PHYSICAL REVIEW LETTERS

VOLUME 45 | 17 NOVEMBER 1980 | NUMBER 20

Quark-Moment Contributions to Baryon Magnetic Moments

Jerrold Franklin

Department of Physics, Temple University, Philadelphia, Pennsylvania 19122

(Received 27 May 1980)

Recent baryon magnetic moment measurements are used to isolate individual quark magnetic moment contributions, including nonstatic effects. The pattern of symmetry breaking observed is that quarks in spin-triplet states preserve SU(3), while quarks in spin-singlet states break SU(3). Relativistic effects are suggested as the dominant symmetry breaking mechanism. The present Σ^- moment determination is incompatible with this analysis.

PACS numbers: 13.40.Fn, 12.40.Cc, 14.20.-c, 14.80.Dq

The recent accurate measurement of the magnetic moment of the Ξ^- baryon,[1] along with other hyperon moment measurements,[2-6] permits, for the first time, a reasonably accurate determination of the contribution of each quark to baryon magnetic moments and a good indication of the extent and mechanism of SU(3) and SU(6) breaking in these quark contributions. This results in a clarification of the pattern of symmetry breaking in the quark model, evidenced by a correlation between the symmetry breaking in baryon masses with that now observed in magnetic moments.

In this note, I use sum rules on baryon moments to isolate SU(3) and SU(6) breaking, nonstatic (exchange, orbital, and relativistic) contributions. This enables one to use the now accurately measured baryon moments to pinpoint the magnitude and pattern of the symmetry breaking. A particularly effective approach is to isolate the contribution of individual quarks (including the nonstatic effects) to each baryon moment. Apart from the Σ^- moment (whose present experimental value[4] is incompatible with this analysis) I find as a general rule that quarks in spin-triplet states are approximately SU(3) symmetric while quarks in spin-singlet states break SU(3). This rule, of course, also violates SU(6). This pattern of symmetry breaking is just what has been observed in baryon-mass calculations in the quark model.

In Table I, I list all the measured hyperon magnetic moments. For purposes of comparison I have also listed static-quark-model predictions[7-9] for these moments using the measured proton, neutron, and Λ^0 moments as input. Table I indicates qualitative agreement with the static quark

TABLE I. Baryon magnetic moments (in nuclear magnetons) in the static quark model.

Baryon	Ref.	Experiment	Theory (Ref. 9)
Λ^0	2	-0.6138 ± 0.0047	-0.61
Σ^+	3	2.33 ± 0.13	2.67
Σ^-	4	-1.41 ± 0.27	-1.09
Ξ^0	1, 5	-1.236 ± 0.014	-1.44
Ξ^-	1	-0.75 ± 0.07	-0.50
(Λ, Σ)	6	$-1.82^{+0.18}_{-0.25}$	-1.63

model, but the experiments are now accurate enough to rule out detailed quantitative agreement.

One point to emphasize about the remarkable qualitative agreement of Table I is that this provides *no* evidence for SU(3) or SU(6) symmetry in the static quark model. In the static quark model, the theoretical predictions of Table I follow only from the assumption that baryons are composed of three spin-$\frac{1}{2}$ quarks obeying effective parastatistics[10] for identical quarks, along with the neglect of relativistic, orbital, and exchange effects.[9] In the absence of these nonstatic effects, no amount of SU(3) or SU(6) breaking could change the theoretical results of Table I. The quantitative differences in Table I thus measure the importance of these nonstatic effects.

In nuclear physics there is a corresponding situation for the three-body ^{3}H and ^{3}He nuclear moments. There little progress has been made and nuclear physicists have had to live with the corresponding discrepancies from static three-body moment predictions. This is in large part due to the difficulties and ambiguities of calculating the nonstatic contributions, especially those due to relativistic and exchange effects.

Although less is known about quark wave functions and dynamics, the situation is in fact much better. This is due, in part, to the fact that there are more (now eight) measured baryon moments to work with. This enables us to form linear combinations of baryon moments that cancel out the nonstatic contributions to the extent that they are SU(3) symmetric. The sum rules can then be used to test the extent of SU(3) symmetry in the quark wave functions.

The resulting sum rules were derived some time ago[11] and have been compared to earlier experiments.[12] The order-of-magnitude agreement of these sum rules has been improved by the new Ξ^- measurement, but, because of improved accuracy, there still is evidence for SU(3) breaking. SU(6) symmetry breaking would not affect these sum rules [see Eqs. (1)–(3) of Ref. 12].

