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Hybrid Quantum Oscillations in a Surface Space-Charge Layer

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The transconductance of an accumulation layer on n-type InAs is studied in a magnetic field $\vec{H}$ parallel to the layer. Structures are observed, nonperiodic in $1/H$, corresponding to mixed electric and magnetic subbands. The effect promises a simple, sensitive method of probing the shape of the self-consistent potential in surface space-charge layers with multiply occupied subbands.

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We report the observation of quantum oscillations in a surface space-charge layer for the crossed-field geometry. In this geometry the magnetic field $\vec{H}$ is applied parallel to the space-charge layer, in contrast to the usual configuration in which both electric and magnetic fields are normal to the layer. For the usual configuration, electric and magnetic quantization is decoupled, leading to complete quantization of the two-dimensional electron gas.[1] For the crossed-field geometry, electric and magnetic quantization is mixed, giving hybrid subbands whose dispersion depends on the relative strengths of electric and magnetic fields.[2] The number of observable oscillations in our experiment corresponds to the number of electric subbands occupied at $H=0$; thus a space-charge layer with multiply filled subbands is best suited for studying the effect.

The accumulation layer on low-concentration n-type InAs is such a multiply filled system.[3,4] Because of the small (isotropic) effective mass ($m^*=0.024m_e$)[5] the space-charge layer is ~ 100 Å thick in the usual density range ($\sim 10^{12}$ cm^{-2}). In relatively modest magnetic fields ($\lesssim 10$ T), the cyclotron radius achieves a comparable value, so that a perturbation approach[6] is not expected to be applicable.

Effects of a parallel magnetic field on a space-charge layer have been observed in other experiments. In a fixed, high-density accumulation layer on n-type InAs, Tsui[7] used a tunneling technique to observe the decrease of the binding energy of the (single) bound state with increasing magnetic field. In this high-density case (i.e., strong electric field) perturbation theory[6] predicted the observed H^2 dependence. Beinvogl, Kamgar, and Koch[8] observed the (plasma-shifted) intersubband resonant frequency to increase with magnetic field for an accumulation layer on (100) Si. Their result, expected qualitatively from perturbation theory,[6] was analyzed quantitatively by Ando.[2,9] Although the effect of H on the occupied ground-state subband can be treated as a small perturbation, the excited (unoccupied) subbands are severely distorted by the magnetic field. In PbTe this strong-field condition is easily realized for all subbands, and recently[10] inversion-layer cyclotron resonance was shown to behave three dimensionally in a parallel magnetic field. The present experiment is capable of describing features of the hybrid subbands not directly accessible by these other methods.

Our sample is a (100) epitaxial layer of InAs ($n\approx 2\times10^{15}$ cm^{-3}) overlaid with a SiO_2 insulating layer and a gate electrode. Further details maybe found elsewhere.[3,4] We monitor the low-temperature (4.2 K) transconductance $d\sigma/dV_G$ of the induced accumulation layer as a function of V_G and H. (σ is the channel conductivity and V_G is the "gate" voltage applied between the semiconductor and the gate electrode.) $\vec{H}$ is applied at $\sim 45°$ to the current direction.

The observed oscillations are illustrated in the first figure. The transconductance is shown in Fig. 1(a) as a function of H for selected values of gate voltage and, alternatively, in Fig. 1(b) as a function of V_G for fixed magnetic fields. In both cases, the observed oscillatory features are seen to vary systematically with H and V_G. If we mark the transconductance minima, we arrive at the plot of Fig. 2. Contrary to the familiar Shubnikov–de Haas (SdH) oscillations, the minima are not periodic in $1/H$, nor are they periodic in V_G (i.e., subband density) as in the conventional geometry with $\vec{H}$ perpendicular to the layer.

We identify the oscillations with the mechanism sketched in Fig. 3. In the absence of a magnetic field, the dispersion relation for each bound state i with energy E_i (depending on V_G) for motion perpendicular to the layer is[6]

$$E = E_i + \hbar^2 k_x^2/2m^* + \hbar^2 k_y^2/2m^*, \tag{1}$$

where the parabolic terms (we ignore nonparabolic effects) reflect free-electron-like motion in the layer plane. A magnetic field in the y direction, however, breaks the symmetry between positive and negative k_x values because of the dependence of the energy on the position of the "orbit center" (most easily envisioned for flatband) relative to the sample surface.[11] Furthermore, the subband minima are shifted with respect to $k_x = 0$ and raised in energy such that the energy difference between minima of successive subbands increases with H.[2,8] As a result, fewer subbands may accommodate a given total number N_s of induced electrons per unit area with a magnetic field than without. (N_s is fixed by the device capacitance and the difference between gate voltage and flatband voltage.) Thus, depending on gate voltage and the magnetic field strength, a given hybrid subband may pass through the Fermi level. At the corresponding value of V_G and H, the transconductance will change since, at the Fermi level, both the density of states and the average scattering rate change rapidly. We expect the average scattering rate to change

