in double charge exchange (DCE) stems from at least two important features. First is the ability to reach nuclei very far from the line of stability,[1,2] and second is the possibility that two-nucleon or pairing correlations within nuclei can be probed in a unique way with the DCE process.[3,4] Previous reports of DCE have been based only on experiments with pion beams[1,3,5] via the reactions (π^+,π^-) and (π^-,π^+), and many questions have been raised about these processes. Heavy ions open a new approach to DCE, which, while addressing the features that make the DCE phenomenon so unique, may shed some additional light on the pion process as well. Equally interesting is the investigation of the