tailed calculations are clearly necessary to understand their structure.

These experiments placed severe restrictions on the quality of the electron beam and we are grateful for the help of Dr. M. McAshan, Dr. T. Smith, and Dr. J. Turneaure. The assistance of J. Arruda-Neto and D. Hoffmann during the latter stages of data taking is acknowledged. One of us (A. M. S.) would like to thank Dr. J. Barrette for valuable discussions. This work was supported by the National Science Foundation under Contract No. PHY-79-05286, and by the U. S. Department of Energy under Contract No. DE-AC02-76CH00016.

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Determination of the Transition Charge Density of the Octupole Vibration in ^{208}Pb

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The transition charge density for the octupole vibration of ^{208}Pb has been determined with an unprecedented accuracy for an inelastic transition. A comparison with some of the best theoretical calculations shows a persistent discrepancy in the interior of the nucleus which offers a measure of the limitations of the various theoretical approaches considered.

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We present in this Letter the experimental determination of the transition charge density of the octupole vibration (2.615 MeV) of ^{208}Pb along with recent theoretical calculations. The accuracy in the determination of the density achieved is unprecedented for an inelastic level and it can only

be compared with the recent precision measurements of ground-state charge distributions. Moreover, the agreement with theoretical calculations done in a completely self-consistent fashion is remarkably good and indicates a significant qualitative step in our understanding of the nucleus. Nevertheless, the precision of the experimental result shows that improvements in the theoretical calculations are still needed.

^{208}Pb, the heaviest doubly magic nucleus, is an important testing ground for many nuclear-structure calculations. The recent precise determination of the ground-state charge distribution of ^{208}Pb and its disagreement with theory[1] shed new light on our understanding of nuclear structure and provided a new impetus for further theoretical investigations. The determination of the transition charges associated with various transitions in this nucleus provides an equally important test for nuclear dynamics. The collective octupole vibration of ^{208}Pb at 2.615 MeV which has a strength of ~32 Weisskopf units is such an example. Its significance has been realized quite early and it has been investigated repeatedly both theoretically and experimentally.[2-6]

Previous experiments showed deviations from the simple surface-peaked nature in the density. Such a structure in the interior of the nucleus was also predicted by theories but showed large discrepancies with experiment. The significance of these discrepancies was limited by the statistical accuracy as well as by the insufficient range of the data in momentum transfer. The advent of new electron accelerators where high beam intensities and high energy resolution became routinely available made it possible to extend these measurements. The new data include cross sections as low as 1.4×10^{-10} mb/sr.

In our analysis we have used all the previous available (e, e') measurements,[3-6] the $B(E3)$ value as measured by Coulomb excitation,[7] and our own (e, e') measurements taken at the Centre d'Etudes Nucléaires de Saclay and at the Massachusetts Institute of Technology (M.I.T.). The data taken at Saclay were obtained at the HE1 end station.[8] The momentum range of $1.74<q<3.4$ fm^{-1} was obtained by varying the detector angle between $\theta=38°$ and $\theta=79°$ with an incident energy of 502.0 ± 0.3 MeV. The scattered electrons were detected with use of the standard focal-plane equipment. The data taken at M.I.T. were obtained by use of the Bates high-resolution energy-loss system.[9] The momentum range of $0.55<q<2.6$ fm^{-1} was explored by varying the incident energy between 52 and 335 MeV at $\theta=90°$ and 70 to 225 MeV at $\theta=160°$. In both experiments 99.14% isotopically enriched metallic foils were used with special cooling systems to allow maximum utilization of high incident-beam currents.

The cross sections were corrected for radiative effects. Absolute normalization was obtained by referencing the elastic cross sections to those calculated from the previously determined ^{208}Pb ground-state charge distribution.[1] Systematic errors including normalization are estimated not to be larger than 5%.

Data from 250 electron-scattering measurements have been used in this analysis. As can be seen in Fig. 1, the data from various laboratories are generally compatible with each other. There is some disagreement beyond statistics in the important high-momentum-transfer region between the data taken at Saclay and those by two of us at Stanford University,[4] but we believe this comes primarily because of the difficulty of the Stanford system[4] to resolve the 3^- from the 5^- state at this region. Most of the data are given in the literature as ratios of $\sigma(3^-)/\sigma$(elastic) with only statistical errors quoted. We added to the errors of those data 5% of the value of the cross section in quadrature to account for systematic errors.

