"Statistical Yrast Line" in Heavy-Ion Fusion Reactions

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Fusion cross sections measured at high energies are parametrized in terms of a "statistical yrast line." This line lies parallel to the usual yrast line (given by the rigid-body moment of inertia, $\frac{2}{5}Mr_0^2A^{5/3}$) but is shifted upward by an additional energy ΔQ. Most experimental fusion cross sections for systems with $A \lesssim 80$ are well reproduced with values of $r_0 = 1.20 \pm 0.05$ fm and $\Delta Q = 10 \pm 2.5$ MeV.

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In recent years considerable experimental and theoretical effort has been focused on the study of heavy-ion fusion reactions.[1] For lighter heavy-ion systems, it has been established that the fusion cross section σ_f is determined mainly by the penetration of the Coulomb and centrifugal barriers in the region of low incident energy (Region I), where σ_f is nearly equal to the total reaction cross section σ_R^{tot}. Above a certain incident energy (Region II), as can be seen in Fig. 1(a), σ_f becomes smaller than σ_R^{tot}. The absolute value of σ_f and its variation with respect to the incident energy often show a strong dependence on the entrance channel.

Although σ_f can be expressed in terms of a critical angular momentum L_{cr} by use of a sharp-cutoff model, we do not yet have a clear understanding of the physical origin of L_{cr}. One of the well-known interpretations of L_{cr} assumes the existence of a critical distance of separation, $R_{\text{cr}} = r_{\text{cr}}(A_1^{1/3} + A_2^{1/3})$, where $r_{\text{cr}} = 1.0 \pm 0.07$ fm, which the ions must reach in order for them to fuse.[5] In this case, fusion cross section also should depend on the value of the total potential V_{cr} at R_{cr}. However, the systematics of r_{cr} and V_{cr} are not well established. Shell effects on the critical distance have been related to systematic variations in σ_f for p-shell and sd-shell nuclei.[6] There are, however, serious exceptions such as the $^{15}N + ^{12}C$ system.[7]

An alternative consideration of L_{cr} which has been long discussed is that σ_f might be limited by the yrast line of the compound nucleus, L_y, namely $L_{\text{cr}} = L_y$.[8] However, in order to justify the R_{cr} concept and disprove "the yrast-line-limit model," Glas and Mosel[9] have shown that the experimentally determined L_{cr} values are always smaller than the calculated yrast line. It seems to us that the difference between L_{cr} and L_y in Region II is physically reasonable, as will be explained in the following.

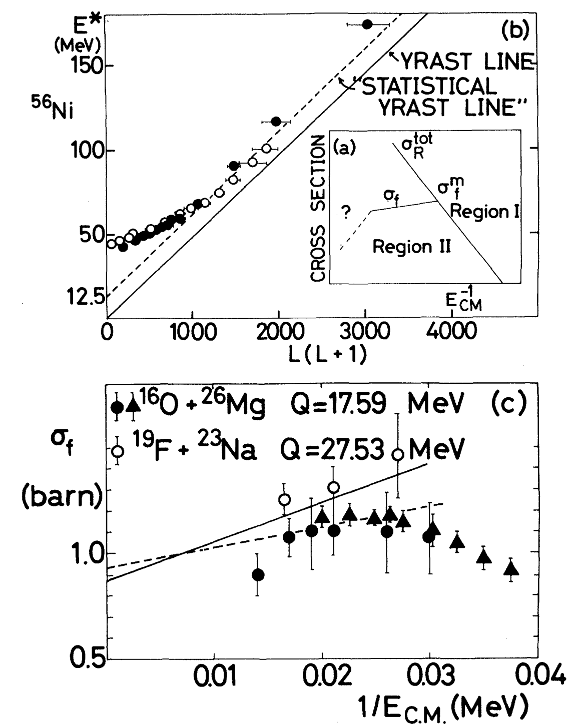

FIG. 1. (a) Schematic representation of σ_f and σ_R^{tot} vs $E_{\text{c.m.}}^{-1}$. (b) Extracted L_{cr} curves from σ_f for ^{56}Ni compound system (Ref. 2). The closed circles are extracted from $^{16}O + ^{40}Ca$ and the open ones are from $^{32}S + ^{24}Mg$. The statistical yrast line (dashed line) is calculated by using Eq. (3) with the parameters of $r_0 = 1.15$ fm and $\Delta Q = 12.5$ MeV. (c) The σ_f curves for the ^{42}Ca compound system (Refs. 3 and 4) calculated by using Eq. (3) with $r_0 = 1.20$ fm and $\Delta Q = 7.5$ MeV. The solid line is calculated for the $^{19}F + ^{23}Na$ system and the dashed line for the $^{16}O + ^{26}Mg$ system.

