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Correlations and Specific Heat of the SU(2) Lattice Gauge Model

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Monte Carlo calculations of the specific heat of the four-dimensional SU(2) lattice gauge model show a sharp peak where Creutz discovered a crossover in the value of the string tension. In this region is also found a rapid increase in the correlation length and a slowing down in the Monte Carlo convergence towards equilibrium.

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It is generally believed that the SU(N) lattice gauge models in four dimensions do not exhibit phase transitions, giving a simple picture for the confinement of static quarks, in accordance with the Wilson criteria.[1,2] Recently Creutz[3,4] has given evidence in support of this conjecture by evaluating the string tension for SU(2) and SU(3) gauge models using Monte Carlo simulations on finite lattices, and finding a rapid crossover between the expected strong coupling and asymptotic freedom limits. Smooth matching between these two domains had previously been discussed from strong coupling expansions of the β function.[5] The relation between the lattice and the continuum scales has been obtained analytically by Hasenfratz and Hasenfratz[6] in agreement with Creutz's numerical results. Other tests of quark confinement ideas by Monte Carlo calculations have been carried out by applying 't Hooft's twisted boundary conditions[7] on SU(2) lattice gauge models.[8]

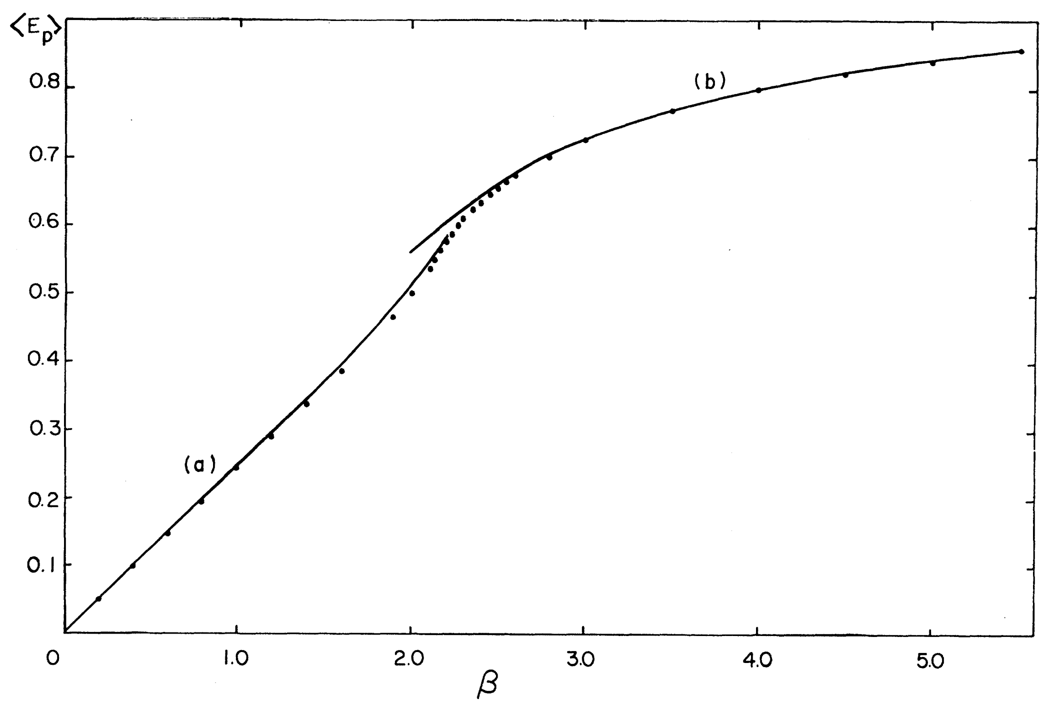

FIG. 1. The SU(2) average plaquette energy $\langle E_p \rangle$ as a function of β for a lattice of size 4^4. The curves labeled a and b are obtained from the strong and weak coupling expansions, Eqs. (2) and (3).