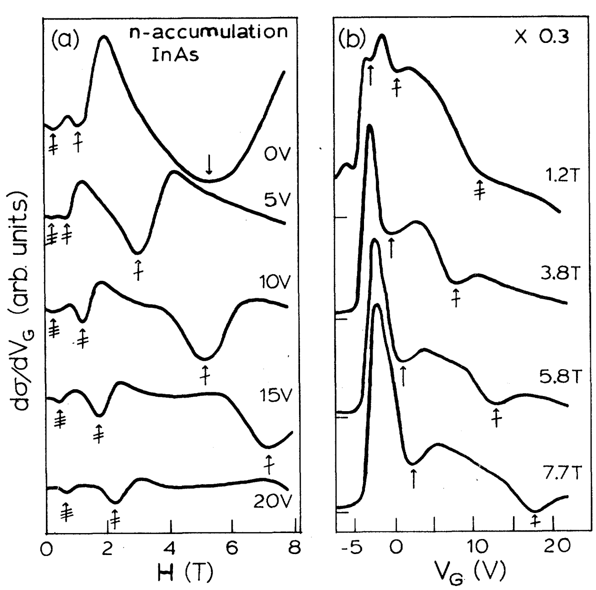

FIG. 1. Parallel–magnetic-field transconductance (a) as a function of field at fixed gate voltage and (b) as a function of gate voltage at fixed field. The number of slashes on each arrow corresponds to subband index i.

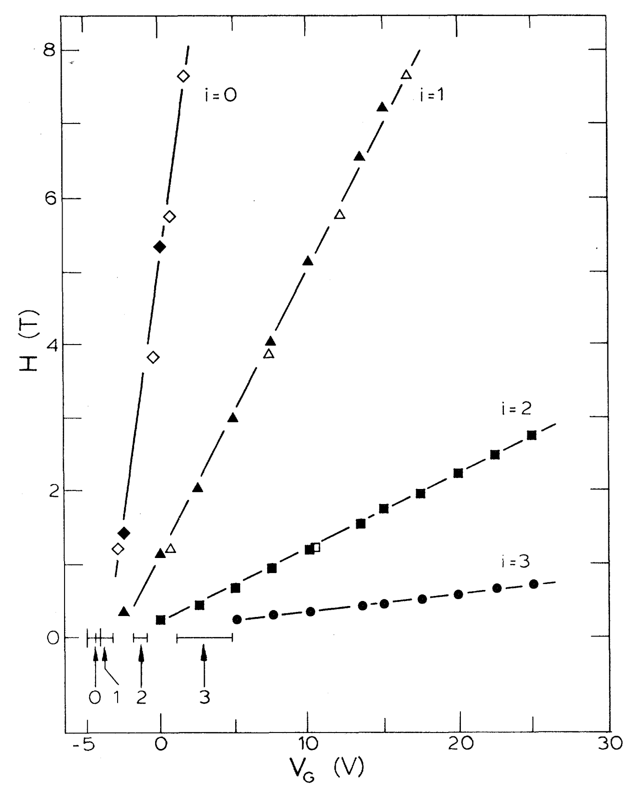

FIG. 2. Dependence of the transconductance minima on magnetic field and gate voltage. Filled symbols are taken from H sweeps, unfilled from V_G sweeps. At $H = 0$ the threshold voltages $V_T(0, i)$ and their uncertainties are indicated for each subband i. These have been obtained from analysis of normal-field SdH oscillations in our sample.

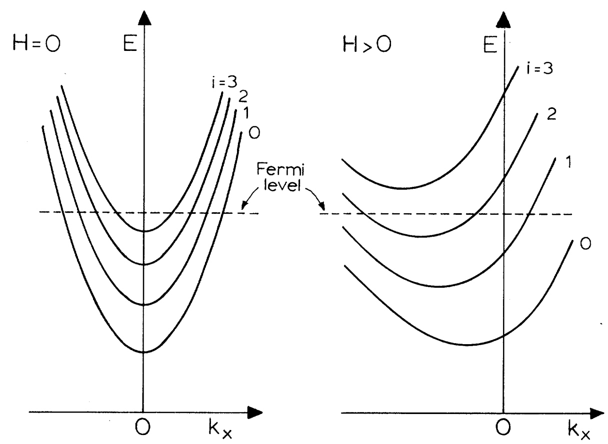

FIG. 3. Schematic subband dispersions ($k_y = 0$) with and without a magnetic field at constant gate voltage. The observed transconductance minima are identified with the passing of a hybrid subband through the Fermi level. The subband spacing (~ 100 meV) is increased by the parallel magnetic field.

since the scattering rate varies between subbands[3] (and even within a given hybrid subband, depending on k_x) due to the dependence of surface scattering on wave-function extent.

Which feature of our $d\sigma/dV_G$ structure is to be associated with the coincidence of Fermi level and the bottom of a hybrid subband is best left to detailed theoretical treatment. This question corresponds to the determination of the phase in the usual SdH effect. In the hybrid case, since dispersion and thus density of states depend on both H and V_G, the "phase" could in principle vary. We have chosen the minima because they are easily discernible for both H and V_G sweeps and because one naively expects $d\sigma/dV_G$ to rise when a new subband begins to be populated.

