represented by weakly coupled gluons, but the nature of the gauge excitations responsible for the observed sharp increase in the specific heat in the transition region remains to be elucidated.

We would like to thank R. Brower and P. Hasenfratz for many stimulating discussions. This work was supported in part by the National Science Foundation.

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Magnetic Electron Scattering from ^{181}Ta

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(Received 30 June 1980)

Transverse electron scattering form factors from the ground-state rotational band of ^{181}Ta have been measured to study the single-particle contribution to the magnetization current density. The data are compared with a Hartree-Fock calculation by use of density matrix expansion with filling approximation.

PACS numbers: 25.30.Cg, 21.10.Ky, 27.70.+q

This Letter reports the first measurements of the transverse electron scattering form factor from the ground-state band of a heavy odd-even rotational nucleus. In electron scattering, the nucleus interacts with the electromagnetic field of the electron via its charge, current, and magnetization. In strongly deformed nuclei, the charge scattering provides information about the collective properties of nuclei; whereas the magnetic scattering, which is transverse, results principally from one or a few nucleons and is a measure of single-particle aspects. These results are used to study the single-particle contributions to the ground-state magnetization and to the transition current densities and are compared directly with Hartree-Fock (H-F) calculations.

At $\theta=\pi\pm\delta\theta$, the electron scattering cross section can be written in first Born approximation as

$$\frac{d\sigma}{d\Omega}\approx\left(\frac{Z\alpha\hbar c}{2E_0}\right)^2\{\tfrac{1}{4}(\delta\theta)^2|F_L|^2+[1+\tfrac{3}{8}(\delta\theta)^2]|F_T|^2\},$$

where Z is the atomic number of the target nucleus, α, θ, and E_0 are the fine structure constant, the scattering angle, and the incident energy, respectively; and F_L and F_T are, respectively, the longitudinal and transverse form factors. The finite angular aperture of the apparatus ranged from 6.48×10^{-3} to 5.75×10^{-2} rad.

The present experiment was performed at the 400-MeV electron scattering facility of the Bates Electron Accelerator. The scattered electron spectra were measured at 180° scattering angle. The high-resolution energy-loss spectrometer, whose performance and design are described in detail elsewhere,[1] was used along with the 180° scattering system[2] to obtain these data over the effective momentum transfer range 0.7 fm^{-1} $< q_{eff} < 2.5$ fm^{-1}. Electron beam currents of up to 50 μA were incident on tantalum and tungsten targets. Two different target thicknesses of tantalum (9.88 and 21.7 mg/cm^2) were used to obtain the data at low- and high-momentum transfer, respectively. The energy resolution over the entire momentum range studied allowed a clear separation among the lower members of the ground-state rotational band.

Because of finite solid angle at 180°, the measured cross section includes a contribution from the longitudinal form factors. This contribution was subtracted by measurements on even-even tungsten targets for every energy under the same experimental conditions. Measurements at the same energy with two different target thicknesses demonstrated the adequacy of the subtraction technique with respect to the multiple scattering effect. In the range of momentum transfer where these subtractions are important, the intrinsic Coulomb multipoles[3] of ^{181}Ta are almost identical to those of the even-even tungsten isotopes. This was established by measurements at 90° on these isotopes. In subtracting the Coulomb contributions from the tantalum measurements appropriate combinations of the multipoles were made using Clebsh-Gordon coefficients as described in Ref. 4. The three tungsten targets used were foils of WO_3 evaporated onto a ^{12}C backing and were obtained from Oak Ridge National Laboratory. The three targets were, respectively, enriched in the isotopes ^{182}W, ^{183}W, and ^{184}W with the abundances being 94.32%, 82.5%, and 94.87%. The target thicknesses were determined by comparing elastic electron scattering measurements from these targets to those from a natural tungsten target of known thickness and abundance. These target thickness measurements, at five different energies, were done at low-momentum transfer at 90° laboratory angle. The two even-even targets had a small amount of ^{183}W impurity (2.54% and 1.82%, respectively). However, several measurements in the region of low-momentum transfer with the ^{183}W target showed that no effect would be observable in these even-even targets because of the presence of ^{183}W.