By making the further assumption that the moments of u and d quarks are in the 2:(−1) ratio of their charges, we can use the procedure of Ref. 11 to solve for individual quark moment contributions (including the nonstatic contributions) in terms of measured baryon moments. This is an important step because it enables us to separate the SU(3) and SU(6) breaking aspects of the nonstatic contributions for each quark and to draw conclusions about the symmetry of the quark wave functions. These equations have been derived in Ref. 12 and can be written as

$$\mu_d(n) = -\tfrac{1}{4}(2p+n) = -0.92\,, \tag{1a}$$

$$\mu_d(\Sigma^-) = -\tfrac{1}{4}(\Sigma^+ - \Sigma^-) = -0.93 \pm 0.07\,, \tag{1b}$$

$$\mu_d'(p) = p + 2n = -1.03\,, \tag{1a$'$}$$

$$\mu_d'(\Xi^-) = \Xi^0 - \Xi^- = -0.49 \pm 0.07\,, \tag{1b$'$}$$

$$\mu_s(\Xi) = \tfrac{1}{4}(\Xi^0 + 2\Xi^-) = -0.68 \pm 0.04\,, \tag{2}$$

$$\mu_s'(\Sigma) = -\Sigma^+ - 2\Sigma^- = +0.49 \pm 0.56\,, \tag{2$'$}$$

where the physical baryon moments are represented by the baryon symbol, while the notation $\mu_i(B)$ represents the type-i-quark contribution to the magnetic moment of baryon B. The prime on a quark-moment contribution (and the corresponding equation) indicates that it corresponds to the odd quark in a baryon with two like quarks and one odd quark. The absence of a prime generally indicates that it is one of the two like quarks.

Equations (1) and (2) were derived by forming, in each isotopic multiplet, linear combinations of baryon moments that exactly cancel out (including orbital and relativistic effects) the contribution of either the two like quarks or of the odd quark to the baryon moments. The equations have been normalized so that, in the absence of nonstatic effects, they would provide measures of the actual magnetic moments of nonrelativistic quarks in s states. However, I emphasize that as used here they are "quark-moment contributions" including nonstatic effects and not just "quark moments." Although exchange effects of the canceled quarks have not been removed from Eqs. (1) and (2), they do cancel out when quark contributions are related as in Eqs. (4) and (5).

Besides the above quark contributions, the cancellation of nucleon quark contributions to the Λ^0 moment can be used to give another measure[13] of μ_s:

$$\mu_s(\Lambda) = \Lambda^0 = -0.61\,. \tag{3}$$

The pairs of Eqs. (1a) and (1b) and Eqs (1a′) and (1b′) test SU(3) symmetry of the baryon wave functions.[14] That is, SU(3) symmetry for the nonstatic effects would imply

$$\mu_d(n) = \mu_d(\Sigma^-) \text{ and } \mu_d'(p) = \mu_d'(\Xi^-)\,. \tag{4}$$

The spin independence assumption that would lead to SU(6) symmetry would result in primed quark moment contributions equaling the corresponding unprimed contributions. That is, SU(6) symmetry for the nonstatic effects would imply

$$\mu_d(n) = \mu_d'(p), \quad \mu_d(\Sigma^-) = \mu_d'(\Xi^-),$$

$$\mu_s(\Xi) = \mu_s'(\Sigma). \qquad (5)$$

Before drawing other conclusions, we note that the positive s'-quark–moment contribution of Eq. (2′) is incompatible with almost any conceivable quark model. The Σ^- moment is the most difficult to measure and has been determined chiefly by a very difficult atomic fine-structure experiment.[4] A new measurement is being undertaken.[1,15] Unless the new result decreases the Σ^- magnitude by at least 1 standard deviation, any reasonable quark model of magnetic moments would be destroyed by Eq. (2′).

The most striking SU(3)-symmetry correlations in Eqs. (1)–(3) are those that indicate $\mu_d(n) \simeq \mu_d(\Sigma^-)$ and $\mu_s(\Xi) \simeq \mu_s(\Lambda^0)$. The near equality of $\mu_d(n)$ and $\mu_d(\Sigma^-)$ (which would not be changed much by even a large decrease in the magnitude of Σ^-) corresponds to the fact that, in each case, the two d quarks are in the same (triplet) spin state and whether the third, unlike quark is u or s does not have much effect on the d-quark wave function. This is approximately borne out in model calculations with otherwise large SU(3) breaking.[16]

The near equality of $\mu_s(\Xi)$ and $\mu_s(\Lambda^0)$ can be understood by the fact that the strange quark in the Λ^0, although in a mixed spin state, still is predominantly (3:1) spin triplet, while the s quarks in the Ξ are in pure spin-triplet states. It has been known for some time that the spin-triplet quark forces are approximately SU(3) symmetric.[9,17] This has been related to the success of the Gell-Mann–Okubo formula and the equal-spacing decuplet rule. It also follows for the SU(3) breaking expected from quantum chromodynamic (QCD) considerations[18] or in any theory where the SU(3)-breaking force is a spin-spin interaction. This approximate symmetry is also borne out in model calculations with this type of SU(3) breaking.[16]

