Coherent Isobar Production in Peripheral Relativistic Heavy-Ion Collisions

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Theoretical estimates of total cross sections for the coherent production of isobars in a relativistic heavy-ion projectile with concomitant excitation of the target to a giant resonance at two appreciably different energies/nucleon are presented and comparisons are made to estimated experimental total cross sections for pion production.

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In the last few years, experiments on pion production at forward angles in relativistic heavy-ion collisions have been undertaken. For example, Papp *et al.*[1] have detected charged pions at 2.5° in the collision of 2.1-GeV/nucleon α-projectile nuclei at a carbon target and Benenson *et al.*[2] have measured charged pions at 0° with heavy ions at 400 MeV/nucleon. A possible mechanism for the production of bosons in the forward direction might be the formation and decay of isobars in the projectile along with fragmentation resulting from peripheral collisions of relativistic heavy ions.[3] The main intent of this note is to estimate the total cross section for the formation of isobars that are produced coherently in the projectile ion and to discover the degree of importance associated with the projectile periphery. If the estimates are comparable to the data, then a more detailed examination would be warranted. It was pointed out[4] that the one-pion–exchange interaction is ideal for producing the isobar $\Delta(1236)$ in a projectile ion, say ^{16}O, which subsequently decays emitting a pion while exciting a target ion, say ^{12}C, to its $T=1$, $J^{\pi}=1^{+}$ giant resonance isobaric-analog states near 15 MeV. In that work, the amplitude for exciting only a single projectile nucleon to an isobar was calculated with no consideration of coherent isobar production. Also, only the target periphery was approximated whereas the projectile-ion was assumed to be infinite in extent. In this work both the target and projectile are localized and approximations are made that essentially assume that collisions take place in the peripheral regions of both the target and projectile-ions.

In their theoretical calculations, Feshbach and Zabek[5] have used a Weisäcker-Williams approach[6] where the summed nuclear field of the target nucleons in the relativistic heavy ion is folded with the target density and then transformed into a virtual quasiphonon field transferring an impulse of energy and momentum to the projectile heavy ion as seen in the projectile reference frame. We extend the spin-independent formalism of Ref. 5 to include spin effects, coherent target excitations, and the peripheral effects of the projectile ion. This work uses absorption of "bosons" to excite isobars in a coherent manner whereas in Ref. 5, a "boson" is absorbed on a nucleon pair. We therefore use a similar trigger mechanism, but excite a completely different final state. A more exact approach would be to calculate the one-pion–exchange interactions and higher-order processes between target and projectile nucleons, but first it is desired to obtain a rough estimate to see if the cross section is within experimental sensitivity before embarking on a more ambitious program. The following approach is proposed not only because of recent interest in pion production, but also because important coherent effects may enter into peripheral processes.

The frequency spectrum for the production of the $\Delta(3,3)$ isobar in the projectile ion and excitation to the giant resonance in the target ion is given by time-dependent perturbation theory as

$$F(\omega)=(1/2\pi)\int dt \exp(i\omega t)\langle \Psi_\beta \Theta_\mu \varphi_m u | V | u \Psi_\alpha \Theta_\lambda \varphi_l\rangle, \tag{1}$$

where the motion of the impact parameter between centers of mass, $\vec{b}_{\mathrm{c.m.}}$, is given by the initial and final states $\varphi_l(\vec{b}_{\mathrm{c.m.}})$ and $\varphi_m(\vec{b}_{\mathrm{c.m.}})$. The target states are described by space-spin states $\Theta_\lambda(\vec{\xi}_t, \vec{\sigma}_t)$ and $\Theta_\mu(\vec{\xi}_t, \vec{\sigma}_t)$ and the space-spin projectile states are given by $\Psi_\alpha(\vec{\xi}_p, \vec{\sigma}_p)$ and $\Psi_\beta(\vec{\xi}_p, \vec{S}_p, \vec{\sigma}_{p'})$ where $\vec{\xi}_t$ and $\vec{\xi}_p$ describe the totality of positions in the target and projectile ions and the spin coordinates are denoted by $\vec{\sigma}_t$ and $\vec{\sigma}_p$. The pth nucleon in the final state has transition spin[7] $\vec{S}_p$ with all other spins given by $\vec{\sigma}_{p'}$, where $p'\neq p$. The speed of the target in the longitudinal direction is v and the energy transferred by the phononlike spectrum to a projectile nucleon is $\hbar\omega$. It is assumed that enough energy ($\sim 2m_\pi c^2$) is transferred to produce the isobar, but is small compared to the incident energy so that harmonic perturbation theory can be applied.