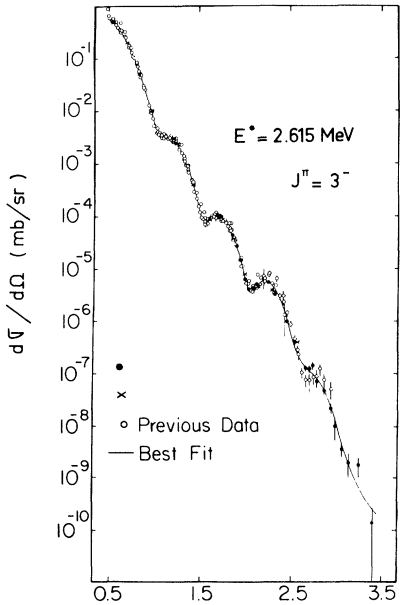

FIG. 1. Experimental cross sections recalculated to an incident energy of 502 MeV. The solid line represents the best fit obtained with the Fourier-Bessel analysis. Solid circles, new data taken at Saclay; crosses, new data taken at M.I.T.; open circles, previous data (Refs. 3–5).

In Fig. 1 we also present the best fit obtained in distorted-wave Born approximation by a Fourier-Bessel analysis.[10] In analyzing our data we assumed that the current responsible for the transition is of irrotational character. It should thus be possible to describe the transition by a nuclear current derived from a transition charge density via the continuity equation. This assumption was tested and justified by our 160° data to the level of accuracy of the experiment. The fit shown was produced by the phase-shift code HEINEL which allowed the transition charge to vary in order to minimize χ^2. The best fit was obtained with a cutoff radius of 11 fm and varying fifteen coefficients in the Fourier-Bessel expansion. The last three coefficients were not sensitive to the data and were determined from constraints on the form factor beyond q_{max}, the maximum momentum transfer measured. A χ^2 of 1.7 per point was obtained with a transition charge radius of 7.334 ± 0.011 fm. The transition probability was found to be $6.12 \times 10^5\ e^2 \cdot \mathrm{fm}^6 \pm 2.2\%$, compatible with the average of the Coulomb excitation measurements.[7]

The error band of the transition charge corresponding to the best fit is shown in Fig. 2. The error envelope includes both the statistical and an estimate of the completeness error. The latter arises from the lack of knowledge of the form factor beyond $q_{max} = 3.4\ \mathrm{fm}^{-1}$. It was estimated following Dreher *et al.*,[11] and making an exponential extrapolation for the uncertainty in the cross section beyond q_{max}. The improvements in the determination of the transition charge density are due to two factors. The large number of data at low momentum transfer reduce the statistical contribution to the error. The cross section at q_{max} is sufficiently small to reduce significantly the completeness error. The small incompatibility of the two sets[4,9] of high-q data did not influence the extracted transition density and the discrepancy lies within the stated error.

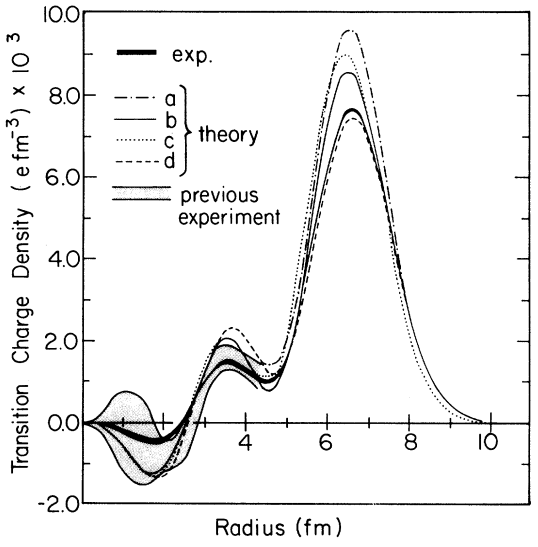

FIG. 2. Comparison of experimental transition charge density of the 3^- level at 2.615 MeV in ^{208}Pb with various theoretical predictions: curve *a*, from Ref. 15; curve *b* from Ref. 13; curve *c*, from Ref. 12; and curve *d*, with use of the Migdal interaction. The shaded area in the interior represents the experimental uncertainty from the previous analysis (Ref. 16). Note that below $r = 4.2$ fm, curves *a* and *d* are indistinguishable.

