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η Decay and the Quark Structure of the ϵ

R. Aaron and H. Goldberg
Department of Physics, Northeastern University, Boston, Massachusetts 02115
(Received 4 April 1980)

The decay rate for $\eta \to 3\pi$ is calculated with use of the u_3 tadpole piece of the quark Hamiltonian, but without assumptions of chiral perturbation theory. The calculation is performed within the framework of the bag model; however, the results are independent of bag parameters and depend only on (1) the light quark mass difference $|m_u - m_d|$ and (2) the quark structure of the $\epsilon(700)$. Comparison of the present calculated η decay rate with experiment shows that most theoretical estimates of $m_u - m_d$ imply a substantial four-quark component in the $\epsilon(700)$.

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A convincing (and successful) calculation of the amplitude for $\eta \to 3\pi$ remains an elusive goal of the particle theorist. The problems associated with past calculations are well known,[1] and we sketch them briefly: Let $\mathcal{H}'$ be the Hamiltonian density responsible for the decay, and define the (dimensionless) Feynman amplitude

$$T(E_+,E_-,E_0) = \langle \pi^+\pi^-\pi^0 | \mathcal{H}' | \eta \rangle, \qquad (1)$$

where the pion energies E_+, E_-, and E_0 are defined in the c.m. system. In order to avoid the (experimentally *un*observed) Sutherland suppression[2] of the amplitude at $E_0 = 0$, it was proposed[3] to identify $\mathcal{H}'$ with the u_3 tadpole[4] associated with $\Delta I = \frac{1}{2}$ mass differences. In quark language,

$$\mathcal{H}' = (m_u - m_d)(\bar{u}u - \bar{d}d)/2, \qquad (2)$$

where m_u and m_d are the current quark masses. With $\mathcal{H}'$ given by Eq. (2), one can show that T vanishes for $E_\pm = 0$ (i.e., $E_0 = m_\eta/2$), and hence may be parametrized

$$T = A(1 - 2E_0/m_\eta), \qquad (3)$$

which gives a good representation of the experimental Dalitz plot. It then remained to evaluate $A = T(m_\eta/2, m_\eta/2, 0)$; experimentally, $A = 0.65 \pm 0.13$.[5]

From this point, the calculational procedures become ambiguous. Contraction of Eq. (1) [using (2)] with all three pions soft brings one to an ill-defined point relative to the real Dalitz plot. The resulting amplitude is, in any case, small by a factor of $\sim\sqrt{3}$.[1] The case for chiral perturbation theory[6] in the square of the η four-momentum is also difficult to maintain in the face of possibly rapid momentum dependences of the matrix element due to Kogut-Susskind ghosts.[7]

In this paper we present a calculation of the amplitude A which circumvents the above problems. Although it is performed within the framework of the Massachusetts Institute of Technology (MIT) bag model,[8] and utilizes the concept of pole dominance of $A = T(m_\eta/2, m_\eta/2, 0)$ by the $\epsilon(700)$,[9] it will be seen that the answer *does not depend on the bag parameters*, and depends only *very weakly* on the relatively uncertain width of the ϵ, for m_ϵ in the range 650–800 MeV.

We proceed to outline our calculation. From Eqs. (1) and (2), upon contracting the π^0, we obtain the standard result

$$A = i[(m_u - m_d)/(2F_\pi)]\bar{A},$$
$$\bar{A} = \langle \pi^+\pi^- | \bar{u}\gamma_5 u + \bar{d}\gamma_5 d | \eta\rangle_{E_+ = E_- = m_\eta/2}, \quad (4)$$

where A is defined by Eq. (2). As proposed by Jaffe,[10] we assume that there is a bound quark state (the ϵ) at ~ 680 MeV. Given the proximity of m_η to this energy, one may hope to obtain a good approximation for $\bar{A}$ by pole dominating with the ϵ. Thus we write[11]

$$\bar{A} \cong \frac{g_{\epsilon\pi^+\pi^-}\langle \epsilon | \bar{u}\gamma_5 u + \bar{d}\gamma_5 d | \eta\rangle}{m_\epsilon^2 - m_\eta^2 - i\Gamma_\epsilon (m_\epsilon m_\eta)^{1/2}}. \quad (5)$$

Because of the large width of the ϵ, we use a symmetric form of the Breit-Wigner amplitude to minimize bias.

