

FIG. 1. Depression of one of the phonon branches against percent strain for nearest-neighbor Lennard-Jones 6-12 potential.

that the coherency strains are, in fact, relieved periodically by some interface dislocations, leaving, however, a semicoherent interface which still leads to effects of the type described above.

Finally, preliminary x-ray diffraction data on arc-cast specimens and x-ray powder pattern data on annealed specimens suggest, that for various compositions, there can be a definite lattice contraction in the iridium layers and a probable expansion of the  $YIr_2$  layers of the eutectic mixture. Since yttrium atoms are larger than iridium atoms, these results could not originate from a simple extended solubility or homogeneity range. Previous powder patterns from arc-cast specimens<sup>1</sup> failed to reveal any crystallinity in either eutectic phase. We believe that this resulted from strains introduced during powder fabrication.

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## Observation of Inelastic Impurity Scattering of Electrons in the Resistivity of Potassium-Rubidium Alloys

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The resistances of K Rb alloys have been measured by new high-precision techniques (1 part in  $10^7$ ) from 180 mK to 4 K. Below 1.3 K a term proportional to the residual resistivity and to  $T^2$  is positively identified. It is believed that this is the long sought for contribution to the resistivity from inelastic impurity scattering of electrons.

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In 1964 Taylor,<sup>1</sup> following the lead of Koshino,<sup>2</sup> calculated the effect on the electrical resistivity of the scattering of electrons by the thermal motion of impurities. His conclusion was that at sufficiently low temperature

$$\rho = \rho_0 + A_I \rho_0 T^2 + B T^5 , \qquad (1)$$

where  $\rho_0$  is the residual resistivity (proportional to concentration *c*) and

$$A_{I} = \pi^{2} \hbar^{2} K_{av}^{2} / 2M k \Theta^{3} , \qquad (2)$$

where  $K_{\rm av}$  is an averaged scattering vector of value close to  $2k_{\rm F}$  ( $k_{\rm F}$  is the Fermi wave vector),

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*M* is the host ionic mass, and  $\Theta$  is the Debye temperature. Taylor concluded "that the  $T^2$  term would be hidden by the lattice resistance except at temperatures below  $\Theta/40$ . Its magnitude would be less than  $10^{-5}\rho_0$  and would not generally be observable."

Since then, techniques in low-temperature measurements have improved to the extent that we are now able to make measurements<sup>3</sup> with a precision of ~1 part in  $10^7$ . With these improvements the effects Koshino and Taylor predicted should become clearly observable.

Besides its intrinsic interest the unequivocal measurement of this term is important for two reasons. First, claims have been made<sup>4,5</sup> that the Koshino-Taylor term has been observed. Panova, Zhernov, and Kutaitsev, for example, observed a contribution to the resistivity of Al Ag and Al Au alloys proportional to the concentration and  $T^2$  between 1.5 and 10 K. However, the contribution is  $\sim 1000$  times that estimated from Eq. (2) for Al. If they are correct then this term is a very significant one which could be misinterpreted as electron-electron scattering in other systems, "pure" potassium in particular. Second, Kaveh and Wiser<sup>6,7</sup> suggest that it may be responsible for the lack of saturation of  $\Delta \rho(T)$ (departure from Matthiessen's rule) at large  $\rho_0$ , a fact which has been held against the isotropization theory of  $\Delta \rho(T)$ . Again confirmation that the theory gives the correct order of magnitude is essential.

Kagan and Zhernov<sup>8</sup> and Kaveh and Wiser<sup>6</sup> list other terms, including  $T^4$  and  $T^5$  terms, contributing to  $\rho$ . To positively identify the Koshino-Taylor term it is essential to perform experiments at temperatures where all  $T^4$  and  $T^5$  terms are negligibly small.  $A_I$  calculated from Eq. (2) for potassium is  $\sim 1.37 \times 10^{-5}/K^2$ . More recently Kus and Taylor<sup>9</sup> have performed a more detailed calculation which yields a value of  $\sim 1.33 \times 10^{-5}/K^2$ for  $A_I$ . These values suggest that the Koshino-Taylor term will be substantially greater than the ideal resistivity for pure potassium<sup>10</sup> only for T<~1.3 K. Order of magnitude calculations<sup>11</sup> indicate that the other  $T^4$  and  $T^5$  terms will also be negligible in this range. In addition to low temperatures, high precision (~1 part in  $10^7$ ) is required for  $\rho$  measurements.