In this paper we study the nature of the transition between the strong and weak coupling regimes by evaluating correlations between the SU(2) plaquettes by the Monte Carlo techniques of Ref. 3. We have computed the correlations between all nearest-neighbor plaquettes and the sum of all plaquette-plaquette correlations which corresponds to the lattice specific heat C,

$$C \equiv \beta^2 \frac{d}{d\beta}\langle E_p\rangle = \beta^2 \sum_{p'} [\langle E_p E_{p'}\rangle - \langle E_p\rangle^2], \tag{1}$$

where E_p is the energy of the pth plaquette,[1,2] and $\beta = 4/g^2$, where g is the bare coupling constant.[3] If the correlation length is short we expect that this sum is saturated by the contribution of nearest-neighbor plaquettes, while a growing correlation length is indicated by the increasing contribution of more distant plaquettes and a dependence on the finite size of the lattice. We have carried out Monte Carlo computations on lattices of size L^4, where $L = 4$, 5, and 6.

In Fig. 1 we show our results for the average plaquette energy $\langle E_p\rangle$ as a function of β in agreement with the calculation of Creutz.[3] For comparison we have plotted also a strong coupling expansion, curve a, and a weak coupling approximation, curve b, where for curve a,

$$\langle E_p\rangle = (\tfrac{1}{4}\beta + \tfrac{1}{48}\beta^3 + \tfrac{7}{1536}\beta^5)/(1 + \tfrac{1}{8}\beta^2 + \tfrac{1}{192}\beta^4) \tag{2}$$

and for curve b,

$$\langle E_p\rangle = 1 - \tfrac{3}{4}\beta^{-1} - 0.13\beta^{-2} - 0.29\beta^{-3}. \tag{3}$$

Although the coefficients of the weak coupling expansion can be computed analytically, this is a nontrivial task beyond the first two terms.[10] We have obtained the coefficients of the β^{-2} and β^{-3} terms in Eq. (3) approximately by a least-squares fit to Monte Carlo data for $4 \le \beta \le 10.5$.