The total potential V is the sum of all transition potentials[8,9] for $NN \to N\Delta$ between target and projectile nucleons. The transition potential is of the form of a one-pion-exchange potential containing a spin-dependent and tensor term. In principle, the tensor term should be calculated because even though the spin term sometimes dominates cross sections,[10] there are cases where the tensor term becomes comparable or even dominates cross sections by an order of magnitude.[11] However, the tensor term couples orbital and spin angular momenta which negates our simplifying assumption of simple products of uncoupled space and spin single-particle states and further requires additional terms due to the $2l+1$ multipole components needed to describe the $l=2$ tensor effect. Therefore, for the sake of obtaining a rough order-of-magnitude estimate quickly, only the spin-dependent term was calculated. The total potential is taken as

$$V = \sum_{p,t}^{A_p, A_T} \vec{S}_p \cdot \vec{\sigma}_t v_{p,t}(\xi, b), \qquad (2)$$

where the separation of the pth and tth nucleon is measured by the longitudinal and transverse distances ξ and b. The geometry for the collision as measured in the projectile frame is shown in Fig. 1. With use of the same parametrization as in Ref. 5 which allows for analytic integrations, the projectile-target nucleon-nucleon spatial interaction is taken to be a separable Gaussian form,

$$v_{pt}(\xi, b) = -(2v_0)\exp(-\xi^2/2\beta^2)\exp(-b^2/2\beta^2). \qquad (3)$$

Since the $\pi N\Delta$ coupling constant in the transition potential is a factor of 2 greater than the $N\pi N$ coupling constant in the one-pion-exchange potential,[8] the strength v_0 of the nucleon-nucleon potential is enhanced to $2v_0$ for the nucleon-nucleon transition potential. The values for the parameters are $v_0 = 105.4$ MeV at 2.1 GeV, 50.46 MeV at 400 MeV and $\beta = 0.461$ fm.

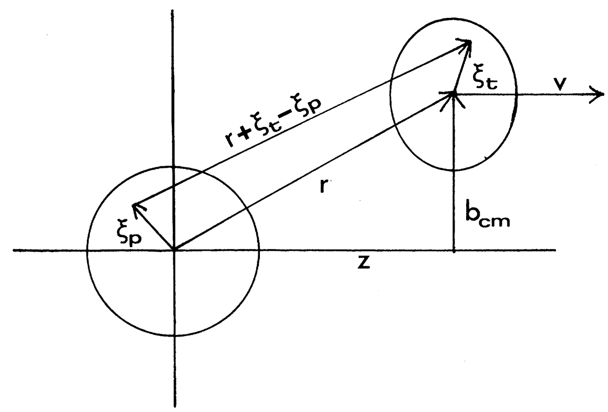

FIG. 1. Geometry for relativistic heavy-ion collisions as measured in the center-of-mass rest frame of the projectile. The relative speed of the centers of mass is v in the z direction. The centers of mass are measured by the center-of-mass impact parameter $b_{\mathrm{c.m.}}$, the internal coordinates for the target and projectile are denoted by ξ_t and ξ_p and $r+\xi_t-\xi_p$ measures the separation between a target and projectile nucleon.