The observed cross sections from the even-even tungsten targets were purely Coulomb within the uncertainties of the experiment. For example, $|F_T(q)|^2$ for the first 2^+ excitation of tungsten was less than 3×10^{-8} at $q_{eff} = 1.1$ fm^{-1}. This result was derived by comparing the measurement

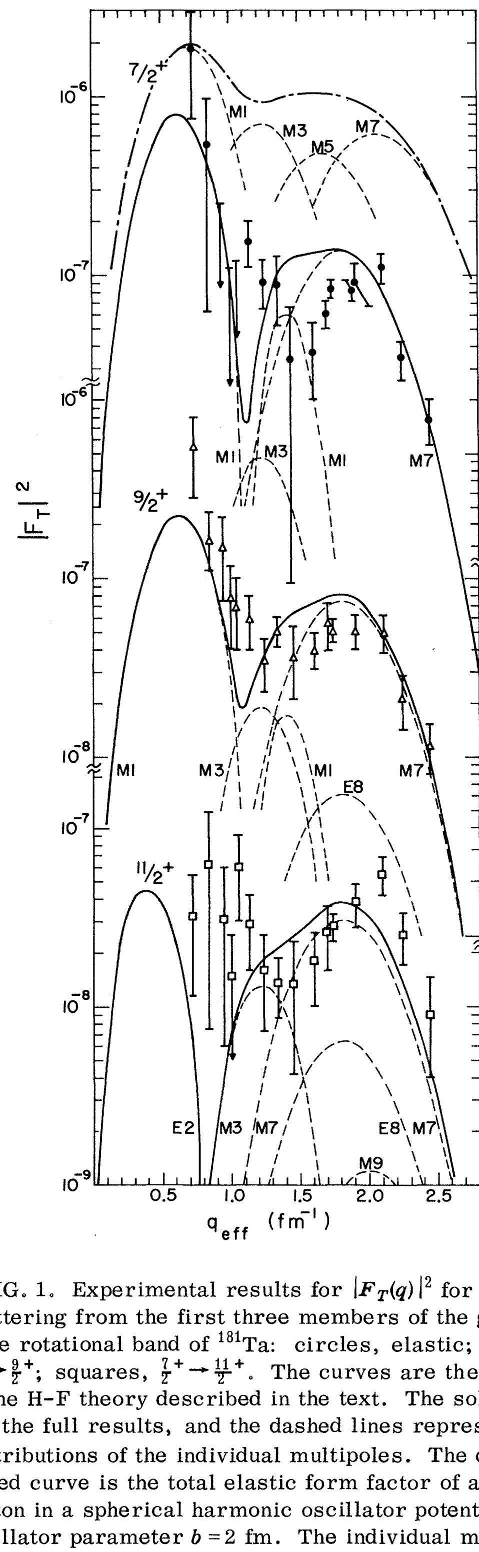

FIG. 1. Experimental results for $|F_T(q)|^2$ for electron scattering from the first three members of the ground-state rotational band of ^{181}Ta: circles, elastic; triangles, $\frac{7}{2}^+ \to \frac{9}{2}^+$; squares, $\frac{7}{2}^+ \to \frac{11}{2}^+$. The curves are the result of the H-F theory described in the text. The solid lines are the full results, and the dashed lines represent the contributions of the individual multipoles. The dash-dotted curve is the total elastic form factor of a $g_{7/2}$ proton in a spherical harmonic oscillator potential with oscillator parameter $b = 2$ fm. The individual multipoles, $M1$–$M7$, are shown as dashed curves.

of the elastic scattering cross section from the tungsten isotopes at $\theta=180°$ with the predictions of a phase-shift calculation by use of a charge density that reproduced the elastic cross section at $\theta=90°$. This established the effective angle of the spectrometer or the deviation from exactly 180°. The 2^+ cross section was then determined at the effective angle assuming only charge scattering. The distorted-wave Born approximation calculation in this case used the same elastic scattering charge density and a transition charge that reproduced the 2^+ cross section at $\theta=90°$. This result agreed with the measurement at the 180° scattering angle, establishing the Coulomb nature of the 2^+ cross section. A similar calculation established the 4^+ excitations in tungsten to be pure Coulomb. In addition, the contribution due to the finite mass of the electron has been evaluated and found to be small.

All measurements in this experiment were absolute, and no normalization was made to any other data. As a verification of the calibrations of the apparatus, thin foils of ^{9}Be and ^{27}Al were mounted to the back of the tantalum targets. The cross sections obtained for ^{9}Be are in good agreement with the measurements at other laboratories,[5] and the cross section measurement for ^{27}Al are in excellent agreement with other measurements[6] made in this laboratory. These targets as well as the carbon backings of the tungsten targets also allowed an accurate determination of the bombarding energy by the recoil kinematics.

The transverse form factor at 180° for the $\frac{7}{2}^+$, $\frac{9}{2}^+$, and $\frac{11}{2}^+$ members of the ground-state rotational band of ^{181}Ta are shown in Fig. 1, where $|F_T|^2$ has been plotted as a function of effective momentum transfer, q_{eff}.[7] The error bars indicated are statistical. Above the momentum transfer of 1.7 fm^{-1} no Coulomb subtraction was needed. Below 1.7 fm^{-1}, however, Coulomb subtractions were performed as described above. Other members of the ground-state rotational band are not presented, since they could not be resolved from other states.

