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Nuclear-Structure Effects on Parity Nonconservation in Light Nuclei

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The nucleon-nucleon parity-nonconserving potentials given by Desplangues, Donoghue, and Holstein (DDH) are used to calculate matrix elements between states of opposite parity in ^{10}B, ^{16}O, ^{18}F, ^{19}F, ^{20}Ne, and ^{21}Ne. The sensitivity of the parity-nonconserving matrix elements to various approximations in the microscopic shell-model wave functions is investigated. The final results using the DDH estimates for the weak meson-nucleon coupling constants based on the Weinberg-Salam theory are several times larger than experiment.

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Gauge theories of weak interactions predict nonleptonic weak interactions between quarks which are manifest in the well-known strangeness-changing ($\Delta S = 1$) decays of hadrons. All observed $\Delta S = 1$ decays involve only charged weak currents but both charged and neutral weak currents are expected to contribute to $\Delta S = 0$ weak interactions. Nonleptonic $\Delta S = 0$ weak interactions give rise to parity mixing in the energy levels of hadrons and nuclei which can be experimentally investigated by measuring small parity nonconservation in their electromagnetic and strong decays.[1]

The investigation of parity nonconservation in the decay of light nuclei has been extensively pursued because fairly reliable many-body wave functions exist for those nuclei and because there are several situations where the mixing between particular low-lying levels gives rise to measurable effects.[1] A theoretical interpretation of these data is important for the extraction of $\Delta S = 0$ nonleptonic weak-interaction coupling constants.

In this paper we report on the results of the first systematic calculation of parity-nonconserving (PNC) matrix elements for light nuclei using microscopic nuclear wave functions. Our emphasis is on the reliability and sensitivity of the results to the approximations which must be made in order to construct shell-model wave functions with finite degrees of freedom. For the nucleon-nucleon weak interaction we use the PNC potential given by Desplanques, Donoghue, and Holstein[2] (DDH) in Eq. (115) of their paper together with the "best-value" parameters from Table VII of their paper (we have used $h_\rho{}'^{(1)} = 0$). This potential is equivalent to most of the other PNC potentials which have been proposed.[1] Our matrix elements will be given as a sum of all terms in the DDH potential but we note that the isoscalar and isovector matrix elements are dominated by about a factor of 10 by the potential terms proportional to $h_\rho{}^0$ and f_π, respectively. More com-

plete results will be given elsewhere.[3]

The calculations are carried out in a spherical shell-model basis with the nucleons allowed to occupy the $1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $2s_{1/2}$, and $1d_{3/2}$ orbitals. Within this basis various truncations on the number of ways that the nucleons are partitioned between these orbitals are made in order to keep the Hamiltonian matrices below dimensions of about 200; in particular a $(1s_{1/2})^4$ configuration is assumed in all cases. The PNC matrix element can be expressed in terms of a product of two-body transition densities (TBTD) and two-body matrix elements[4] (TBME):

$$\langle\psi_f|V_{\rm PNC}|\psi_i\rangle=\sum_{\Delta T=0}^{2}\langle\psi_f|V_{\rm PNC}{}^{\Delta T}|\psi_i\rangle,\quad \langle\psi_f|V_{\rm PNC}{}^{\Delta T}|\psi_i\rangle=\frac{(-1)^{T_f-T_z}}{(2J_i+1)^{1/2}}\begin{pmatrix}T_f & \Delta T & T_i\\ -T_z & 0 & T_z\end{pmatrix}\sum_{\substack{\rho_1\le\rho_2\\ \rho_3\le\rho_4\\ JTT'}}(\text{TBTD})\times(\text{TBME}),$$

where

$$\text{TBTD}=-\frac{\langle\psi_f|||\{[a_{\rho_1}{}^{+}\otimes a_{\rho_2}{}^{+}]^{JT}\otimes[\tilde a_{\rho_3}\otimes\tilde a_{\rho_4}]^{JT'}\}^{\Delta J=0,\Delta T}|||\psi_i\rangle}{[(2\Delta T+1)(1+\delta_{\rho_1\rho_2})(1+\delta_{\rho_3\rho_4})]^{1/2}},$$

and

$$\text{TBME}=\langle\rho_1\rho_2JT|||V_{\rm PNC}{}^{\Delta T}|||\rho_3\rho_4JT'\rangle,$$

and where ρ stands for all single-particle quantum numbers.