In this Letter we shall show that the behavior of the fusion cross section at high energies can be parametrized in terms of a "statistical yrast line," $L_y^{\,st}$, of the compound nucleus. The basic assumption of this model is that heavy ions do not fuse at the L_y of the compound nucleus where the nuclear temperature $T=0$ and the level density is low. In order for fusion to occur, the system must lie on or above the statistical yrast line in a region where the level density is high, i.e., $T>0$.

With the assumption that the statistical yrast line $E_{y,st}{}^*$ runs parallel to the yrast line $E_y{}^*$ with an additional energy ΔQ, it can be expressed as

$$E_{y,st}{}^* = (\hbar^2/2\mathcal{I})L_{cr}(L_{cr}+1)+\Delta Q, \qquad (1)$$

where $\mathcal{I}$ is the moment of inertia of the compound nucleus A which is assumed to be equal to that of a rigid body for simplicity: $\mathcal{I}=\mathcal{I}_{rig}=\frac{2}{5}MAR^2$ and $R=r_0A^{1/3}$. By use of a sharp-cutoff model, the fusion cross section is expressed as

$$\sigma_f = (\pi\hbar^2/2\mu E_{c.m.})(L_{cr}+1)^2, \qquad (2)$$

where μ is the reduced mass of two ions and $E_{c.m.}$ is the c.m. energy of the entrance channel. The excitation energy of the compound nucleus, E^*, is the sum $E_{c.m.}+Q$, where Q is the Q value of the system. From Eqs. (1) and (2), we obtain

$$\sigma_f \simeq (\pi\mathcal{I}/\mu)[1+(Q-\Delta Q)/E_{c.m.}]. \qquad (3)$$

This expression contains two parameters, ΔQ and r_0, which are properties of the compound nucleus. On the other hand, the entrance channel determines the values of Q and μ. It is obvious that the slope of σ_f depends on the value of $(\pi\mathcal{I}/\mu)(Q-\Delta Q)$. For the sake of convenience, we define $\sigma_f{}^m$ as the fusion cross section at the crossing point between Regions I and II [see Fig. 1(a)]. Then the variation of $\sigma_f{}^m$ also depends strongly on the difference of Q values between different entrance channels. An important test of this model is whether we can find the systematic values of r_0 and ΔQ.

As a first test we consider two different entrance channels which form the same compound nucleus. In Fig. 1(b), the L_{cr} values are extracted from the σ_f data of $^{16}O+^{40}Ca$ and $^{32}S+^{24}Mg$ systems[2] whose compound nuclei are the same, i.e., ^{56}Ni. Up to $E^*=65$ MeV in Region I, we can see different behavior for the L_{cr} values for the two different systems. The experimental L_{cr} values of the two systems are consistent with the same $L_y^{\,st}$ line of ^{56}Ni in Region II. From the dashed line, we get the values $r_0=1.15$ fm and $\Delta Q=12.5$ MeV.

In Fig. 1(c), we show another example: the $^{16}O+^{24}Mg$ and $^{19}F+^{23}Na$ systems[13,4] which form the compound nucleus ^{42}Ca. The fusion cross sections of these two systems are well reproduced by Eq. (3) with parameters of $r_0=1.20$ fm and $\Delta Q=7.5$ MeV. As was mentioned above, differences in the slopes of σ_f and the values of $\sigma_f{}^m$ for the two systems are naturally explained by the large difference in their respective Q values.

Recently, Saint-Laurent *et al.*[10] studied the $^{16}O+^{16}O$ and $^{20}Ne+^{12}C$ systems at Region II. They found that the L_{cr} values of both systems agree quite well with each other and that the Z distributions of the angle-integrated yields of evaporation residues from both entrance channels are nearly the same at the excitation energies $E^*\simeq 48$ and 78 MeV in ^{32}S. The above examples show that σ_f in Region II is consistent with a statistical-yrast-line prediction.

In order to illustrate the role of ΔQ, we show two examples of a fusion calculation with $\Delta Q=0$ for the $^{14}N+^{12}C$ and $^{18}O+^{12}C$ systems. As can be seen in Fig. 2, it is not possible to reproduce the experimental slope with $\Delta Q=0$.