Clearly, the number of observable minima must be the same as the number of subbands occupied at $H=0$. As indicated in Fig. 2, this is the case. Furthermore, for $H\neq 0$, each $d\sigma/dV_G$ minimum is observed only at gate voltages $V_T(H, i)$ above the $H=0$ threshold voltage $V_T(0, i)$ for each subband i. These latter threshold voltages were obtained from an analysis of normal-field SdH oscillations in our sample.[12] The third subband ($i=3$) had been suspected in an earlier SdH study[3]; it has since been clearly identified in cyclotron resonance.[13]

The dispersion curves for the hybrid subbands must, of course, be calculated self-consistently since the electrostatic potential readjusts itself in the presence of the magnetic field to satisfy both the Poisson and the Schrödinger equations.[2] We hope our results will stimulate such a calculation (also for $H=0$) for InAs.

In lieu of self-consistency we can estimate the shift $\Delta V_T(H, i) = V_T(H, i) - V_T(0, i)$ in the voltage at which the subband i is populated. In a strong magnetic field, the intersubband spacing is increased by an amount on the order of $\hbar\omega_c$, where ω_c is the cyclotron frequency. This shift allows $\Delta N_s \approx \omega_c m^*/\pi\hbar$ to be transferred out of a given subband. Here we have used the usual expression for the two-dimensional density of states, neglecting spin splitting (not resolvable here) and the change of the density of states in a parallel magnetic field.[14] The expected threshold shift for each subband is then $\Delta V_T(H, i) \approx \Delta N_s/(dN_{s,i}/dV_G)$, where $N_{s,i}$ is the carrier density in subband i. We take the values for the population rates, $dN_{s,i}/dV_G$, from the SdH results.[3,12] These are for the zeroth to third subband, respectively, (each accurate to 10% or better): 1.11×10^{11}, 0.32×10^{11}, 0.10×10^{11}, and 0.024×10^{11} $\mathrm{cm}^{-2}/\mathrm{V}$. The sum of these values gives the total induced charge determined from the measured device capacitance: $(1.58\pm0.08)\times10^{11}$ $\mathrm{cm}^{-2}/\mathrm{V}$. At $H=1$ T, the corresponding estimates for $\Delta V_T(H, i)$ are 0.4, 1.6, 4.8, and 24 V. The respective experimental values from Fig. 2 are (extrapolating for $i=3$) 1.5, 4, 10, and 35 V. It thus appears that much of the variation of the threshold shifts among the different subbands is due to their different population rates. The estimated magnetic-field–induced change in subband spacing is substantiated by a "triangular-well" approximation[8] applied to InAs. Our calculations also suggest that the change in the density of states due to the magnetic field, although less important than the increase in subband spacing, is an additional contribution to the threshold shifts. As discussed by Ando,[2,9] the exact values of the energy shifts are extremely sensitive to the extent of the subband wave functions into the semiconductor and thus to the variation of the self-consistent potential.

In a nondegenerate semiconductor, one expects the mobility turn-on to occur at the threshold voltage for the ground-state subband. In our degenerate sample, however, this appears not to be the case. Within our present interpretation, Fig. 1 shows the threshold voltage for the lowest subband to increase with H although the mobility turn-on remains near −5 V. We suggest the residual $d\sigma/dV_G$ peak to be associated with increased continuum electrons at the surface above the flat-band voltage.

One might expect $d\sigma/dV_G$ to show structure also for $H=0$ at the subband threshold voltages $V_T(0,i)$. Except at the turn-on voltage, -5 V, we observe a smooth variation of $d\sigma/dV_G$. In InSb, such structure has been observed[15,16] and (the minima again) correlated[16] with population onset of the various subbands. Observed shifts of the InSb structures in a parallel magnetic field are ascribed to the same mechanism described here.[17] That the $H=0$ curves show no corresponding structure in InAs may be due to the relatively higher scattering rate in our sample. When the parallel magnetic field is turned on, the majority of electrons are pushed away from the surface (manifested in the dispersion asymmetry in k_x), reducing the effect of surface scattering. This mechanism has been suggested[18] as a cause of negative magnetoresistance in a surface space-charge layer.

Thus in addition to its promise as a sensitive probe of the self-consistent potential, the present effect may be capable of yielding detailed scattering information. We expect this experiment also to be relevant to the evaluation of SdH oscillations in tilted magnetic fields,[12] i.e., for a mixed normal and parallel geometry. An interesting extension of this work would be in PbTe where, because of the high dielectric constant, the cyclotron radius becomes comparable to the space-charge-layer thickness already at ~1 T. Finally we note for the hybrid effect that one may expect the temperature dependence of the oscillation amplitude to relate to the intersubband spacing as it does analogously to the Landau-level separation (i.e., cyclotron mass) for the usual SdH effect. We hope to report on these aspects at a later date.

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