The calculation of the TBTD was facilitated by the development of a new shell-model code at Oxford University by Godwin and Rae.[5] The TBME were calculated with harmonic-oscillator wave functions ($\hbar\omega=14$ MeV which is appropriate for $A\approx16$) with use of the Moshinsky transformation. Short-range correlations (SRC) have been included by multiplying the harmonic-oscillator radial wave functions by the factor $1+f(r)$ given by Miller and Spencer[6] [$\alpha=1.10$ fm^{-2} and $\beta=0.68$ fm^{-2}]. The SRC reduce the π-exchange matrix elements by about 20% to 30% and the ρ-exchange matrix elements by a factor of 3 to 4, results which are consistent with much more elaborate treatments of SRC.[1,7] In the situation where certain orbitals are completely filled in both ψ_f and ψ_i part of the expression for $\langle V_{\rm PNC}\rangle$ can be simplified to a sum of one-body transition densities times one-body matrix elements (OBME). The OBME are calculated in a straightforward way as a sum of two-body matrix elements with ρ_1 and ρ_3 as members of the filled orbits and ρ_2 and ρ_4 as members of the unfilled (valence) orbits. As an example we find

$$\langle(1s_{1/2})^4(1p_{3/2})^8 2s_{1/2}||V_{\rm PNC}||(1s_{1/2})^4(1p_{3/2})^8 1p_{1/2}\rangle$$
$$=-2.9\ (0.6)\ \text{eV}$$

for a valence proton (neutron). The meson-nucleon coupling constants enter into this matrix element in nearly the same proportions as they enter into the parameters X_p (X_n) of Desplanques and Missimer.[2,8]

The partition truncations for the basis states are constructed relative to a vacuum of $(1s_{1/2})^4\times(1p_{3/2})^8(1p_{1/2})^4$, and three different truncations (model spaces) are considered.

(*A*) For $A\ge16$ we use basis states of the form $(1d_{5/2},2s_{1/2})^n$ for one parity and $(1d_{5/2},2s_{1/2})^{n+1}\times(1p_{1/2})^{-1}$ for the opposite parity state where $n=A-16$. Then this basis is expanded in two ways: (*B*) by using $(1d_{5/2},2s_{1/2},1d_{3/2})^n$ and $(1d_{5/2},2s_{1/2},1d_{3/2})^{n+1}(1p_{1/2},1p_{3/2})^{-1}$, and (*C*) by using $(1d_{5/2},2s_{1/2})^{n+m}(1p_{1/2})^{-m}$ and $(1d_{5/2},2s_{1/2})^{n+m'}(1p_{1/2})^{-m'}$, where $m=0$, 2, and 4 and $m'=1$ and 3. The dimension of the matrices would be prohibitively large if expansions *B* and *C* were made simultaneously. We propose to examine the effects of $A\to B$ and $A\to C$ separately and then to extrapolate the result for $A\to B+C$. For ^{10}B only the basis states similar to type *B* are considered, i.e., $(1p_{1/2},1p_{3/2})^n$ and $(1p_{1/2},1p_{3/2})^{n-1}(1d_{5/2},2s_{1/2},1d_{3/2})^1$. For truncation *A* and *B* the effective strong interaction of Millener and Kurath[9] (MK) has been used and for truncation *C* the two effective strong interactions *F* and *Z* of Ref. 10 have been used. Some of the states we consider can be obtained only within one model space, e.g., the 2_1^+ $T=0$ "four-particle, four-hole" state in ^{16}O is obtained only within model space *C*.

For model spaces *A* and *C* it has been shown that the states contain little spuriosity (center-of-mass motion) and that there is no method to eliminate the small component.[11] In contrast, in model space *B* about half of the negative parity states below 10 MeV are predominantly spurious; the spuriosity is largely fragmented only when two states lie within a few hundred kiloelectronvolts of each other. The matrix elements of *R* between the positive and negative parity states

are large only for the spurious states whereas the PNC matrix elements are about the same size for the spurious and nonspurious states. Thus we conclude that a small amount of spuriosity in the wave function should have only a small absolute effect on the PNC matrix element. For model space B it is in general convenient to remove the spurious components by adding a center-of-mass Hamiltonian $H_{\mathrm{c.m.}}$ times a large constant β to the original shell-model Hamiltonian.[12] It turns out that the lowest ^{18}F 0^- and ^{19}F $\frac{1}{2}^-$ states obtained with the MK interaction are predominantly nonspurious and they both lie about 2 MeV below the lowest spurious states. Thus the PNC matrix elements involving these particular states change by very little ($\leq$0.2 eV) when the spurious states are removed by using the MK $+\beta H_{\mathrm{c.m.}}$ Hamiltonian.