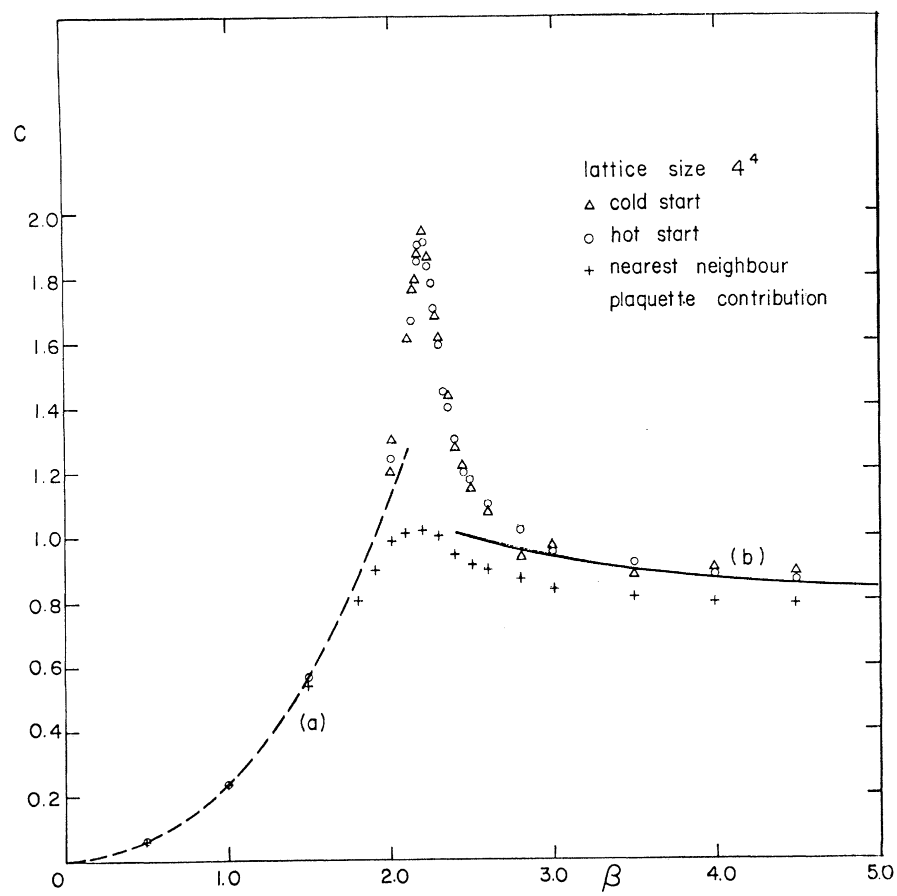

FIG. 2. The specific heat C as a function of β obtained from Monte Carlo runs starting from cold (triangles) and hot (circles) configurations and the contribution to C from nearest-neighbor plaquettes (crosses). The curves labeled a and b are obtained from the derivatives of the strong and weak coupling expansion, Eqs. (2) and (3).

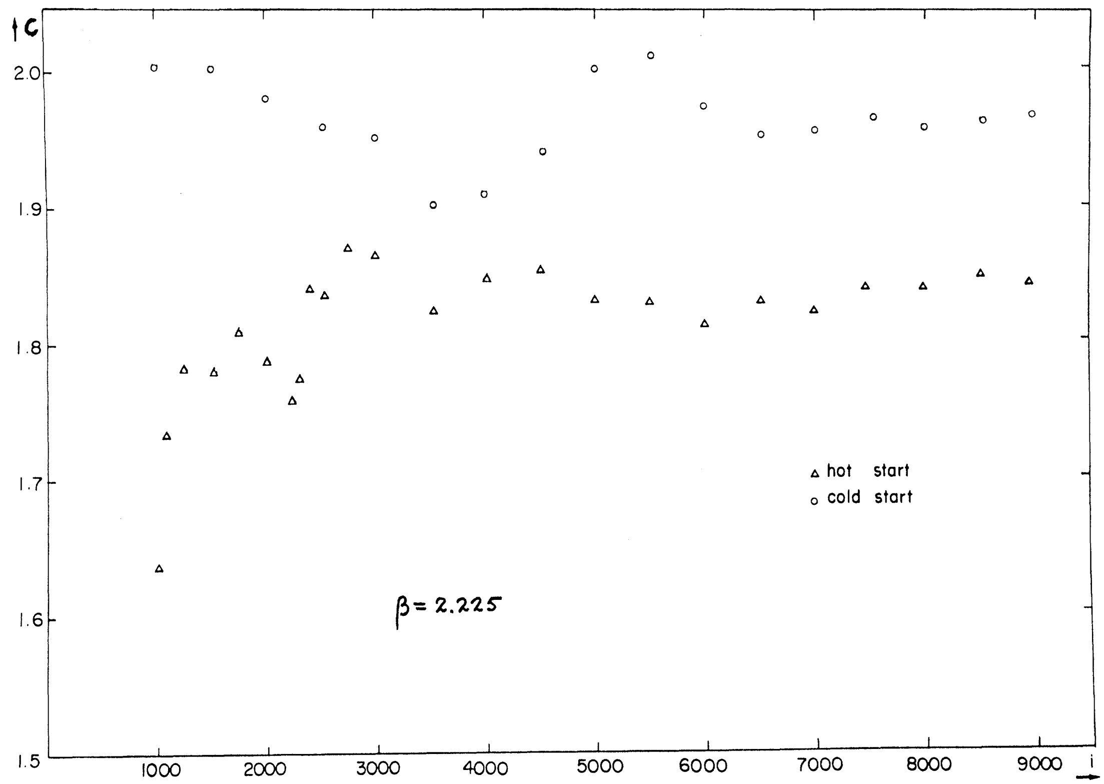

FIG. 3. The evolution of the specific heat C as a function of Monte Carlo iterations for a lattice of size 5^4, starting from cold (triangles) and hot (circles) configurations.