The only strong SU(3) breaking is given by the large difference between $\mu_d'(\Xi^-)$ and $\mu_d'(p)$. This corresponds to the large SU(3) breaking observed in static-quark-model singlet forces.[9] Aside from $\mu_s'(\Sigma)$, which still awaits a more accurate Σ^- measurement, strong SU(6) breaking is seen in $\mu_d'(\Xi^-) \neq \mu_d(\Sigma^-)$. The somewhat smaller, but still significant SU(6) breaking in the nucleon wave functions, seen in $\mu_d(n) \neq \mu_d'(p)$ has been noted and discussed in Ref. 12.

As a general observation, we see that, just as with quark-model masses, the spin-singlet quark-moment contributions break SU(3) symmetry strongly while the spin-triplet quark-moment contributions are approximately SU(3) symmetric. I now argue that one can conclude from this and other factors that the symmetry breaking is predominantly due to relativistic quark-moment contributions and only slightly due to orbital and exchange effects. This is because exchange or orbital contributions would break SU(3) symmetry for all quark contributions, while only relativistic contributions could preserve SU(3) for some quarks and break it for others. Also the orbital calculation would only affect baryon moments to second order and would be quite small even for relatively large orbital components in the wave function.[9,16,19] The orbital effect on the transition moment (Λ, Σ) could be larger because of Λ^0-Σ^0 interference and this (along with Λ^0-Σ^0 mixing)[13] could effect Eq. (6) of Ref. 12. Exchange effects would be expected to be small in QCD because neutral gluons would dominate. In any quark model with a three-quark saturation mechanism, exchange currents due to $\pi^{\pm}$ meson exchange would be strongly damped because this would correspond to a five-quark baryon state. Thus, only the relativistic effects[20] would be large enough and have the selective SU(3) and SU(6) breaking characteristics to explain the agreement and disagreement among Eqs. (1)–(5).

If the symmetry breaking of the quark-moment contributions is predominantly due to relativistic effects, then we might expect the quark-moment contribution to decrease for a larger momentum spread of the quark wave function. Thus $|\mu_d'(p)| > |\mu_d(n)|$ implies that the like quarks in the nucleon have a larger momentum spread than the odd quark. This is just what is observed in deep inelastic scattering as an explanation[21] of why the ratio of the neutron to proton structure functions approaches $\frac{1}{4}$ as x approaches 1.[22]

Also, $\mu_d'(\Xi^-)$ being smaller in magnitude than $\mu_d'(p)$ would be consistent in direction with the type of spin-spin SU(3) breaking expected from QCD.[18] However, the very small (0.49) magnitude of $\mu_d'(\Xi^-)$ indicates much more SU(3) breaking than QCD would imply. Since QCD predicts that the strange-quark spin-spin force is smaller than the nucleon-quark spin-spin force, it would follow from QCD that $\mu_d'(\Xi^-)$ should lie between $\mu_d(n)$ and $\mu_d'(p)$, which is far from the case. Understanding this very large SU(3) breaking in $\mu_d'(\Xi^-)$ and obtaining a more accurate value for

$\mu_s(\Sigma)$ are the two outstanding questions left for baryon–magnetic-moment physics.

My conclusions are the following:

(1) Baryon magnetic moments are best correlated with symmetry breaking in the quark model by solving for the individual quark-moment contributions of Eqs. (1)–(3).

(2) The strange-quark contribution to the Σ moments has the wrong sign with present measurements and a more accurate Σ^- moment determination should change Σ^- by at least 1 standard deviation.

(3) As a general rule the magnetic moment contributions of quarks in spin triplet (or predominantly triplet) states have approximate SU(3) symmetry but quarks in predominantly spin-singlet states break SU(3). This is evidenced by $\mu_d(n) = \mu_d(\Sigma^-)$ and $\mu_s(\Xi) = \mu_s(\Lambda^0)$, but $\mu_d'(\Xi^-) \neq \mu_d'(p)$. Further corroboration (or violation) of this rule depends on an accurate Σ^- moment determination.

(4) The SU(6) breaking for the nucleon moments is characterized by quarks in spin-triplet states having smaller moment contributions than similar quarks in spin-singlet states.

(5) The pattern of SU(3) and SU(6) breaking observed in the quark-moment contributions strongly suggests that relativistic quark magnetic moment contributions are responsible for the symmetry breaking and that exchange currents and orbital effects on baryon moments are small.