A list of the following assumptions is also made: (a) Localized matter densities are assumed for the target and projectile ions; (b) the projectile and target internal wave functions are taken to be unsymmeterized products of space and spin functions; (c) because of the high energies involved, Fermi motion will be ignored; and (d) the classical assumption that the target center-of-mass motion is undeviated throughout the collision or $u^2(z,t) = \delta(z - vt)$ is made.[5]

With the above assumptions and expanding the spin operators into a spherical basis, the amplitude $F_p(\omega)$ for the transition of the pth nucleon spin state $N_{1/2}{}^{m_p}$ to an isobar spin state $\Delta_{3/2}{}^{M_p}$ and the excitation of the closed-shell ground spin states $|0\rangle$ to the normalized μth component of the giant spin resonance $|\mu\rangle = A_t^{-1/2}\sum_t \sigma_t{}^\mu|0\rangle$ is

$$F_p(\omega) = (\varphi_m\Psi_\beta, \mathcal{V}_p{}^\mu(\omega)\varphi_i\Psi_\alpha)\langle\Delta_{3/2}{}^{M_p}|S_p{}^{-\mu}|N_{1/2}{}^{m_p}\rangle, \qquad (4)$$

where the frequency spectrum $\mathcal{V}_p{}^\mu(\omega)$ presented to the pth projectile by the target is

$$\mathcal{V}_p{}^\mu(\omega) = [(-1)^\mu/2\pi v](\delta_{\mu,\pm1}A_T{}^{-1/2})\int(\gamma d^3\xi_T)\rho(\vec{b}_t, \gamma z_T)\int dz\, \exp(i\omega z/v)v_{pt}(|\vec{r} + \vec{\xi}_T - \xi_p|). \qquad (5)$$

In Eq. (5), the target density $\rho(\vec{b}_T, \gamma z_T)$ is measured in the target frame in terms of target coordinates and the γ factor is due to Lorentz contraction of the target density as measured in the projectile frame. The normalization factor $A_T{}^{-1/2}$ comes from the giant spin resonance matrix element for the tth target nucleon and is

$$\langle \mu | \sigma_t^{\mu'} | 0 \rangle = (2/\sqrt{A_T})\delta_{\mu',\mu}\delta_{\mu,-2\mu_t}. \quad (6)$$

Since the target density is proportional to the target baryon number A_T and the normalization goes as $1/\sqrt{A_T}$, the cross section will then be proportional to the coherence factor A_T. However, because of the assumption of periphality, the full impact of this term is not realized because the target integrations from R to ∞ include interactions only in the periphery of the target density. Similar to Ref. 5, the target peripheral approximation allows for a separation of target coordinates or

$$\rho_T \approx \rho_0 \exp(-z_T^2/2a\lambda)\exp[-(b_T - R)/a], \quad (7)$$

where for ^{12}C, the central density $\rho_0 = 0.168$ nucleons/fm^3, the radius $R = 2.30$ fm and the diffuseness[12] $a = 0.421$ fm and for ^{20}Ne, $R = 2.72$ fm and $a = 0.228$ fm.[13] The mean free path of a nucleon in nuclear matter $\lambda = 1.75$ fm effectively limiting the longitudinal penetration of the projectile nucleon in the target. The second factor in Eq. (7) is a rough approximation of this absorption effect. The projectile peripheral approximation consists of letting $\rho_p = fA_p/V_p$ where the ratio f is the mass between the region of the half-density radius out to ∞ to the total mass and replacing the Woods-Saxon mass distribution by an approximately equivalent but analytic uniform distribution plus Gaussian edge.