The basic experimental setup<sup>3</sup> is shown in Fig. 1.  $R_x$  is the specimen connected through a 50  $\mu\Omega$ (residual) resistance R' to the mixing chamber of a dilution refrigerator.  $R_s$  is a reference resistance, which in practice is a near duplicate of  $R_x$ .



FIG. 1. The essential elements of the experiment.  $R_x$  and  $R_s$  are two similar samples. R', Ag-0.1 at.% Au alloy, resistance 50  $\mu\Omega$ .  $I_x$ ,  $I_s$ , currents through  $R_x$  and  $R_s$  when S is balanced.  $H_1$  and  $H_2$  are heaters.

 $R_s$  is thermally connected to but electrically isolated from the mixing chamber. A temperature regulator with its heater on the mixing chamber is used to keep the temperature of  $R_s$  fixed. The measurement begins with a steady heater power in heater 1. Currents  $I_x$  and  $I_s$  are smoothly increased from zero to ~20–50 mA and are adjusted until the SQUID null detector indicates zero. At this balance

$$c = I_s / I_x = R_x / R_s , \qquad (3)$$

where c is measured to a precision of 0.1 ppm using a commercial current comparator. Next the heater power is transferred from heater 1 to heater 2 which causes the temperature of  $R_x$  to rise  $\Delta T$  since heat is now flowing through R'. Hence  $R_x$  increases to  $R_x + \Delta R_x$ , and on balancing the bridge we obtain a new value of c:

$$c' = c + \Delta c = (R_x + \Delta R_x)/R_s.$$
(4)

For sufficiently small  $\Delta T$  it is straightforward to show that

$$\frac{\Delta c}{c\,\Delta T} = \frac{1}{R_x} \frac{dR_x}{dT} \simeq \frac{1}{\rho_x} \frac{d\rho_x}{dT}, \qquad (5)$$

where  $(1/\rho_x)d\rho_x/dT$  is the quantity we wish to measure.

At low temperatures we expect

$$\rho_r = \rho_0 + A T^2 \,, \tag{6}$$

where we now consider A to be made up of two

parts

$$A = A_0 + A_I \rho_0, \qquad (7)$$

where  $A_0$  allows for the possibility of concentration-independent electron-electron scattering. On this basis

$$\frac{1}{\rho_x} \frac{d\rho_x}{dT} = \frac{2A}{\rho_x} T \simeq \frac{2A}{\rho_0} T \tag{8}$$

so that if Eqs. (6) and (7) are correct, for each specimen there should be a linear plot of  $(1/\rho_x) \times d\rho_x/dT$  vs T with slope  $M = 2A/\rho_0$ . This test of the temperature dependence is independent of geometrical factors. To check the  $\rho_0$  dependence of A we write Eq. (8) in the form

$$M = 2A_0 / \rho_0 + 2A_1.$$
 (9)

*M* plotted against  $1/\rho_0$  should be a straight line with slope  $2A_0$  and intercept  $2A_P$ . To obtain  $\rho_0$ it is necessary to know the geometrical form factors. This limits our precision as discussed below.

The alloys were prepared from high-purity potassium and rubidium obtained from Mine Safety Appliances Ltd. The components were melted together in the dried argon atmosphere of a glove box, and extruded from a stainless steel press through stainless steel dies. Potential leads of the same material were cold-welded onto the samples and onto two copper supports to which the superconducting potential leads were connected. The samples were mounted free standing in copper containers which were sealed in the glove box before transferral to the cryostat. The samples all were 3 mm in diameter and had potential probes separated by  $\sim 4$  cm. In Table I we give some properties of the samples. Errors in  $\rho_0$ arise for two reasons. First, there are errors  $\sim 6\%$  in the determination of the geometrical factor. Second,  $\rho_0$  was determined in a subsidiary experiment and exhibited a long-term variation

TABLE I.  $\rho_{0}$ ,  $\rho_{300}/\rho_{4.2}$ , and nominal concentration of KRb alloys.