In the bag model we make the usual static replacement

$$\bar{A} = \langle \epsilon | \bar{u}\gamma_5 u + \bar{d}\gamma_5 d | \eta\rangle \to 2(m_\epsilon m_\eta)^{1/2}\mathfrak{M}_{\text{bag}}, \quad (6)$$

where

$$\mathfrak{M}_{\text{bag}} = \langle \bar{\epsilon} | \int d^3x [\bar{u}(x)\gamma_5 u(x) + \bar{d}(x)\gamma_5 d(x)] | \bar{\eta}\rangle \quad (7)$$

with $\langle \bar{\eta} | \bar{\eta}\rangle = \langle \bar{\epsilon} | \bar{\epsilon}\rangle = 1$, and $u(x)$ and $d(x)$ are operators defined over cavity wave functions.[12] Note that $\mathfrak{M}_{\text{bag}}$ is dimensionless, and hence for zero-mass quarks and equal radii for $|\bar{\epsilon}\rangle$ and $|\bar{\eta}\rangle$, it will have no dependence on the bag parameters. Substituting (6) and (7) into (4), we find

$$|A| = \frac{|m_u - m_d|}{F_\pi}\left(\frac{32\pi}{3}\right)^{1/2}\left(\frac{\Gamma_\epsilon m_\epsilon^2 m_\eta}{(m_\epsilon^2 - m_\eta^2)^2 + \Gamma_\epsilon^2 m_\epsilon m_\eta}\right)^{1/2}\left(1 - \frac{4m_\pi^2}{m_\epsilon^2}\right)^{-1/4}\mathfrak{M}_{\text{bag}}, \quad (8)$$

where we have used the relation

$$\frac{2}{3}\Gamma_\epsilon = \frac{g_{\epsilon\pi^+\pi^-}^2}{16\pi m_\epsilon}\left(1 - \frac{4m_\pi^2}{m_\epsilon^2}\right)^{1/2}. \quad (9)$$

The kinematic factor on the right-hand side of Eq. (8) is only weakly dependent on the ϵ parameters; e.g., for the bag value $m_\epsilon = 680$ MeV, *the factor shows less than a 5% variation as Γ_ϵ varies between 400 and 600 MeV.* There is similar small variation with m_ϵ. For the bag values $m_\epsilon = 680$ MeV, and $\Gamma_\epsilon = 500 \pm 100$ MeV, we write

$$A = \frac{|m_u - m_d|}{3\ \text{MeV}}(0.23 \pm 0.01)\mathfrak{M}_{\text{bag}}, \quad (10)$$

with

$$\mathfrak{M}_{\text{bag}} = \sin\theta\,\mathfrak{M}_{\text{bag}}^{(2)} + \cos\theta\,\mathfrak{M}_{\text{bag}}^{(4)}. \quad (11)$$

The superscripts (2) and (4) and the angle θ in Eq. (11) refer to the quark content of the ϵ; i.e.,

$$|\bar{\epsilon}\rangle = \sin\theta\,|q\bar{q}\rangle + \cos\theta\,|q^2\bar{q}^2\rangle. \quad (12)$$

The first ket in (12) represents the standard p-wave quark model picture of the ϵ and has been described in the bag model.[13] The $q^2\bar{q}^2$ structure has been proposed by Jaffe[10] as a more favorable assignment for the ϵ.

$\mathfrak{M}_{\text{bag}}^{(4)}$ of Eq. (11) is evaluated in a straightforward, albeit tedious, calculation with use of the $q^2\bar{q}^2$ wave function for the ϵ given by Jaffe, and $|\eta\rangle = (u\bar{u} + d\bar{d} - 2s\bar{s})/\sqrt{6}$. For $m_u = m_d = 0$ and $R_\eta = R_\epsilon$,[14] we obtain

$$\mathfrak{M}_{\text{bag}}^{(4)} = 2\sqrt{2}(0.972\sqrt{\tfrac{6}{7}} + 0.233\sqrt{\tfrac{1}{7}}) \simeq 2\sqrt{2}. \quad (13)$$