(6) Except for the Σ^-, the direction of symmetry breaking in the quark-moment contributions of Eqs. (1)–(3) correlates well with relativistic effects suggested by the SU(6) breaking seen in deep inelastic scattering and SU(3) breaking suggested by QCD. However, the magnitude of SU(3) breaking in $\mu_d'(\Xi^-)$ is much larger than QCD could explain.

---

[1]The Ξ^- result is a preliminary number based on a partial sample of the data, as reported by T. Devlin, Bull. Am. Phys. Soc. 25, 572 (1980).

[2]L. Schachinger *et al.*, Phys. Rev. Lett. 41, 1348 (1978).

[3]R. Settles *et al.*, Phys. Rev. D 20, 2154 (1979).

[4]B. L. Roberts *et al.*, Phys. Rev. D 12, 1232 (1975); G. Dugan *et al.*, Nucl. Phys. A254, 396 (1975); T. Hansl *et al.*, Nucl. Phys. B132, 45 (1978).

[5]G. Bunce *et al.*, Phys. Lett. 86B, 386 (1979).

[6]F. Dydak *et al.*, Nucl. Phys. B118, 1 (1977).

[7]G. Morpurgo, Ann. Phys. (N.Y.) 2, 95 (1965); W. Thirring, in *Proceedings of the Third Schladming Conference on Nuclear Physics*, edited by P. Urban (Springer, Berlin, 1965), Acta. Phys. Austriaca Suppl. II, p. 205.

[8]H. R. Rubinstein, F. Scheck, and R. H. Socolow, Phys. Rev. 154, 1608 (1967).

[9]J. Franklin, Phys. Rev. 172, 1807 (1968).

[10]O. W. Greenberg, Phys. Rev. Lett. 13, 598 (1964).

[11]J. Franklin, Phys. Rev. 182, 1607 (1969).

[12]J. Franklin, Phys. Rev. D 20, 1742 (1979).

[13]Equation (3) is not as general as Eqs. (1) and (2) because only relativistic effects, but not exchange or orbital effects, cancel out of the nucleon quark contributions. Also, Λ^0-Σ^0 mixing [see A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. D 12, 147 (1975)] would modify Eq. (3), changing $\mu_s(\Lambda^0)$ to -0.57.

[14]Don Lichtenberg (private communication) has pointed out that Eq. (4) could be broken even for SU(3)-symmetric wave functions by unequal–quark-mass effects in the orbital contributions. I do not consider this effect important, however, since it is a correction to a correction. Similarly, unequal-mass effects would lead to some SU(3) breaking in the wave functions even for SU(3) symmetric forces. That that effect is small has been demonstrated in a model calculation in J. Franklin, Phys. Rev. D 21, 241 (1980).

[15]M. Eckhause *et al.*, "A Precision Measurement of the Magnetic Moment of the Negative Sigma Hyperon by the Exotic Atoms Technique" (unpublished).

[16]See Franklin, Ref. 14.

[17]Ya. B. Zeldovich and A. D. Sakharov, Yad. Fiz. 4, 283 (1967) [Sov. J. Nucl. Phys. 4, 283 (1967)]; P. Federman, H. R. Rubinstein, and I. Talmi, Phys. Lett. 22, 208 (1966); H. R. Rubenstein, Phys. Lett. 22, 210 (1966).

[18]See De Rújula, Georgi, and Glashow, Ref. 13.

[19]N. Isgur and G. Karl, Phys. Rev. D 21, 3175 (1980).

[20]Relativistic effects on magnetic moments were first investigated by G. Breit, Phys. Rev. 71, 400 (1947); R. G. Sachs, Phys. Rev. 72, 91 (1947); H. Primakoff, Phys. Rev. 72, 91 (1947). Relativistic effects for quark-model magnetic moments have been discussed by N. N. Bogolubov, B. Stiminski, and A. Tavkhelidze, Joint Institute of Nuclear Research, Dubna, Report No. JINR D-1968 (unpublished), and No. JINR-D-2015, 1965 (unpublished), and No. JINR-P-2141, 1965 (unpublished); H. J. Lipkin and A. Tavkhelidze, Phys. Lett. 17, 331 (1965); J. Franklin, Ref. 9; and in the context of the quark bag model by T. DeGrand, R. L. Jaffe, K. Johnson, and J. Kiskis, Phys. Rev. D 12, 2060 (1975); R. H. Hackman, N. G. Deshpande, D. A. Dicus, and V. L. Teplitz, Phys. Rev. D 18, 2537 (1978); N. Isgur and G. Karl, Ref. 19.