Nominal at. % Rb	ρ <sub>0</sub> (4.2 K) (nΩ cm)	ρ <sub>300</sub> /ρ <sub>4.2</sub>
2.24	387	18.6
0.83	142	50.5
0.32	56	129.0
0.13	24	304.0
0.05	10	720.0
0.00		~ 4000.0

with time. We estimate the total uncertainty in  $\rho_0$  to be ±10%. The 2.24-at. % sample was made by mixing a 1-g ampoule of Rb as supplied by the manufacturers with a 20-g ampoule of K. The remaining samples were made by diluting the 2.24-at. % sample, in the glove box, on the basis of volume. For our purposes  $\rho_0$  is more important than c, but for comparison our 2.24-at. % sample gives a value of 0.17  $\mu\Omega$  cm/at. % compared with 0.13  $\mu\Omega$  cm/at.% obtained for samples used by Guénault and MacDonald.<sup>12, 13</sup> The nominal concentrations for alloys other than the 2.24at. % sample were derived from the 0.17  $\mu\Omega$  cm/ at. % figure and the measured value of  $\rho_0$ .

The germanium thermometers were recalibrated against National Bureau of Standards, Superconducting Reference Materials (SRM 767, 768) with interpolation provided by using the Curie law susceptibility of a paramagnet.<sup>3</sup> Errors in  $\Delta T$ are ~1 mK. Typically  $\Delta T/T$  ranged from 0.1 for the higher temperatures to 0.36 at the lower temperatures where larger relative gradients were necessary to obtain measurable values of  $c_{\circ}$  No error will arise from these large  $\Delta T/T$  values if  $d\rho/dT$  is  $\infty T_{\circ}$ 

In Fig. 2 we show  $(1/\rho)d\rho/dT$  plotted against T for T < 1.3 K, for the various alloy concentrations, and see that the linear relation is obtained. To



FIG. 2.  $(1/\rho) d\rho/dT$  plotted against T for  $T \le 1.3$  K. Numbers appended to curves are nominal concentrations. Typical errors are shown for the 0.13% sample. Curve *a* represents  $\rho^{-1} d\rho/dT$  for electron-phonon scattering in units of  $10^{-7}$  K<sup>-1</sup> (0.32% sample).

show that the phonon contribution is indeed negligible in this temperature range, we also show in curve *a* of Fig. 2 the exponential part of our fit of the higher-temperature data, for the 0.32-at. % sample, by the equation<sup>14</sup>

 $\rho = \rho_0 + AT^2 + BT \exp(-\Theta^*/T).$ 

(Note that the scale of curve *a* is  $10^{-2}$  that of the remaining graphs in Fig. 2.) To test the  $\rho_0$  dependence of *A* we plot  $M (= 2A/\rho_0)$  vs  $1/\rho_0$  in Fig. 3. In this graph the vertical error bars represent the standard deviations of the slopes obtained by least-squares fitting of the data in Fig. 2. The rather large error bars on  $1/\rho_0$  introduce an appreciable uncertainty (~15%) in the slope and therefore in  $A_0$ , but they produce comparatively little effect (~3%) on the intercept and  $A_I$ .

Our final results for the dilute alloys of rubidium in potassium are

 $A_{I} = (8.5 \pm 0.3) \times 10^{-6} / \text{K}^{2}$ 

 $A_0 = (2.2 \pm 0.31) \times 10^{-13} \Omega \text{ cm/K}^2$ .

These are to be compared with  $A_I = 13.7 \times 10^{-6}/\text{K}^2$ from the Taylor expression in Eq. (2) and  $A_I = 13.3 \times 10^{-6}/\text{K}^2$  from Kus and Taylor.<sup>9</sup>  $A_0$  can be compared with experimental values<sup>14, 15</sup> ranging from 0.5 to  $3 \times 10^{-13} \Omega$  cm/K<sup>2</sup>. The situation is complicated by two factors. First, Kaveh and Wiser<sup>16</sup> assert that  $A_0$  depends on the density of dislocations, which may account for the sample dependence in the experimental studies quoted above. Second, MacDonald *et al.*<sup>17, 18</sup> obtain  $A_0 = 1.7 \times 10^{-13}$  $\Omega$  cm/K<sup>2</sup>, assuming that  $A_0$  is largely determined by phonon-mediated interactions rather than the



FIG. 3. *M*, the slope of graphs in Fig. 2, plotted against  $1/\rho_0$ .

Coulomb interaction assumed in earlier work.<sup>17,19</sup> Both these factors require confirmation. In particular, further measurements on potassium below 1 K and in well-defined states of strain are required.

In summary, we have measured  $T^2$  dependence of the resistivity of KRb alloys in a temperature region where complicating effects of phonon scattering are avoided. The order of magnitude and dependence on residual resistivity are strong evidence that this term arises from the inelastic impurity scattering as predicted by Koshino and Taylor.

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