Furthermore, a standard $q\bar{q}$ model[13] for the ϵ yields[15]

$$\mathfrak{M}_{\text{bag}}^{(2)} = 0. \quad (14)$$

From Eqs. (10)–(14), we see that in the approximation of ϵ saturation, *the η does not decay unless the ϵ has a $q^2\bar{q}^2$ component.* From (10), (11), and (13) the condition for agreement with experiment ($A = 0.65 \pm 0.13$) is

$$|m_u - m_d|\cos\theta = 3.0 \pm 0.6\ \text{MeV} \quad (15)$$

and consistency is achieved for

$$|m_u - m_d| \geq 2.4\ \text{MeV}. \quad (16)$$

Several values for $|m_u - m_d|$ (at a renormalization mass scale $\mu \sim 1$ GeV) have appeared in the literature: (a) Pagels and Stokar,[16] in conjunction with the chiral perturbation theory analysis of Langacker and Pagels,[17] obtain $m_u - m_d = -2.5 \pm 2.4$ MeV [implying from Eq. (15) that $\cos\theta$ may equal 1, i.e., no $\bar{q}q$ component to the ϵ]. (b) An analysis of ρ-ω^0 mixing by Langacker[18] yields $(m_d - m_u)/2m_s = 0.010 \pm 0.002$, which for $m_s = 150 \pm 50$ MeV gives $m_u - m_d = -(3.0 \pm 1.0)$ MeV. (c) Weinberg's analysis[19] gives $m_u - m_d = (-3.3$

MeV)/Z_m^*, where $Z_m^* = \langle \tilde{h} | \int \bar{s}(x) s(x) d^3x | \tilde{h} \rangle$ and $\tilde{h}$ is a hadron state containing one strange quark (normalized to $\langle \tilde{h} | \tilde{h} \rangle = 1$). In the bag model $Z_m^* \simeq 0.5$, and thus Weinberg obtains $m_u - m_d = -6.6$ MeV, implying [from (15)] that $\cos\theta = 0.45$.
(d) Bag-model fits to electromagnetic mass differences[20] give $m_u - m_d$ in the range -2 to -5 MeV. All of these estimates are model dependent. Nevertheless, if we accept as brackets 2 MeV $< |m_u - m_d| < 5$ MeV, we may conclude that agreement with experiment is achieved for $0.5 \leq \cos\theta \leq 1$.

To summarize, we have obtained a consistent picture of the decay $\eta \to 3\pi$ based on the u_3 tadpole, which has the following features: (1) The calculation does not involve the usual extrapolations in energy variables over a range $\sim m_\eta$. (2) On the assumption that the 2π final state in the amplitude $\bar{A}$ [Eq. (4)] is saturated with the $\epsilon(700)$, we find that the decay does not proceed unless the ϵ has a sizable $q^2\bar{q}^2$ component. (3) The decay amplitude is linear in the current quark mass difference $|m_u - m_d|$, and the value of $|m_u - m_d|$ and the $q^2\bar{q}^2$ mixture in the ϵ are strongly correlated [Eq. (15)]. In particular, values of $|m_u - m_d|$ (renormalized at a mass scale $\sim m_\epsilon$) of less than 6 MeV imply a substantial $q^2\bar{q}^2$ component in the ϵ wave function. (4) Consistency of the present analysis with experiment places a lower bound of 2.4 MeV on the mass difference $|m_u - m_d|$. (5) The decay amplitude depends only very weakly on Γ_ϵ, the width of the ϵ, as long as $\Gamma_\epsilon \gtrsim 300$ MeV. (6) Although the calculation was performed in the framework of the MIT bag model, the dimensionlessness of the relevant matrix element $\mathfrak{M}_{\text{bag}}$ [Eq. (7)] makes the results insensitive to the bag parameters, as long as these produce the observed masses in a manner consistent with $R_\epsilon \sim R_\eta$.

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[15]The vanishing of $\mathfrak{M}_{\text{bag}}^{(2)}$ can be traced to the spin flip implemented on the struck quark by the transition operator $\bar{u}\gamma_5 u + \bar{d}\gamma_5 d$. For the static case ($\vec{P}_\eta = \vec{P}_\epsilon = 0$), the amplitude for this to occur (with the unstruck quark remaining unflipped) is zero, because of the spin-0 nature of the η and ϵ. *This result holds in any $q\bar{q}$ model.* ($\mathfrak{M}_{\text{bag}}^{(4)}$ is not zero because $\bar{u}\gamma_5 u + \bar{d}\gamma_5 d$ acts as a pair creation operator in this case.) We would like to thank Professor E. Golowich for correcting an earlier version of this work (where $\mathfrak{M}_{\text{bag}}^{(2)}$ was given as $\ll \mathfrak{M}_{\text{bag}}^{(4)}$, but nonzero), and for discussion on this point.