The results are given in Table I. The overall sign in each case is arbitrary but the sign of the isoscalar and isovector components are fixed relative to each other. There are several cases where the wave functions have a high overlap when going from A to B to C and thus the relative signs of the results connected by lines in Table I are fixed. From the ^{18}F and ^{19}F results it is seen that the expansions $A \rightarrow B$ and $B \rightarrow C$ each separately reduce the matrix element by about a factor of 2. The C-model-space results are not very sensitive to the differences in the wave functions obtained with the F and Z interactions for the lowest states considered; however, the matrix elements involving more highly excited states of a given spin are more sensitive to the choice of strong interactions and hence are not reliably determined by the present calculations. The reductions from $A \rightarrow B$ and $A \rightarrow C$ can be qualitatively understood by examining the one-body transition densities. For $A > 16$ in model space A it is clear that $\langle \pi = + || a^+(2s_{1/2}) \otimes \tilde{a}(1p_{1/2}) || \pi = - \rangle = 0$; however, in model space C this is not zero because of the many-particle many-hole states in the $\pi = +$ state and this reduces the overall matrix elements in ^{18}F and ^{19}F and even leads to a sign change in

TABLE I. Theoretical and experimental parity-nonconserving matrix elements $\langle JTT_z | V_{\mathrm{PNC}} | JT'T_z \rangle$.

	J(T)	J(T')	Model space (interaction) A(MK)	B(MK+βH_{cm})	C(F)	C(Z)	$\lvert\langle V_{pv}\rangle\rvert$ (eV) theory	experiment
^{10}B	$2^+(1)$	$2^-(0)$		(0,-0.20)[a,c]			0.2	(d)
^{16}O	$2_1^+(0)$	$2^-(0)$			(1.07,0)[7.82][b]	(0.92,0)[7.39]	0.7	0.3±0.1[e]
	$2_2^+(0)$				(-.46,0)[11.85]	(-0.45,0)[10.28]		
	$2_3^+(0)$				(0.16,0)[12.85]	(-0.75,0)[11.78]		
	$2_4^+(0)$				(0.60,0)[14.10]	(-0.32,0)[12.71]		
^{18}F	$0^+(1)$	$0^-(0)$	(0,2.33)	(0,1.45)	(0,0.65)	(0,0.81)	1.0	0.14±0.41[f]
^{19}F	$1/2^+(1/2)$	$1/2^-(1/2)$	(-1.21,-1.43)	(-0.90,-1.02)	(-0.68,-0.79)	(-0.69,-0.94)	1.5	0.47±0.14[g]
^{20}Ne	$1^+(1)$	$1_1^-(0)$			(0,0.29)[6.10]	(0,0.45)[6.57]		
		$1_2^-(0)$			(0,0.40)[10.08]	(0,0.78)[10.08]		
		$1_3^-(0)$			(0,-0.009)[12.77]	(0,-0.21)[12.50]	<0.8	<1.2[h]
		$1_4^-(0)$			(0,-0.26)[13.45]	(0,0.11)[14.12]		
^{21}Ne	$1/2^+(1/2)$	$1/2^-(1/2)$	(0.94,-0.37)		(-0.13,0.62)	(0.08,0.40)	<0.5	0.02±0.03[i]

[a] The PNC matrix elements are given in units of (i)electronvolts in the form (S,V), where $\langle V_{\mathrm{PNC}} \rangle = S + V$, $S = \langle V_{\mathrm{PNC}}{}^{\Delta T=0} \rangle$, and $V = \langle V_{\mathrm{PNC}}{}^{\Delta T=1} \rangle$.

[b] Theoretical excitation energy in megaelectronvolts.

[c] The "best-value" parameters from Table VII of Ref. 2 are used.

[d] Experiments for ^{10}B have been planned (Ref. 13).

[e] Extracted from experimental values of Γ_α, $\gamma_\alpha{}^2$, and calculated penetration factors; the parity forbidden width is found to be dominated from mixing with the 6.92-MeV 2^+ state in ^{16}O (Ref. 14).

[f] $|P_\gamma(^{18}\mathrm{F})| = |\langle V_{\mathrm{PNC}} \rangle| \times 49 \times 10^{-4}$ eV^{-1}; $P_\gamma = (-7 \pm 20) \times 10^{-4}$ from Ref. 15.

[g] $|A_\gamma(^{19}\mathrm{F})| = |\langle V_{\mathrm{PNC}} \rangle| \times 1.8 \times 10^{-4}$ eV^{-1}; $A_\gamma = (-85 \pm 26) \times 10^{-6}$ from Ref. 16.

[h] Reference 17, mixing between $1^+, T = 1$ state at 11.23 MeV and $1^-, T = 0$ state at 11.26 MeV.

[i] $|P_\gamma(^{21}\mathrm{Ne})| = |\langle V_{\mathrm{PNC}} \rangle| \times 950 \times 10^{-4}$ eV^{-1}; $P_\gamma = (23 \pm 29) \times 10^{-4}$ from Ref. 17.