Theoretical calculations[8] are also shown in Fig. 1 (dashed curves for individual multipoles and solid curves for their sum) for the first three members of the ground-state rotational band of ^{181}Ta. This calculation has been done in the "pair-filling approximation" to allow the application of H-F calculation to odd-A nuclei. In this approximation, the states of angular momentum projection on the symmetry-axis (Ω and $-\Omega$) are assumed to be equally populated for the odd nucleon. Based on this model, the core contributions to the transverse form factors are quite small as compared to the single-particle contributions, except for the transverse $E2$ multipole in the region of low-momentum transfer for the $\frac{11}{2}^+$ state. The total transverse form factor is an incoherent sum over the possible multipoles of the transverse electric and magnetic form factors arising from single-particle and collective contributions. Neglecting the small core part, the cross section reduces to a sum of the single-particle intrinsic form factors $F_K{}^{ML}(q)$, $F_{2K}{}^{ML}(q)$, and $F_{2K}{}^{EL}(q)$ according to the expression,

$$[\sigma_{I_i\to I_f}(q)]_{\text{s.p.}}=\left(\frac{Z\alpha\hbar c}{2E_0}\right)^2\Big[\sum_{L=\text{odd}}|\langle I_i,K;L,0|I_f,K\rangle F_K{}^{ML}(q)+\langle I_i,-K;L,2K|I_f,K\rangle F_{2K}{}^{ML}(q)|^2 + \sum_{L=\text{even}}|\langle I_i,-K;L,2K|I_f,K\rangle F_{2K}{}^{EL}(q)|^2\Big].$$

It should be pointed out that all the ML multipoles in the elastic scattering are considerably reduced compared to those of a $g_{7/2}$ proton in the spherical shell model, as shown by the dashed-dot curve in Fig. 1.

These Hartree-Fock calculations have been done in plane-wave Born approximation. Nevertheless, the effects of the distortion can be well reproduced by plotting the experimental results as a function of the effective momentum transfer. As seen, the theoretical calculation is in good qualitative agreement with the present measurements. In detail, however, there are a number of obvious discrepancies between theory and experiment as well as some notable agreement. The theory predicts that the high-q region is dominated by the $M7$ multipole and yields form factors which are reasonably well reproduced experimentally in the case of the ground state and the $\frac{9}{2}^+$ state. In the $\frac{11}{2}^+$ case the agreement is less satisfactory. For the lower q side of the $M7$ lobe, the theory predicts substantially more cross section than is observed in all three form factors. In the region of intermediate q, where a number of multipoles are contributing, the theory is substantially out of line when compared to the experimental results. At low q in the elastic and $\frac{9}{2}^+$ cases, the form factor is dominated by the $M1$ moment and the agreement is reasonably good. However, it should be noted that the H-F prediction for the static dipole moment is approximately one-half the experimental value. As pointed out in Ref. 8, the choice of $g_s^{\rm eff} = 0.48$ for the proton can reproduce the experimental value of the static dipole moment. At the same time this choice of g has only a small effect ($\lesssim 10\%$) on the $M1$ form factor. This may be understood from the fact that at the peak of the $M1$ form factor the convection current contribution is 4 times larger than the spin magnetization part.

The results of Rad *et al.*,[3] Bertozzi,[9] Creswell *et al.*,[9] and Sasanuma and co-workers[10] demonstrate the ability of the H-F technique to predict the lower multipole Coulomb densities of axially symmetric nuclei, and show that on a microscopic level ^{181}Ta is a very good rotational nucleus. The lack of detailed agreement of the calculations with the higher Coulomb multipole densities and with the present data is not understood. These discrepancies clearly indicate the need to investigate the validity of the specific assumptions in the H-F calculations as well as the sensitivity of the results to the details of the nucleon-nucleon force if detailed agreement with the experimental measurements is to be achieved.

This work was supported by the U.S. Department of Energy, Contract No. DE-AC02-76ERO-3069. One of us (M.V.H.) is a recipient of a Weizman Foundation Fellowship.

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Detailed Test of the Interacting Boson Approximation in a Well-Deformed Nucleus: The Positive-Parity States of ^{168}Er

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(Received 25 September 1980)

The interacting boson approximation has been applied to the deformed nucleus ^{168}Er. The parametrization used corresponds to a description close to the SU(3) limit of the model. The calculation, which is the most detailed test of the interacting boson approximation to date, correctly reproduces the complete sequence of positive-parity collective bands below the pairing gap and provides an excellent overall description of their decay properties.

PACS numbers: 21.60.Ev, 21.60.Fw, 27.70.+q

The interacting boson approximation (IBA)[1] has now been applied to a large number of nuclei with widely varying structure, and has met with a great deal of success in predicting many different properties[2] but no detailed test has yet been made in a deformed nucleus. The underlying SU(6) group structure of the model basis leads[1] to three limiting symmetries, SU(5), SU(3), and O(6),