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Correlations and Specific Heat of the SU(2) Lattice Gauge Model

B. Lautrup[(a)] and M. Nauenberg[(b)]
CERN, CH-1211 Geneva 23, Switzerland
(Received 16 September 1980)

Monte Carlo calculations of the specific heat of the four-dimensional SU(2) lattice gauge model show a sharp peak where Creutz discovered a crossover in the value of the string tension. In this region is also found a rapid increase in the correlation length and a slowing down in the Monte Carlo convergence towards equilibrium.

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It is generally believed that the SU(N) lattice gauge models in four dimensions do not exhibit phase transitions, giving a simple picture for the confinement of static quarks, in accordance with the Wilson criteria.[1,2] Recently Creutz[3,4] has given evidence in support of this conjecture by evaluating the string tension for SU(2) and SU(3) gauge models using Monte Carlo simulations on finite lattices, and finding a rapid crossover between the expected strong coupling and asymptotic freedom limits. Smooth matching between these two domains had previously been discussed from strong coupling expansions of the β function.[5] The relation between the lattice and the continuum scales has been obtained analytically by Hasenfratz and Hasenfratz[6] in agreement with Creutz's numerical results. Other tests of quark confinement ideas by Monte Carlo calculations have been carried out by applying 't Hooft's twisted boundary conditions[7] on SU(2) lattice gauge models.[8]

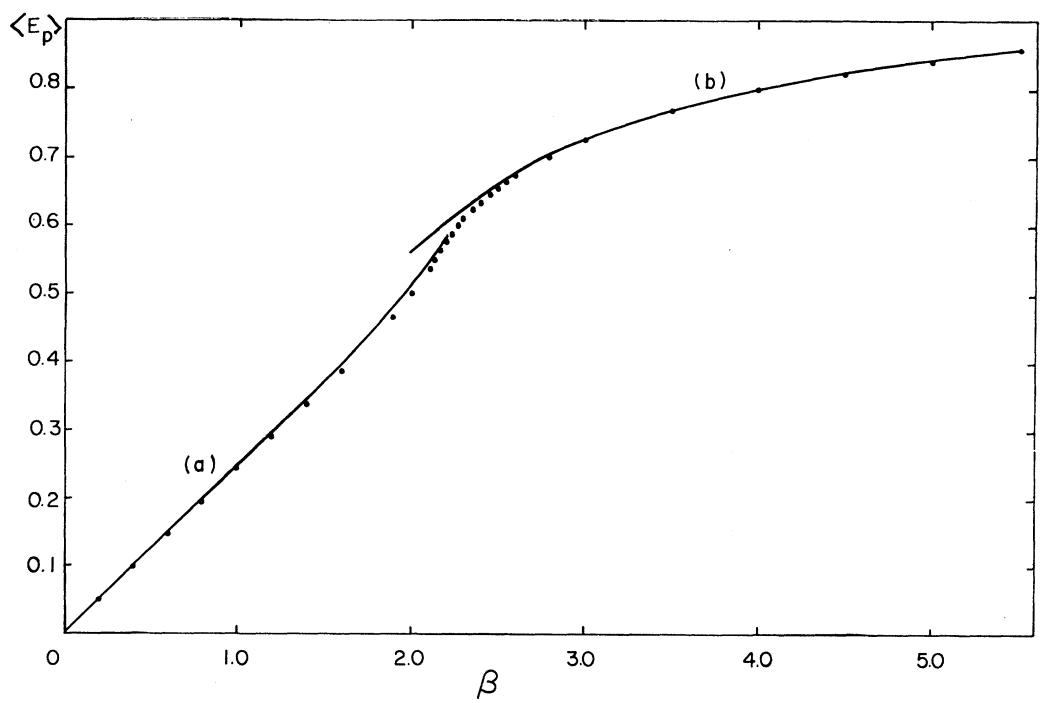

FIG. 1. The SU(2) average plaquette energy $\langle E_p \rangle$ as a function of β for a lattice of size 4^4. The curves labeled a and b are obtained from the strong and weak coupling expansions, Eqs. (2) and (3).