A large number of theoretical calculations of the 3^- transition charge density exist nowadays. We exhibit in Fig. 2 four of them.

The prediction of Gogny and co-workers[12] is a result of a fully self-consistent calculation of collective excitations in the framework of the random-phase approximation (RPA). In this approach a Hartree-Fock-Bogoliubov calculation is performed for ^{208}Pb with use of a density-dependent effective interaction of finite range. An RPA calculation is performed using the particle-hole (p-h) Hartree-Fock (HF) basis with the same effective interaction which was also used to extract the mean field. It thus represents one of the most fundamental self-consistent calculations available today to describe ^{208}Pb. The overall agreement with the experimental result is good, and the structure is reproduced well in the phasing. Still there is a disagreement which lies outside the experimental error. The $B(E3)$ value is rather high ($6.37 \times 10^5\ e^2 \cdot \mathrm{fm}^6$) and too much structure is predicted like in the case of the ground-state charge distribution.[1]

The calculations by Bertsch and Tsai[13] are also fully self-consistent. Both, the HF calculation for the ground-state and the RPA calculation use the same zero-range Skyrme interaction.

The calculations with use of the zero-range Migdal interaction are not fully self-consistent since the generation of the p-h basis was done from a HF single-particle potential with use of a different interaction. The strength of the 3^- state was used to adjust the parameters of the interaction. Thus the agreement in the surface peak is somehow constrained. The structure in the interior is qualitatively very good in the phasing. The better agreement in the interior compared to earlier calculations by Ring and Speth[2] is due to the bigger p-h basis of fourteen $\hbar\omega$ utilized in this

calculation.

Another way to describe collective excitations is the approach of the "particle vibration coupling."[14] This scheme describes successfully various properties of collective excitations in the lead region. We include in Fig. 2 a calculation by Hamamoto.[15] Once again we observe that the phasing of the structure in the transition charge is very good but that too much structure is predicted in the interior. This might be explained by the coupling between surface mode and compression mode that is expected to be important at small radii. It is not contained in the present model. In the region of the surface where larger deviations are seen, the surface tension might be an important effect, which is also not included in the present formulation of the problem.

In all theoretical predictions shown the negative lobe in the center of the nucleus is predicted too large by a factor of 2 or more. This lobe is dominated by the proton $2f_{7/2}3s_{1/2}^{-1}$ and the $(3p, 2d^{-1})$ p-h components. The ground-state charge distribution suggests a strong depletion of the proton $3s$ and $3d$ orbits. This depletion should reduce the amplitude of these p-h configurations and thereby the amplitude of the negative lobe in this transition density.

However small the disagreements are, they offer us a measure of the various shortcomings of the theoretical calculations. Since the general approach seems to be sound, one expects that the discrepancies arise because of the use of various approximations. The error introduced by basis truncation is expected to be rather small since very large bases have been used. The calculation of Gogny and co-workers should be less sensitive to basis truncation because their force is of finite range. The discrepancy between theory and experiment cannot be accounted easily by a single theoretical shortcoming. It might be the effect of various second-order processes not adequately taken into account. Some of them are, for example, meson-exchange currents, relativistic effects (kinetic and potential dependent), dispersion and recoil effects, and inclusion of multiparticle-multihole configurations. The inclusion of these effects does not go beyond the simplest version of a theory that accounts for fluctuations of the one-body density matrix around the HF equilibrium. As suggested by Gogny, a more refined theory would introduce, for instance, "dressed" particles (quasiparticles in the Landau sense) instead of the "bare" of the HF basis. In such a complete picture the coupling between particles (or holes) and the vibration themselves would arise naturally.

Our new result shows the need for further refinements in present calculations.

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