Evaluating the transition spin matrix elements for the pth projectile nucleon which are Clebsch-Gordan coefficients, summing over all projectile amplitudes and performing the usual sums and averages over all spin states, the total cross section σ is calculated. The spin-dependent cross section $\sigma_\mu(\omega)$ contains the coherent sum of transitions for A_p nucleons each having a spin projection m_p transforming to an isobar of spin projection M_p concomitant with the excitation of the closed target to a giant spin resonance of projection μ. If we assume isobar formation and decay, this cross section is given as

$$\sigma_\mu(\omega) = \frac{(fA_p)^2 V_p}{(2\pi\hbar)^3}\int d(Mc^2)\iint d^3p_\Delta d^2k \left|\frac{F_\Delta^{\mu}(\omega)}{\hbar}\right|^2 \left[\frac{\Gamma/2\pi}{(M_\Delta c^2 - Mc^2)^2 + (\Gamma/2)^2}\right], \quad (8)$$

where an isobar of rest mass Mc^2 and momentum $\vec{p}_\Delta$ is formed after the absorption of the quasiphonon of momentum $\hbar\vec{q}$ which is made up of transverse and longitudinal momenta as $\hbar\vec{q} = \hbar\vec{k} + \hbar(\omega/v)\hat{z}$ and the factor $|F_\Delta^{\mu}(\omega)|^2$ contains a momentum-conserving δ function, $\delta(\hbar\vec{q} - \vec{p}_\Delta)$. For the sake of obtaining an approximate estimate to the total cross section it is further assumed that the resonance width is infinitesimal or the term in brackets is replaced by the δ function, $\delta(M_\Delta c^2 - Mc^2)$, which assumes no decay of the isobar. The total cross section is then an estimate of coherent isobar formation in the projectile and is given by

$$\sigma = 1.24[(fA_p)^2/A_T][\rho_0^2/(\hbar v)^2](2\pi R\lambda)(\pi a^2)4|t(0)|^2 \exp(\beta/a)^2 \operatorname{erfc}(\beta/a)\exp\{-[(a\lambda/\gamma^2) + \beta^2](\omega/v)^2\}, \quad (9)$$

where the transfer of a quasiphonon of threshold energy $\hbar\omega = (M_\Delta - M_N)c^2$ is assumed. The numerical factor contains the coherent sum of isobar spin amplitudes and sums over target final spin states. The factor A_P^2 is the projectile-ion coherence factor which greatly enhances the cross section over that of single isobar excitation; however, because of the peripheral fraction $f = 0.25$, the cross section is brought down considerably. The coherent production then involves only the "surface" nucleons and the projectile peripheral approximation is important in limiting the magnitude of the total cross section. The geometric factor $2\pi R$ goes as $A_T^{1/3}$ which is characteristic of a peripheral interaction, and because of the target peripheral approximation, there remains a residual "penalty" factor of $1/A_T$. The Fourier Transform of the nucleon-nucleon transition strength gives $t(0) = 2(2\pi)^{3/2}\beta^3 v_0$.

An estimate is made for peripherial pion production in the forward direction from the data of Papp *et al.*[1] From their data on negative-pion production at 2.5° (laboratory) by 2.1 GeV/nucleon α beams on a ^{12}C target, by assuming most of the pions produced in the peripheral reaction are contained in this forward cone and assuming a factor of 4 for the production of negative pions by an ^{16}O beam over that of an α beam and an additional factor of 3 for production π^0's and π^+'s, the total experimental cross section is estimated to lie roughly between 4 and 6 mb. This calculation produces a total cross section of approximately 4 mb. An estimate of the total cross section is also made using the 0° charged-pion production at 400 MeV/nucleon from the data of Benenson *et al.*[2] if we assume a flat distribution obtained by averaging the $\pi^+\pi^-$ distributions and integrating out to 150 or 200 MeV pion laboratory energy and including a factor of 3 for all pion charge states. The same forward-cone assumption is also made. This data is not as appropriate since the target-mass dependence is approximately $A_T^{2/3}$ and the "Ne" target state may be primarily dominated by an orbital recoupling than by a spin-flip mechanism[14]; however, the data should be an upper-limit test of the theory at a much lower energy where it contains central reactions as well as peripherals. The estimate from the data for the total cross section is from 0.6 to 0.8 mb as compared to a theoretical calculation of 0.1 mb. Since the theory is comparable of these experimental estimates, we are encouraged to improve the theory by including the tensor term, pion absorption effects,[15] and Coulomb corrections, and generalizing to an i-spin formalism.

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