^{21}Ne. From Ref. 10 it appears that both the F and Z interactions produce too much excitation from the $1p_{1/2}$ shell which implies that the $A \rightarrow C$ reduction should only be about half that obtained with the F and Z interactions. The $A \rightarrow B$ reduction is due to destructive interference between terms involving the $a^+(1p_{1/2}) \otimes \tilde{a}(2s_{1/2})$ and $a^+(1p_{3/2}) \otimes \tilde{a}(1d_{3/2})$ one-body transition densities. All of the above considerations have gone into the final theoretical results for the total matrix element given in the last columns of Table I. Because of the $A \rightarrow C$ sign change for ^{21}Ne only a theoretical upper limit is justified.

These results are compared with the experimental matrix elements as deduced from measured electromagnetic and strong decay properties.[14,15,17,18] The magnitudes of the PNC matrix elements can be related to the measured A_γ and P_γ by using two-state mixing relations based on measured electromagnetic strengths[15,16,18] (see the footnotes to Table I). The measured parity-forbidden α decays in ^{16}O and ^{20}Ne can also be related to the magnitudes of the PNC matrix elements.[14,17] For ^{19}F, $A_\gamma = 2\delta(M1/E1)$, where δ is the Rose-Brink[19] mixing ratio. The calculated sign of A_γ in all model spaces is in agreement with experiment. Since A_γ depends strongly on $h_\rho{}^0$, this result supports the DDH sign for $h_\rho{}^0$ based on the $SU(6)_W$ model which differs in sign from the previously used factorization approximation.[2] However, the calculated sign in this case is not absolutely reliable because the theoretical $E1$ matrix elements involved are hindered and result from delicate cancellations between large single-particle components. (Similar, but not so severe cancellations, also occur for the $M2$ and PNC matrix elements.) We hope in the future to be able to calculate the ^{18}F isospin-forbidden $E1$ matrix element as well as to improve the calculations for hindered allowed $E1$ matrix elements by including isospin mixing in the wave functions.

From Table I it is seen that the DDH predictions for the magnitudes of the PNC matrix elements are several times larger than experiment. The ^{16}O $\Delta T = 0$ matrix element is dominated by the parameter $h_\rho{}^0$ for which we obtain a value of $[(-)0.5 \pm 0.2] \times 10^{-6}$ compared with the DDH "best-value" prediction of -1.14×10^{-6}. The ^{18}F $\Delta T = 1$ matrix element is dominated by the parameter f_π for which we obtain a value of $\leq (+)0.25 \times 10^{-6}$ compared with the DDH "best-value" prediction of 0.45×10^{-6}. Further measurements for ^{18}F are needed in order to improve the determination of the important parameter f_π. In addition, measurements sensitive to the $\Delta T = 1$ matrix element in ^{10}B have been planned,[13] but the small matrix element we obtain in this case indicates that it will be a difficult experiment.

The ^{19}F matrix element is dominated by the linear combination of parameters $X_p = 5.5 f_\pi - 0.7 h_\rho{}^1 - 1.4 h_\omega{}^1 - 1.7 h_\rho{}^0 - 1.6 h_\omega{}^1$ which Desplanques and Missimer have found for odd proton nuclei,[2,8] in particular, $|\langle V_{PNC} \rangle|(^{19}\mathrm{F}) \approx 0.30 |X_p|$ MeV. The comparison with the experimental matrix element gives $X_p = (1.7 \pm 0.5) \times 10^{-6}$ which can be compared with the value of $X_p = (3.1 \pm 0.5) \times 10^{-6}$ deduced from ^{175}Lu and ^{181}Ta (Ref. 8) and the DDH "best-value" prediction of 5.0×10^{-6}. Our wave functions for ^{21}Ne are too uncertain at present (because of the extreme $A \rightarrow C$ reduction) to justify a quantitative comparison between experiment and theory.

In conclusion, our investigation of the nuclear structure effects on parity nonconservation has provided a crucial link in the chain which is needed to go from the fundamental $\Delta S = 0$ quark-quark weak-interaction theory to measurable parity-nonconserving observables in nuclei. The values which we have deduced for the PNC meson-nucleon coupling constants are 2–3 times smaller than the DDH "best values" but are still within the reasonable range proposed by Desplanques, Donoghue, and Holstein.[2] The recent quantum chromodynamics calculation of f_π by Guberina *et al.*[20] is consistent with our empirical upper limit but the quantum chromodynamics results of Buccela *et al.*[21] and Körner *et al.*[22] are several times larger than our empirical upper limit.

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