The specific heat C is obtained from the correlation sum, Eq. (1), starting from ordered (cold start) and disordered (hot start) configurations. For a lattice of size 4^4 the results for C obtained from approximately 5000 iterations at each value of β are shown in Fig. 2, together with the contribution to C of nearest-neighbor plaquettes only. The most striking results are the occurrence of a sharp peak in C at $\beta \cong 2.2$ and a rapid increase in the correlation length near this peak.[11] We note that this peak occurs precisely where Creutz found a rapid crossover in the string tension between the strong and weak coupling regimes. Outside the location of this peak the Monte Carlo results are in excellent agreement with the strong coupling expansion (curve a) and weak coupling approximation (curve b) obtained by evaluating $\beta^2 d\langle E_p\rangle/d\beta$ from Eqs. (2) and (3), respectively. We have also obtained values of C near $\beta = 2.2$ for lattices of size $L = 5$ and 6, which indicate that the peak does not increase, in sharp contrast with our previous results for the U(1) lattice gauge model.[12] However, there is a small shift in C, giving additional evidence of a growing correlation length in this region. Furthermore, the convergence of the Monte Carlo process starting from ordered and disordered configurations is considerably slowed down in this region as shown in Fig. 3. This behavior is similar to that found in the U(1) model where critical slowing down can be attributed to the occurrence of competing phases in the lattice.[12]

It is apparent from our results that the nature of the transition between the strong and weak coupling regime in the SU(2) lattice gauge model is of a novel and nontrivial character which has not been found in any known spin systems, with the possible exception of the Heisenberg model in two dimensions.[13] The correspondence in the location of the specific-heat maximum and the rapid crossover of the string tension indicates that this crossover is due to a bulk behavior of the lattice, and not to a roughening transition as has been recently speculated by several authors.[14,15] Scaling theory implies that the string tension is proportional to the inverse square of the correlation length which is increasing rapidly in this crossover region. For large β it is clear from the energy and specific-heat curves, Figs. 1 and 2, that the SU(2) lattice gauge excitations are well represented by weakly coupled gluons, but the nature of the gauge excitations responsible for the observed sharp increase in the specific heat in the transition region remains to be elucidated.

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Magnetic Electron Scattering from ^{181}Ta

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Transverse electron scattering form factors from the ground-state rotational band of ^{181}Ta have been measured to study the single-particle contribution to the magnetization current density. The data are compared with a Hartree-Fock calculation by use of density matrix expansion with filling approximation.

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This Letter reports the first measurements of the transverse electron scattering form factor from the ground-state band of a heavy odd-even rotational nucleus. In electron scattering, the nucleus interacts with the electromagnetic field of the electron via its charge, current, and magnetization. In strongly deformed nuclei, the charge scattering provides information about the collective properties of nuclei; whereas the magnetic scattering, which is transverse, results principally from one or a few nucleons and is a measure of single-particle aspects. These results are used to study the single-particle contributions to the ground-state magnetization and to the transition current densities and are compared directly with Hartree-Fock (H-F) calculations.

At $\theta=\pi\pm\delta\theta$, the electron scattering cross section can be written in first Born approximation as

$$\frac{d\sigma}{d\Omega}\approx\left(\frac{Z\alpha\hbar c}{2E_0}\right)^2\left\{\tfrac{1}{4}(\delta\theta)^2|F_L|^2+\left[1+\tfrac{3}{8}(\delta\theta)^2\right]|F_T|^2\right\},$$

where Z is the atomic number of the target nucleus, α, θ, and E_0 are the fine structure constant, the scattering angle, and the incident energy, respectively; and F_L and F_T are, respectively, the longitudinal and transverse form factors. The finite angular aperture of the apparatus ranged from 6.48×10^{-3} to 5.75×10^{-2} rad.

The present experiment was performed at the 400-MeV electron scattering facility of the Bates Electron Accelerator. The scattered electron spectra were measured at 180° scattering angle. The high-resolution energy-loss spectrometer,