The observed σ_f for each system ($A\leq 80$) in Region II may be fitted with Eq. (3) by choosing appropriate values of r_0 and ΔQ. Reasonable fits are obtained for all of the systems with values of $r_0=1.20\pm0.05$ fm and $\Delta Q=10\pm2.5$ MeV. In order to show the validity of our present model, the experimental values of $\sigma_f{}^m$ determined at the crossing point between Regions I and II for the various systems are compared in Fig. 3, with values calculated for $r_0=1.20$ fm and $\Delta Q=10$ MeV. The calculated values reproduce quite well the general trend of the experimental data. Thus, the anomalous behavior of reactions such as $^{15}N+^{12}C$ and $^{19}F+^{23}Na$ can be understood as a Q-value effect, and need not be explained solely as the effect of valence nucleons[6] on R_{cr}.

The physical basis of a "statistical yrast line" (i.e., the need for a shift in energy of ΔQ) can be understood as follows: The scattering matrix of the entrance channel which involves the coupling with the compound state is evaluated by use of the formalism of the scattering theory of Mahaux and Weidermüller.[15] The partial widths which come from the coupling of the heavy-ion scattering state to the compound state are estimated by use of results of a microscopic study between composite nuclei.[16] It is found that the transmission coefficient to form the compound nucleus is

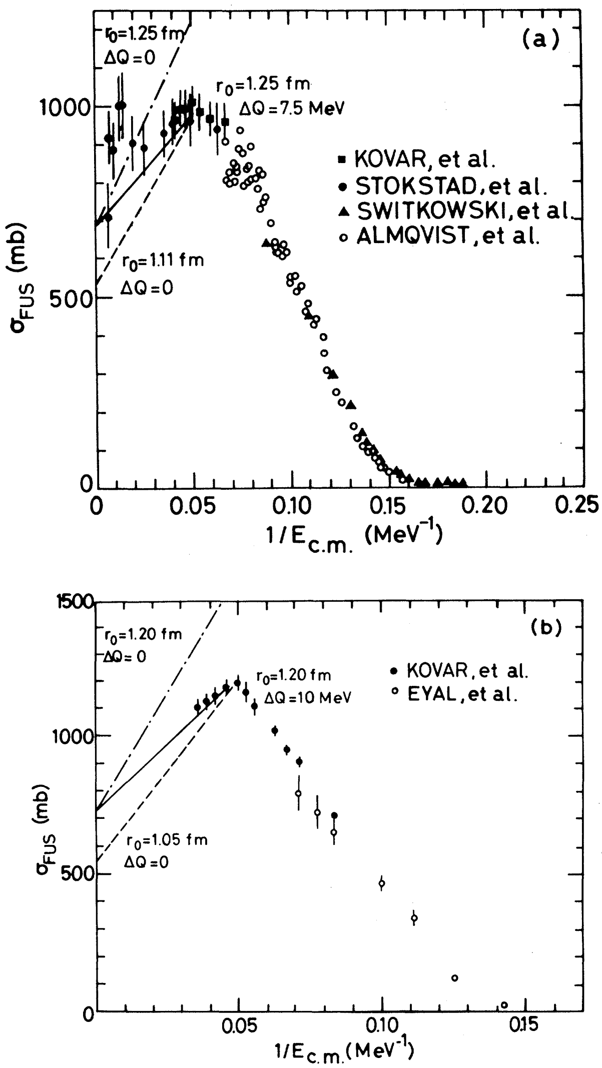

FIG. 2. (a) The σ_f curves for the $^{14}N + ^{12}C$ system (Refs. 11 and 12) calculated from Eq. (3). The solid line is calculated with $r_0 = 1.25$ fm and $\Delta Q = 7.5$ MeV. The dashed line with $r_0 = 1.11$ fm and $\Delta Q = 0$. The long dashed line with $r_0 = 1.25$ fm and $\Delta Q = 0$. (b) The same calculated curves from the $^{18}O + ^{12}C$ system (Refs. 11 and 13). The solid line is calculated with $r_0 = 1.20$ fm and $\Delta Q = 10$ MeV, the dashed line with $r_0 = 1.05$ fm and $\Delta Q = 0$, and the long dashed line with $r_0 = 1.20$ fm and $\Delta Q = 0$.

about one-half for most all of the systems ($A \leq 80$) at $\Delta Q \simeq 10$ MeV above the usual yrast line. In this way, the "statistical yrast line" can be interpreted as the beginning of strong absorption into the compound nucleus. The calculation and its discussion will be published in greater detail in a forthcoming paper.[17]

Although most of the existing data are well re-

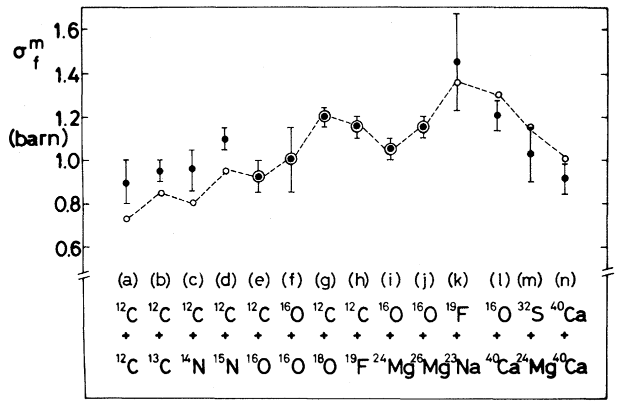

FIG. 3. Closed circles are the measured values of σ_f^m for various systems at crossing point between two regions. The experimental data are taken from Ref. 11 for the systems (*a*)–(*h*), from Refs. 3 and 4 for (*i*)–(*k*), from Ref. 2 for (*l*) and (*m*), and from Ref. 14 for (*n*). Open circles are the calculated values at the same point by use of Eq. (3) with $r_0 = 1.20$ fm and $\Delta Q = 10$ MeV. The dashed line is a guide for the eyes.

produced by this model, we find two cases, $^{16}O + ^{27}Al$ (Ref. 18) and $^{16}O + ^{10}B$,[19] which do not agree well with the present systematics. The fusion cross sections of the $^{16}O + ^{27}Al$ system which have been recently remeasured,[20] however, show a different trend from the previous one[18] and are in rather good agreement with the present prediction.

In summary, we have shown that heavy-ion fusion cross sections at high energies may be interpreted in terms of a statistical yrast line in the compound nucleus. This interpretation accounts rather well for the systematic trends in the magnitudes of cross sections for fusion in the lighter heavy-ion systems.

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Measurement of $p + d \rightarrow {}^3\mathrm{He} + \gamma$ and Comparison with the Inverse Reaction

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Seven differential cross sections of the reaction $p + d \rightarrow {}^3\mathrm{He} + \gamma$ have been measured at $T_p = 450$ and 550 MeV between 52° and 92° (θ_γ c.m.). ^{3}He's were analyzed by the SPES I spectrometer in coincidence with photons detected by Cherenkov counters. The results are about twice the cross sections of the inverse reaction measured recently by Hegerath *et al.* and by Argan *et al.* The data are consistent, however, with the $\gamma + {}^3\mathrm{He} \rightleftarrows p + d$ data of Heusch *et al.*

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A serious controversy concerning the differential cross section of the reaction $p + d \rightarrow {}^3\mathrm{He} + \gamma$ and its inverse has emerged recently. In 1976 Heusch and co-workers[1] reported results on $d\sigma(\gamma + {}^3\mathrm{He} \rightarrow p + d)$. These data are in good agreement with the only existing measurement at intermediate energy of the inverse reaction, namely nine data points obtained by Heusch *et al.*[2] The agreement was presented as a sensitive test of time-reversal invariance (TRI), notwithstanding the fact that two other measurements of $\gamma + {}^3\mathrm{He} \rightarrow p + d$ did not agree with the results: The cross section measured at $\theta_p = 60°$ and 90° by Argan *et al.*,[3] are half those of Ref. 1, while the data at $\theta_p = 90°$ obtained by Picozza *et al.*[4] are some 40% higher. Recently, Hegerath *et al.*[5] have made three series of measurements of $\gamma + {}^3\mathrm{He} \rightarrow p + d$ and results are internally very consistent. Their data are in excellent agreement with the of results of Argan *et al.* covering twelve comparative data points, thereby challenging the validity of the test of detailed balance in the system $\gamma + {}^3\mathrm{He} \rightleftarrows p + d$ reported in Ref. 1. Figure 1 illustrates the existing controversy in $\gamma + {}^3\mathrm{He} \rightleftarrows p + d$ at $\theta_p = 60°$ and 90°.

A general, model-independent way for evaluating the sensitivity of various tests of detailed balance to a possible violation of TRI is not known at present. There is a paucity of precise tests[6] of TRI at intermediate energies[7] which involve the electromagnetic interaction of hadrons. Only three systems have been investigated. The reactions $\pi^- + p \rightleftarrows \gamma + n$ are very sensitive to TRI violations of the isovector and hypothetical isotensor currents, but the tests[8] are hampered by the absence of free neutron targets. The reac-

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